# Comparative analysis of senior high school prospective mathematics teachers and students' conceptual misunderstanding in algebra 

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#### Abstract

The aim of this study is to compare the performance and conceptual misunderstanding of senior high school prospective mathematics teachers and students in algebra with an intent to statistically find out whether there is a significance difference in students and prospective teachers' ability to identify and rectify algebraic conceptual misunderstanding. A test consisting of 21 tasks was used for data collection. A sample of 170 consisting of 60 prospective senior high school mathematics teachers from mathematics education department of University of Education Winneba and 110 senior high school students from four selected senior high schools in Ashanti Region of Ghana. The study employed convenience, purposive, and simple random sampling as sampling techniques. The result showed that the overall performance of prospective teachers and students' performance on the algebra was generally poor. That is, their ability to identify and rectify algebraic conceptual misunderstanding was poor. However, the prospective teachers' overall performance was comparatively better than the students. The independent $t$-test conducted between the prospective teachers and students on the test scores gave the results (mean $[M]=20.67$, standard deviation $[S D]=11.98$ ) for students and ( $M=33.91, S D=14.78$ ) for prospective teachers with $t(168)=-6.32, p=0.00$ indicating that statistically there is difference between the mean scores of the two groups signifying that the prospective teacher's ability to identify and rectify algebraic conceptual misunderstanding outperformed the students.


Keywords: comparative analysis, conceptual misunderstanding, prospective mathematics teachers

## INTRODUCTION

## Background to the Study

The difficulty in mathematics by students has been identified to have many causes. These includes the perception people have about mathematics (it is a difficult subject, which is for only brilliant students, to these people, to do mathematics, it is a "calling from God"), it is also perceived to have a limited prospect in the job market. Allen (2007) outline four perceptions the society have about mathematic in general and its role in our world. These were, as follows:

1. Mathematics is incorrectly viewed as a collection of rigid rules and mysterious procedures that seem to be unrelated to each other and require total mastery with little or no understanding.
2. Mathematics is perceived by many to be difficult and demanding and is considered to be a subject in which it is socially acceptable to do poorly.
3. Mathematical thinking is regarded as essentially unimportant to people that do not actually "do" mathematics in their employment capacity.
4. The pervasive role of mathematics is underestimated in the world of everyday living.

Other perceptions were on the school you attend, the teacher who teaches the subject mathematics, the pedagogy involved in teaching and learning. These perceptions were called misconception about mathematics by Allen (2007). Conceptual misunderstanding on some concepts in mathematics, contributes to the difficulty in mathematics by students (Aidoo et al., 2016; Allen, 2007; Bintu, 2018; Chamundeswari, 2014; Don, 2011).

The central goal of Ghana education is to develop the ability to think critically and reasoning (CRDD, 2012). This goal is more explicit in the subject mathematics and science. Mathematics as a subject has a tremendous authority but it continues to pose a major threat and problem for many students. Studies have shown how Ghana's performance in mathematics is poor at both

Table 1. Ghana's mathematics performance in TIMSS 2007 \& TIMSS 2011

|  |  | Overall score |  |
| :--- | :---: | :---: | :---: |
| Country |  | Year |  |
|  | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 1 1}$ |  |
| Republic of Korea | 597 | 613 |  |
| Singapore | 593 | 611 |  |
| Malaysia | 474 | 440 |  |
| Tunisia | 420 | 425 |  |
| Botswana | 364 | 397 |  |
| Ghana | 309 | 331 |  |
| Qatar | 307 | 410 |  |
| TIMSS | 500 | 500 |  |

Table 2. Countries achievement in content domain, TIMSS 2007 \& TIMSS 2011

| Country | Numbers |  | Algebra |  | Geometry |  | Data \& chance |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 1 1}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 1 1}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 1 1}$ | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 1 1}$ |
| Republic of Korea | 583 | 618 | 596 | 617 | 587 | 612 | 580 | 616 |
| Singapore | 597 | 611 | 579 | 614 | 578 | 609 | 574 | 607 |
| Malaysia | 490 | 451 | 454 | 430 | 477 | 432 | 469 | 429 |
| Tunisia | 425 | 431 | 423 | 419 | 437 | 426 | 411 | 398 |
| Botswana | 366 | 392 | 394 | 407 | 325 | 381 | 384 | 391 |
| Ghana | 310 | 321 | 358 | 358 | 275 | 315 | 321 | 296 |
| TIMSS | 500 | 500 | 500 | 500 | 500 | 500 | 500 | 500 |

Table 3. Proportion of students obtaining WASSCE grades A1 to C6 (qualifying grade for tertiary education in Ghana) from 2007 to 2015

| Subject | $\mathbf{2 0 0 7}$ | $\mathbf{2 0 0 8}$ | $\mathbf{2 0 0 9}$ | ${ }^{\mathbf{2 0 1 1}}$ | $\mathbf{2 0 1 2}$ | $\mathbf{2 0 1 3}$ | $\mathbf{2 0 1 4}$ | $\mathbf{2 0 1 5}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Social studies | 75.1 | 60.1 | 77.5 | 82.2 | 87.1 | 57.4 | 57.3 | 51.6 |
| English language | 27.9 | 49.4 | 43.9 | 75.9 | 68.5 | 65.7 | 45.2 | 50.1 |
| Integrated science | 23.8 | 26.7 | 34.5 | 42.0 | 56.8 | 49.7 | 28.7 |  |
| Mathematics | 25.3 | 26.2 | 28.6 | 43.8 | 49.4 | 36.6 | 32.3 |  |

Note. Source: Fletcher, 2016 \& "No WASSCE was written in Ghana in 2010 because of four-year SHS
international and local level. TIMSS is an international assessment of mathematics and science at the fourth (primary 6) and eight (JHS2) grades. It has been conducted in every four years since 1995. TIMSS used item response theory methods to summarize the achievement on a scale with a mean of 500 , a standard deviation of 100 , and a maximum of 800 . Three of the participatory countries, Republic of Korea, Singapore, and Malaysia were at the same level of development as Ghana when Ghana attained its independence in 1957. Country's performance was compared to the international benchmark. The international benchmarks are score levels, which can be used to compare country performance at different attainment levels. The benchmark scores are 625 for advanced international benchmark, 550 for high international benchmark, 475 for intermediate international benchmark, and 400 for low international benchmark.

Ghanaian students' poor performance in mathematics from the results of trends in international mathematics and science study (TIMSS) in 2003 ranked Ghana among the lowest in Africa and the world. Ghana was ranked $44^{\text {th }}$ out 45 participatory countries in $8^{\text {th }}$ grade (JHS2) mathematics (Anamuah-Mensah et al., 2004). Table 1 shows Ghana's performance in mathematics by grade eight (JHS2) students from the results of TIMSS in 2007 and 2011.

From Table 1, the score of 309 in 2007 and 331 in 2011 by Ghana were among the lowest and statistically were significantly lower than TIMSS scale score average of 500 and were still lower than the low international benchmark score of 400 . The overall performance placed Ghana second from the bottom on overall mathematics performance, doing slightly better than Qatar in 2007. Ghana's performance though improved in TIMSS 2011, still remains among the lowest in Africa and the world since they placed last position in the ranking (Anamuah-Mensah et al., 2009; Mullis et al., 2012).

Table 2 shows TIMSS report on Ghana's performance in the four-content domain (numbers, algebra, geometry, and data \& chance) in the mathematics by grade eight (JHS2) students. The four-content domain (numbers, algebra, geometry, and data \& chance), algebra score of 358 in 2007 and 2011 (Table 2) was relatively better than the country's average, yet less than TIMSS average (Anamuah-Mensah et al., 2009; Mullis et al., 2012). Another report that reveals poor performance in mathematics by Ghana was in the report on the global scale of quality education by Organization for Economic Cooperation and Development (OECD). In the said report that bothered on mathematics and science education, Ghana was placed in the $76^{\text {th }}$ position out of 76 countries involved (OECD, 2015).

In Ghana, the qualified grade for a student to further their education is A1 to C6. A lecture titled "Performance in mathematics and science: Breaking the jinx" by Fletcher (2016) on 14 March 2016 shows how student performance in mathematics and science on WASSCE from 2007 to 2015 is low as compared to the other two core subjects (Table 3).

Table 3 shows proportion of students obtaining WASSCE grades A1 to C6 (the qualifying grade for tertiary education in Ghana) from 2007 to 2015. Fletcher (2016) shows that only about 30\% to 40\% of students who sat for WASSCE from 2007 to 2015 were able to obtain the qualified grades into tertiary institution (Table 3).

Table 4. Percentage of students obtaining WASSCE grades A1 to C6 (qualifying grade for tertiary education in Ghana) \& disqualifying grade from 2008 to 2018

| Subject | $\mathbf{2 0 0 8}$ | $\mathbf{2 0 0 9}$ | $\mathbf{2 0 1 1}$ | $\mathbf{2 0 1 2}$ | $\mathbf{2 0 1 3}$ | $\mathbf{2 0 1 4}$ | $\mathbf{2 0 1 5}$ | $\mathbf{2 0 1 6}$ | $\mathbf{2 0 1 7}$ | $\mathbf{2 0 1 8}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1-C6 | $\mathbf{2 6 . 2}$ | $\mathbf{2 8 . 6}$ | $\mathbf{4 3 . 8}$ | $\mathbf{4 9 . 4}$ | $\mathbf{3 6 . 6}$ | $\mathbf{3 2 . 4}$ | $\mathbf{2 4}$ | $\mathbf{3 5}$ | $\mathbf{4 3}$ | $\mathbf{3 8}$ |
| D7-F9 | 73.8 | 71.4 | 56.2 | 50.6 | 63.4 | 67.6 | 76 | 65 | 57 | 62 |

Note. Source: WAEC

The percentage of students obtaining WASSCE grades A1 to C6 and D7 to F9 in the WASSCE examination from 2008 to 2018 is as shown in Table 4.

A cursory look at the different batches of SHS students who sat for the WASSCE examination organized by WAEC as displayed above reveals that less than $50 \%$ of students who took part in WASSCE acquired the minimum requirement for admission into tertiary institutions in Ghana.

## METHODOLOGY

## Research Design

This study used the employed quantitative research approach. Simple random sampling were employed as sampling techniques. The sample involves 110 second-year senior high schools' students in four selected senior high schools in Kumasi Metropolis and Ejisu Municipal in Ashanti Region of Ghana and 60 final year (level 400) in the Department of Mathematics Education at University of Education, Winneba.

The diagnostic algebraic test was the main test instrument for the study. A two-hour diagnostic algebraic test consisting of 21 items were administered to 170 participants. The test was marked and scored according to prepared marking scheme. Errors made by the participants were also identified. These errors were scrutinized and analyzed to determine the ones due to conceptual misunderstanding. The obtained data set was put into the statistical package for social science (SPSS). The marked and scored test were coded and captured for analysis using SPSS. The data were analyzed and presented using descriptive statistics (i.e., frequency distribution, percentages, mean, median, percentiles, and mode). The researcher employed largely descriptive data analysis to interpret and describe the participants' performance on the algebra test. Their overall performance on the test and each test item. Independent t-test was also used to analyze, describe and compare the performance of the participants. The purpose was to determine whether there were statistically significant differences between student's and prospective teachers' mean score, hence performance on the test.

## RESULTS/FINDINGS

## Overall Performance of Prospective Senior High School Teachers and Students in Algebra

It common to hypothesize that prospective teachers should perform better than students so far as performance in mathematics is concern relative to conceptual misunderstanding in algebra. However, there is the need to prove or disprove this thinking empirically. In other to look at the performance, various test needs to be carried out on these participants (students and prospective teachers). The participants involved in this aspect were senior high school form 2 students who had learnt algebra in junior and senior high school as prescribed by the teaching syllabus for junior and senior high school (Ministry of Education [MoE], 2012), the final year (level 400) mathematics education students taking their final assessments to go and teach mathematics at the senior high school. These prospective mathematics teachers have learnt algebra at secondary level and even higher algebra as prescribed by their courses' content at the university. The tasks were scored out of one up to ten by using the marking scheme base on the skill. The sections that follow present the results on the test carried out to find out their performance on the test.

## The Participants' Difficulties With Algebra on the Test Items

The analysis revealed that there were difficulties with the algebra test items by both prospective teachers and the students and this resulted in general low performance. The low performance as a result of the difficulties encountered by the participants were as a result of errors they made in their work. These errors, when scrutinized indicate an inadequate conceptual understanding in algebra. These errors had influence on the participants' scores on the test items. The analysis revealed that there were zero scores on each of the 21 test items by both the prospective teachers and students. Table 5 shows the distribution of participants' (prospective teachers and students) who scored zero, part and full scores on some of the test items.

From Table 5 some serious revelation can be seen on the difficulties of the participant when it comes to senior high school algebra. It was observed that there were zero scores for both students and prospective teachers on each test item 1 to 3 , which were categorized to be one of the easiest items of the test for the participants, especially for the prospective teachers.

It can be seen from Table 5 that $76 \%$ of the students and $43 \%$ of the prospective teachers scored zero on the simple question as 3 . Surprisingly, $99 \%$ of the students and $98 \%$ of the prospective teachers scored zero on test item 8 . The second question they performed poorly from Table 5 was on question 16, which demanded the participants to write an algebraic expression from a given word problem, $90 \%$ of the students and $72 \%$ of the prospective teachers scored zero. On question 19, another abysmal performance was observed by the participants, $80 \%$ of the students and $62 \%$ of the prospective teachers scored zero.

Table 5. Distribution of students' \& prospective teachers' zero \& full scores on the test

| Q/N | Item | Participants | ZS: n (\%) | PS: n (\%) | FS: n (\%) | MS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Simplify $\frac{2 x}{2 x}$ | Students | 22 (20) | 0 (0) | 88 (80) | 1.60 |
|  |  | P-teachers | 3 (5) | 0 (0) | 57 (95) | 1.90 |
| 2 | Simplify $\frac{2+x}{2+x}$ | Students | 51 (46) | 0 (0) | 59 (54) | 1.10 |
|  |  | P-teachers | 16 (27) | 0 (0) | 44 (73) | 1.50 |
| 3 | Simplify $x\left(\frac{a}{b}\right)$ | Students | 83 (76) | 0 (0) | 27 (25) | 0.50 |
|  |  | P-teachers | 26 (43) | 0 (0) | 34 (57) | 1.10 |
| 4 | Akosua sells oranges. Ama sells three times as many oranges as Akosua. An orange cost GHC0.25. If oranges are of same size, how many variables can be formed from this problem? Give name/s. | Students | 75 (68) | 7 (6) | 28 (26) | 0.70 |
|  |  | P-teachers | 23 (38) | 8 (13) | 29 (48) | 1.10 |
| 5 | Solve for y if $a=\sqrt[3]{\frac{y}{5}}$ | Students | 75 (68) | 1 (1) | 34 (31) | 0.10 |
|  |  | P-teachers | 20 (33) | 0 (0) | 40 (67) | 2.00 |
| 6 | A father's age is four times that of his son. In five years, father will be three times as old as his son. What are their present ages? | Students | 40 (36) | 58 (53) | 12 (11) | 3.00 |
|  |  | P-teachers | 11 (18) | 25 (42) | 24 (40) | 5.00 |
| 7 | What does xy mean? Write your answer in words. | Students | 53 (48) | 0 (0) | 57 (52) | 0.50 |
|  |  | P-teachers | 21 (35) | 0 (0) | 39 (65) | 0.70 |
| 8 | Write an equation using variables $B \& R$ to represent following statement: " $A$ Toyota car manufacturing company, for every four blue cars produced, there are five red cars produced". Let $B$ represent number of blue cars \& red represent number of red car. | Students | 109 (99) | 0 (0) | 1 (1) | 0.01 |
|  |  | P-teachers | 59 (98) | 0 (0) | 1 (2) | 0.02 |
| 9 | Find value of $x$ if $\sqrt{\left(x^{2}+9\right)}=5$ | Students | 28 (26) | 73 (66) | 9 (8) | 2.30 |
|  |  | P-teachers | 4 (7) | 26 (43) | 30 (50) | 3.40 |
| 10 | A shirt cost $c$ cedis each \& a pair of shoe cost $d$ cedis if Mr. Appiah buys 5 shirts \& 4 pairs of shoes. Explain what $5 c+4 d$ means or represent. Simplify further if possible. | Students | 78 (71) | 1 (1) | 31 (28) | 0.60 |
|  |  | P-teachers | 41 (68) | 0 (0) | 19 (32) | 0.60 |
| 11 | Mr. Asare shared some money to his two sons \& a daughter, Yaw, Kojo, \& Afua. Afua received 5 times amount than Yaw, \& 4 less than Kojo received. Amount received by Yaw \& Kojo is Gh\& 22.00. How much did Mr. Asare gave to each child? | Students | 95 (86) | 11 (10) | 4 (4) | 0.40 |
|  |  | P-teachers | 32 (53) | 21 (35) | 7 (12) | 1.60 |
| 12 | Expand \& simplify (P-Q) ${ }^{2}$ if possible. | Students | 37 (34) | 23 (21) | 50 (56) | 1.60 |
|  |  | P-teachers | 13 (22) | 8 (13) | 39 (44) | 2.10 |
| 13 | Kofi's age is subtracted from 10 \& result is multiplied by two2. Write expression for statement. | Students | 30 (27) | 48 (44) | 32 (29) | 1.40 |
|  |  | P-teachers | 22 (37) | 16 (26) | 22 (27) | 1.50 |
| 14 | Write mathematical expression for total amount of money in Kwame's pocket, if he has some 50 cedis notes \& some 20 cedis notes. | Students | 89 (81) | 2 (2) | 19 (17) | 0.70 |
|  |  | P-teachers | 24 (40) | 17 (28) | 19 (32) | 1.80 |
| 15 | When a number is subtracted from 6, result is 2 times 5 less than number. Find the number. | Students | 63 (57) | 37 (34) | 10 (9) | 0.90 |
|  |  | P-teachers | 28 (47) | 19 (28) | 15 (25) | 2.20 |
| 16 | There are $n$ boys in a parade, 8 boys are in each row. Write an algebraic expression to find out how many rows of boys are marching in parade. | Students | 99 (90) | 0 (0) | 11 (10) | 0.20 |
|  |  | P-teachers | 43 (72) | 0 (0) | 17 (28) | 0.60 |
| 17 | Length of sides of a triangle are $x, x+1, \& 7 \mathrm{~cm}$. If perimeter is 56 cm , what is value of $x$ ? | Students | 47 (43) | 0 (0) | 63 (57) | 1.70 |
|  |  | P-teachers | 12 (20) | 3 (5) | 45 (75) | 2.30 |
| 18 | 5 times a number is 8 more than number. Find the number. | Students | 43 (39) | 24 (22) | 43 (39) | 2.10 |
|  |  | P-teachers | 11 (18) | 37 (62) | 12 (20) | 2.50 |
| 19 | Mr. Adu bought 8 books \& 12 pens from a shop. A book cost him Ghধ 0.50 more than a pen. If he spent $\mathrm{GH} \not \subset 94$ altogether, how much did a book \& a pen cost? | Students | 88 (80) | 14 (13) | 8 (7) | 0.50 |
|  |  | P-teachers | 37 (62) | 10 (16) | 13 (22) | 1.10 |
| 20a | Letter $n$ represent a natural number if $(i) \frac{1}{n}(i i) \frac{1}{n-1}$, which one is more? | Students | 83 (75) | 0 (0) | 27 (25) | 0.25 |
|  |  | P-teachers | 29 (48) | 0 (0) | 31 (52) | 0.50 |
| 20b | Explain your answer. | Students | 83 (75) | 21 (19) | 6 (6) | 0.30 |
|  |  | P-teachers | 29 (48) | 14 (23) | 17 (28) | 0.80 |
| 21 | In a soccer match between Kotoko \& Hearts of Oak, tickets were sold for Gh८ 10.00, Gh\&25.00, \& Gh\&50.00. Twice as many paid GhZ25.00 as paid Ghধ50. 00.5 times as many paid Gh申10.00 as paid Gh申25.00. Total gate fee collected amounted to Gh\&15,000.00. Find how many paid Gh\&25.00. | Students | 106 (96) | 4 (4) | 0 (0) | 0.10 |
|  |  | P-teachers | 51 (85) | 5 (8) | 4 (7) | 0.80 |

Note. P: Prospective; ZS: Zero score; PS: Part score; FS: Full score; \& MS: Mean score

Task 10, which required the participants to give the meaning of an algebraic expression, $71 \%$ of the students and $68 \%$ of the prospective teachers scored zero. Also, on question 20, which required the participants to do a quantitative comparison of a fraction involving variable, $75 \%$ of the students and $48 \%$ of the prospective teachers scored zero. Table 5 also revealed that few of the participants scored part of the total score. The participants that attempted the questions few of them arrived at the full score, which implies that all attempts made by the students and the prospective teachers on the questions were having some difficulties, which resulted in errors. The errors, in turn, revealed conceptual misunderstanding of procedural knowledge in algebra.

## Overall Performance of the Participants on the Test

Table 6 depict the analysis of students' and prospective teachers' performance on the test, specifically the percentiles.
Table 6 presents a comparison on the proportion of students and prospective teachers reaching the $25^{\text {th }}$ and $75^{\text {th }}$ percentile scores on the diagnostic test. Table 6 shows that the bottom $25 \%$ of the students were able to identify and rectify not more than $16 \%$ of the algebraic conceptual misunderstandings while the prospective teachers were able to identify and rectify not more than $33 \%$ of the algebraic conceptual misunderstandings.

Table 6. Proportion of students \& prospective teachers reaching quartile scores of their ability to identify \& rectify algebraic conceptual misunderstanding?

| Participants | $\mathbf{P}_{\mathbf{2 5}}(\%)$ | $\mathbf{P}_{50}(\%)$ | $\mathbf{P}_{\mathbf{7 5}}(\%)$ |
| :--- | :--- | :--- | :--- |
| Students | $12(16)$ | $19(25)$ | $26(35)$ |
| Prospective teachers | $25(33)$ | $34(45)$ | $45(60)$ |

Table 7. Distribution of students' \& prospective teachers' descriptive statistics

| Participants | $\mathbf{n}$ | Minimum (\%) | Maximum (\%) | Mean (\%) | Standard deviation |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Students | 110 | $0(0)$ | $65(87)$ | $20.67(28)$ | 11.98 |
| Prospective teachers | 60 | $3(4)$ | $72(96)$ | $33.91(45)$ | 14.78 |

Table 8. Independent t-test for combined group

| Participants | $\mathbf{n}$ | Mean | Standard deviation | df | t-value | p-value |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 110 | 20.67 | 11.98 | 168 | -6.32 | .00 |
| Prospective teachers | 60 | 33.91 | 14.78 |  |  |  |

Examining the results of the prospective teachers' and the students on their conceptual misunderstanding on the test and the percentile scores obtained show that the participants' ability to identify and rectify algebraic conceptual misunderstanding was generally low but the prospective teachers comparatively outperformed the students.

## Statistical Analysis on Students and Prospective Teachers' Ability to Identify and Rectify Algebraic Conceptual Misunderstanding

The result on performance of student and prospective teachers (see Table 5 and Table 6) shows that the overall performance of the participants on the test was poor, yet the prospective teachers' performance was comparatively better than the students' performance. However, since is a research further statistical test needs to be performed. The result of the various test performed are presented so that conclusion can be drawn with concrete evidence rather than using only the difficulties, scores and the percentiles to conclude. The various test performed includes the descriptive statistics (minimum, maximum, mean, and standard deviation) and the inferential statistics (independent t -test).

## The Group Descriptive Statistics on the test

Table 7 shows the group descriptive statistics on the test.
From Table 7, it is realized that the minimum score for the students is zero and that of the prospective teachers is three out of the total score of 75 . The maximum score obtained by the students is 65 whiles the prospective teachers obtain 72 . The mean score for the students is $20.67(28 \%)$, whiles the prospective teachers obtain a mean score of $33.91(45 \%)$. Comparison of the minimum, maximum and the mean score in Table 7 shows that the prospective teachers' performance is better than the students.

## Independent t-Test Analysis Between Groups

In order to conclude on the claim whether there is a significant difference in prospective teachers and student's performance (the ability to identify and rectify conceptual misunderstanding), an independent $t$-test was conducted to find out whether there is significant difference in the performance of the groups statistically. This section presents the analysis of the group mean using independent t-test.

The following assumptions underlying the use of a t-test were met:

1. The study was made up of independent and dependent variables. Specifically, the groups (participants) being independent variable and the test scores as the dependent variable
2. Independent variable consists of two groups (prospective teachers and students).
3. The sample in the groups were randomly selected and the samples were independent.
4. The dependent variable was approximately normally distributed. A Shapiro-Wilks'(students being $p=0.07>0.05$ and prospective teachers $\mathrm{p}=0.85>0.05$ ) (Razali \& Wah, 2011) and visual inspection of normal $\mathrm{Q}-\mathrm{Q}$ showed that the test scores were approximately normally distributed for both prospective teachers and students, with a skewness of 0.44 (standard error $[\mathrm{SE}]=0.235$ ) and kurtosis of -0.226 ( $\mathrm{SE}=0.46$ ) for the students and a skewness of 0.05 ( $\mathrm{SE}=0.30$ ) and kurtosis of -.13 ( $\mathrm{SE}=0.60$ ) for the prospective teachers (Doane \& Seward, 2011).
5. The Levene's test was used to check equal variances. The Levene's test result, $\mathrm{p}=0.07>0.50$, was significant indicating that variances are equal.
Table 8 shows whether statistically there is a significant difference in their conceptual misunderstanding.
The results in Table 8 show that there is statistically significant difference between the mean scores of students (mean $[M]=20.67$, standard deviation $[S D]=11.98$ ) and prospective teachers' group ( $M=33.91, S D=14.78$ ); $t(168)=-6.32, p=0.00$. That is, statistically there is difference between the mean score of the two groups signifying that the prospective teacher's overall performance is better than the student's overall performance. In order words the prospective teachers have a better ability to identify and rectify algebraic conceptual misunderstandings.

## DISCUSSION

The analysis shows that the poor performance by the participants was as a result of the difficulties the participants had with the algebraic test items. It was observed that there was no question without a zero score on the part of the prospective teachers and the students. Also, with the participants that attempted the questions a few of them arrived at the full score as well as part score leaving majority scoring zero marks. These difficulties were also as a result of the participants' conceptual misunderstanding in senior high school algebra. This confirms studies that students and prospective teachers have a conceptual misunderstanding in the algebraic concept (Adu, 2016; Booth et al., 2014; Don, 2011; Makonya, 2011; Nadirah \& Yusof, 2013). The finding is also consistent with statements in the West Africa Examination Council's (WAEC) chief examiners' reports that most candidates normally make errors, which depict some form of conceptual misunderstanding in answering algebraic problems in the West Africa senior secondary certificate examination (WAEC, 2008, 2011, 2012, 2013, 2014, 2015).

The finding also shows that, majority of the participants performed poorly on the algebraic test. The result that the bottom $25 \%$ of the students who were able to identify and rectify $16 \%$ of the algebraic conceptual misunderstandings while the prospective teachers were able to identify and rectify $33 \%$ of the algebraic conceptual misunderstandings, shows that the overall performance of the participants was not good. That is the ability of the prospective teachers and the students to identify and overcome conceptual misunderstanding in algebra was poor. The poor performance of the students on algebraic tests confirms a similar study by Adu (2016) and Don (2011) on the poor performance of the student algebraic concept. The performance of the prospective teachers also confirms a study on student teachers' conceptual knowledge of algebra by Tanisli and Kose (2013). Yet with $25 \%$ score for the students and $45 \%$ score for the prospective teachers at the $50^{\text {th }}$ percentile means the number of prospective teachers that were able to identify and overcome the conceptual misunderstanding in algebra was higher than that of the students. This shows that the prospective teacher ability to identify and rectify algebraic conceptual misunderstanding is comparatively better than the student's ability to identify and rectify algebraic conceptual misunderstanding in algebra.

Again, the prospective teachers group mean ( $M=33.92$, $S D=14.78$ ), which is statistically significantly different at $5 \%$ level of significant from the students mean ( $M=20.67, S D=11.99$ ) shows that prospective teachers' performance in algebra is better than that of the students. This means the prospective teachers' ability to identify and rectify conceptual misunderstanding is better than that of the students. This is possible because the prospective teachers have gained more experience and teaching of algebraic concept than the senior high school, hence empirically shown and support the claim made earlier that the performance of prospective teachers is better than the student.

## CONCLUSIONS

Prospective teachers are required to have basic algebraic concepts and in-depth knowledge and understanding to face this task of being good mathematics teachers. Again, to enable the prospective teachers to enter the teaching field with confidence and enthusiasm to perform their duties as algebra and mathematics educators they need in-depth knowledge and understanding of the subject they are going to teach. Teachers are required also to have good algebraic conceptual understanding and knowledge to teach with confidence, enthusiasm and devoid of a conceptual understanding that will be transferred to the students as educators of mathematics (Makhubele, 2014). The findings of the study revealed that the prospective teachers' and students' overall performance on the algebraic test was poor, yet the overall performance of the prospective teachers was better than the performance of the students, that is, at the $75^{\text {th }}$ percentile, the students score was $35 \%$ whiles the prospective teachers score was $60 \%$. This shows the prospective teachers ability to identify and rectify algebraic conceptual misunderstanding outperformed that of the students. However, an independent sample $t$-test result showed that there was statistically significant difference between the mean scores of students ( $\mathrm{M}=20.67, \mathrm{SD}=11.98$ ) and prospective teachers' group ( $\mathrm{M}=33.91, \mathrm{SD}=14.78$ ); $\mathrm{t}(168)=-6.32, \mathrm{p}=0.00$. That is, statistically there is difference between the mean score of the two groups signifying that the prospective teachers' overall performance was better than the student's overall performance. Comparatively, the number of prospective teachers that were able to identify and overcome the conceptual misunderstanding in algebra was more than that of the students.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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    Ethical statement: The authors stated that since the aim of this study was to do comparative analysis of conceptual misunderstanding of senior high school prospective mathematics teachers and students in algebra, a formal letter was written to the Department of Mathematics Education, UEW for introductory letter accompanied with a brief proposal of what the aim and the purpose of the research. A consent form was also developed for the schools where the data was to be collected. Authors further stated that all the appropriate quarters for approval and consent at each level directed by the headquarters were also given, a detailed explanation and the aim of the study was given before the data was collected. The consent form was filled in by all the heads of the senior high schools involved in the study. A letter was written to thank all the quarters who approved of the data collection, and they also requested a briefing of the findings which was noted.

