

Designing a teaching model based on the Realistic Mathematics Education (RME) approach and its application in teaching calculus

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ABSTRACT

The trend of applying mathematical knowledge to solve practical problems has become a top priority goal of many mathematics educations in the world, including Vietnam. The pioneer in the concept of “realistic mathematics education” is Freudenthal (1991). He believes that “math teaching needs to be connected with situations related to everyday life, to general society to be of value to learners”. Freudenthal’s (1991) thought opened the way for the formation of a very developed theory of mathematics education in the Netherlands, called realistic mathematics education (RME), with two basic principles that mathematics must not only be linked with the real but also mathematics needs to be viewed as a human activity. By applying this theory, in this article, we have designed a teaching model based on RME, the feasibility of the model has been verified by us through two experimental teaching periods in the classroom. With positive results, we believe that the model that we have proposed according to RME’s approach can meet the goal of mathematics education in Vietnam in the current period and in the coming 21st century, and it needs to be widely studied and applied in math teaching in Vietnam.

Keywords: realistic mathematics education, RME, realistic problem-solving competency, teaching calculus, application of deviation, mathematical modeling, mathematization

INTRODUCTION

Mathematics is not only a purely theoretical science but also plays an active role in human cognitive activities. With its object characteristics, mathematics has penetrated more and more deeply into different scientific fields, holding a special position in many sciences; and thus, cover a wide range of practical activities. The creative role of mathematical thinking in perception is shown quite clearly in that, mathematics is seen as an indispensable tool for the sciences in discovering and finding out the nature of events, objects, and phenomena of the objective world.

The main goal of learning math in schools is that students have the full range of mathematical skills to pursue higher education and solve problems in everyday life. These mathematical skills include problem-solving, reasoning, communicating, connecting, and representing mathematics, as well as high-level thinking skills, such as critical and creative thinking (Fauzan, 2013). Mathematics education promotes innovative thinking and cultivates the ability to reason correctly in problem-solving. Mathematics education is an active, dynamic, and ongoing process; activities in mathematics education help students develop the ability to reason, think logically, systematically, critically, thoroughly, and have an objective and open attitude when solving problems. Social reality shows, educational activities must be carried out according to the principle that learning goes hand in hand with practice, education must combine with productive labor, theory must strive closely with practice...”, from that educational method must promote the positivity, self-discipline, initiative, and creative thinking of learners; foster learners’ ability to self-study, ability to work in groups; practice skills to apply knowledge to solve practical problems.

When we consider the practical principle of RME theory, we agree with Freudenthal’s (1991) view. According to this principle, Freudenthal (1991) said that mathematics must be related to reality, the problems posed to learners must be suitable to the life of society, the material must be conveyed are human activities. As a result, learners can develop mathematical reasoning skills through teacher guidance to rediscover mathematical formulas or concepts. Building on the foundation is said to design learning based on RME can help learners develop mathematical reasoning skills (Laurens et al., 2018; Veloo et al., 2015).

In the high school math curriculum, the content of derivatives and related problems is one of the main topics in the calculus subject. Derivatives have many applications in problem solving, from elementary to advanced math, and is an important tool to help effectively solve many problems, including those with practical and interdisciplinary content. In the system of exercises with practical content, extreme exercises with practical and interdisciplinary content occupy a particularly important position in training a number of mathematical competencies such as mathematical reasoning ability, computing competency, mathematical

communication capacity, and problem solving ability to achieve the goal of optimizing practical human activities. Solving exercises of this type on the one hand strengthens students' knowledge of applying mathematics to practice, and on the other hand, shows an interdisciplinary perspective, a teaching perspective that is receiving great attention today.

It can be said that the thinking activities of high school students develop strongly. They were able to think critically and abstractly more independently and creatively. Highly developed ability to analyze, synthesize, compare, and abstract helps students to comprehend all complex and abstract concepts. They like to generalize, like to learn and try to explain the general laws and principles of everyday phenomena, of the knowledge to be absorbed. It is clear that high school students are growing up, preparing to directly participate in productive labor, contributing to social development; they have the ability to work independently and take responsibility in choosing a career. Therefore, right from the time they are still in school, equipping learners with the ability to adapt to reality and daily life through the ability to solve practical problems is a task need special attention and timely support.

Thus, it is necessary to innovate from the content to the method of teaching and learning mathematics at the high school level in the direction associated with practice. Approaching and applying advanced math education methods into teaching practice will help them form and develop their math competencies to meet the requirements of high school students' educational goals: state and answer questions when thinking and solving problems; use reasoning, inductive, and deductive methods to understand different ways of solving problems; establish a mathematical model to describe the situation, thereby providing a solution to the mathematical problem posed in the established model; implement and present solutions to the problem and evaluate the solutions implemented, reflecting the advantages and disadvantages as well as the value of each solution; generalize the problem into a similar problem; have the ability to use tools and means of learning mathematics in understanding, exploring and solving math problems.

One problem is how to teach and learn calculus topics in high schools more effectively. In the math program at high school, the topic "application of derivatives" in the calculus program of grade 12 has many topics related to reality, exploiting the practical context. The article systematizes a number of studies on RME theory, thereby applying this theory to the design of a learning model based on RME. Finally, based on that model, we design learning activities through the topic "application of derivatives" with the hope that it can contribute to improving the effectiveness of teaching and learning some elements of calculus at high schools in Vietnam today.

The research results are arranged, as follows. First, some research results on RME theory, including the characteristics of RME, teaching, and learning principles based on RME are presented. Then, we propose a model of lesson design based on RME, and a teaching model based on RME. The model we proposed was conducted in a pedagogical experiment at a high school in Vietnam. After that, the discussion of the results reflected in the experimental teaching is presented. Finally, some conclusions and recommendations are presented.

SUMMARY OF RME

What is Realistic Mathematics Education (RME)?

RME is a teaching and learning theory in mathematical education first introduced and developed by the Freudenthal Institute in the Netherlands. This theory has been applied in many countries around the world such as Great Britain, Germany, Denmark, Spain, Portugal, South Africa, Brazil, USA, Japan, and Malaysia. Currently, RME theory is mainly defined by Freudenthal's (1991) view of mathematics. His two important views were that mathematics must be connected with reality and mathematics as a human activity:

1. Mathematics must be close to children and relevant to every day-to-day life situation. However, the word "reality" refers not only to the connection with the real world, but also to real, problematic situations in the student's mind. This means that, for problems presented to students, the context can be real-world but this is not always necessary, the context can also come from the imaginary world of the student, fairy tales, or the formal world of mathematics, as long as the problems are real in student's mind. de Lange Jzn (1996) states that such situations can also be viewed as applications or modeling.
2. He emphasized mathematics as a human activity. Mathematics education is organized as a guided re-invention (re-creation) process, where students can experience a similar process to the one in which mathematics was invented. Furthermore, the principle of reproducibility can also be inspired by informal processes or solutions. Informal student strategies can often be understood as intended for the formation of more formal processes. In this case, the concepts of mathematization serve as a guide to the reinvention process.

What are the Characteristics of RME?

The combination of van Hiele's (1959) three levels, Freudenthal's (1991) "didactic" phenomenology, and Treffers' (1978) "mathematization" process leads to the following five basic features of RME:

1. Phenomenological exploration or the use of contexts;
2. The use of models or bridging by vertical instruments;
3. The use of students' own productions and constructions or students' contribution;
4. The interactive character of the teaching process or interactivity; and
5. The intertwining of various learning strands.

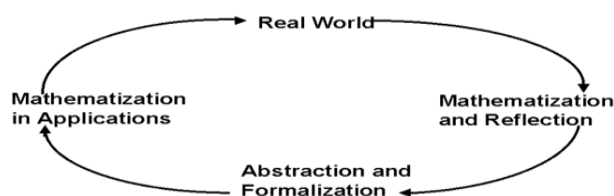


Figure 1. Concept and applied mathematization (de Lange Jzn, 1996)

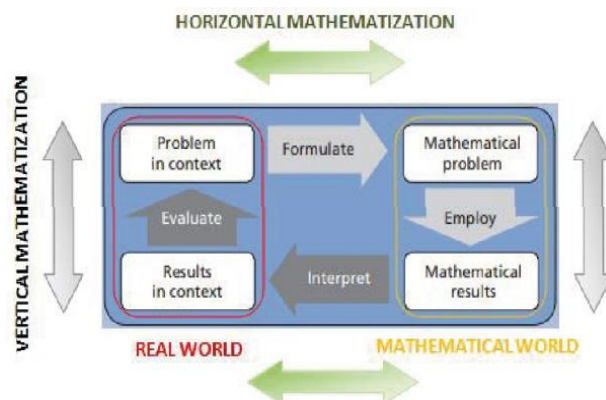


Figure 2. Horizontal & vertical mathematization in the mathematization cycle (Franchini et al., 2017)

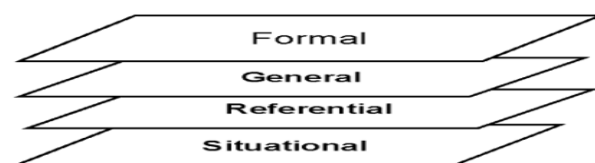


Figure 3. Levels of models in RME (Gravenmejer, 1994)

Phenomenological exploration or the use of contexts

RME is about context, not application. The RME curriculum is built around contexts capable of generating powerful and flexible mathematical models (Turmudi et al., 2014). Contexts can be taken from the real world, from fiction, or from a mathematical field with which the student is already familiar. It is important that students are able to imagine and participate in these situations.

In RME, the starting point of instruction must be “real” for the student; allow them to immediately join the situation. This means that instruction should not begin with a formal conceptual system. The phenomena in which concepts appear in practice should be the source of concept formation. The process of deriving the appropriate concept from a particular situation is stated by de Lange Jzn (1987) as the “mathematization of the concept”. This process will force students to explore the situation, find and identify relevant math content, discover rules, perform math operations, and develop a “model” that leads to a mathematical concept. By reflecting and generalizing students develop a more complete concept. Students can and will then apply mathematical concepts to new areas of the real world, and by doing so, students are reinforcing the concept. This process is called in-app mathematization (see **Figure 1**).

There are two types of mathematization explicitly formulated by Treffers (1987) as horizontal mathematization and vertical mathematization. In horizontal mathematization, students present mathematical tools that can help organize and solve a problem set in a real-world situation. Horizontal mathematization may include: defining or describing specific mathematics in a general context, mathematizing, constructing and visualizing a problem in terms of different ways, discover relationships, detect patterns, rules, convert a real-world problem into a math problem, and convert a real-world problem into a known math problem. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. The following activities are examples of vertical mathematics: representing a relation in a formula, proving the regularity, refining and adjusting the model, using different models, combining and integrate models, build mathematical models and generalize.

Freudenthal (1991) states that “horizontal mathematization” involves moving from the world of life into the world of symbols, while “vertical mathematization” means moving in the “world of symbols”. But he adds that distinction between the two is not always clear-cut. **Figure 2** shows the horizontal and vertical mathematization in the mathematization cycle (Franchini et al., 2017).

The use of models or bridging by vertical instruments

The term model refers to situational models and mathematical models developed by students themselves. This means that students develop patterns in problem solving. At first, the model is a situational model familiar to students. Through a process of generalization and formalization, the model eventually becomes an entity of itself. It can be used as a model for mathematical reasoning. The four levels of modeling in the design of RME-based lessons are depicted in **Figure 3**.

The use of students' own productions and constructions or student's contribution

Student-created work may be an essential part of assessment. For example, students may be asked to write an essay, do a trial, collect data and draw conclusions, design exercises that can be used in a test, or design tests for other students in class.

The interactive character of the teaching process or interactivity

Interaction between students and students and between students and teachers is an essential part of RME (de Lange Jzn, 1996; Gravemeijer, 1994; Gravemeijer & Cobb, 2006). Interaction manifests in discussion, cooperation, and evaluation. These are essential elements in a constructive learning process, in which the informal methods of students are used as leverage to achieve formal ones. Under this principle, students are engaged in explaining, debating, agreeing and disagreeing, and questioning the alternatives of their classmates and instructors or teachers.

The intertwining of various learning strands

In RME (de Lange Jzn, 1996; Gravemeijer, 1994), integration of mathematical circuits or units is essential. It is often referred to as the holistic approach, in that regard learning chains cannot be treated as separate entities; instead, the interweaving of learning chains is exploited in problem solving. One of the reasons for forming this principle is that the application of mathematics is difficult if mathematics is taught only "vertically", i.e. if different subjects are taught separately, ignoring cross relationships.

Teaching Principles of RME

RME is undeniably a product of the times and inseparable from the worldwide reform movement in math education that took place in the previous decades. Therefore, RME has much in common with current approaches to mathematics education in many countries around the world, including Vietnam. However, RME involves some core principles for teaching mathematics that cannot be changed in connection with RME. Most of these core teaching principles were explicitly stated by Treffers (1978), but have been revised over the years, including by Treffers (1978) himself.

The activity principle

In RME, students are seen as active participants in the learning process. It also emphasizes that the best way to learn math is to do it, as evident in Freudenthal's (1991) view of mathematics as a human activity, as well as in Treffers' (1978) idea of mathematization.

The reality principle

In RME, this principle is expressed in two aspects: First, the principle of practice represents the importance attached to the goals of mathematics education including the ability to apply mathematics of students in solving problems from "real-life". Second, it means that math education should start from problem situations that make sense to students, which gives them the opportunity to attach real meaning to the mathematical constructs they are trying to understand while solving problems. Instead of starting with teaching abstract concepts or definitions directly, in RME teachers should start with problems in different contexts. Through mathematization, students are placed on a pathway to implementing solutions involving informal contexts as an initial step in the learning process.

The level principle

This principle emphasizes that, in math learning, students move through various levels of understanding: from solutions involving informal contexts, through creating mathematical operations, to gain insight into how concepts and strategies are related. Models are crucial for bridging the gap between "informal mathematics", in relation to context, and "formal mathematics". To make this connection, models must be transformed from a "model of" (for a particular situation) to a "model for" (for other, but equivalent types of situations).

The intertwining principle

According to this principle, areas of mathematical content such as arithmetic, geometry, measurement, and data processing are not considered chapters of the separate curriculum but integrated with each other. Students are provided with a wide variety of math problems that they can use a variety of mathematical tools and knowledge.

The interactivity principle

This principle indicates that learning mathematics is not only an activity of each individual learner but also a social activity. Therefore, RME encourages whole class discussions or group work, providing opportunities for students to share their strategies and inventions with others. This way, students can get ideas for improving or adjusting their strategies. Furthermore, the interaction helps students achieve higher levels of understanding as well as develop mathematical reasoning and critical thinking abilities.

The guidance principle

In RME, this principle refers to Freudenthal's (1991) idea of "guided reinvention" of mathematics. Specifically, teachers must take an active role in students' learning and educational programs must contain situations capable of acting as a lever to achieve change in students' understanding. For this to happen, teaching and programs must be based on a long and consistent trajectory of teaching and learning.

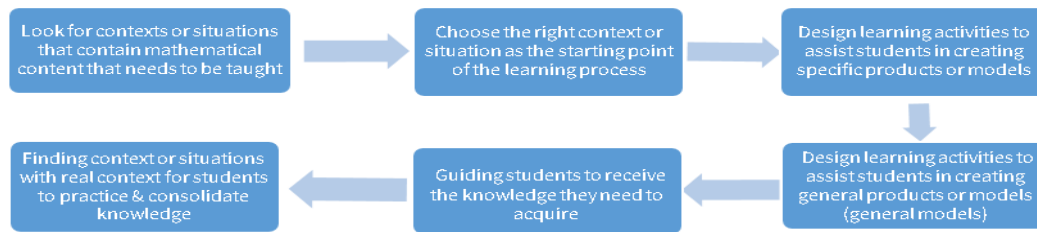


Figure 4. Design RME-based lesson

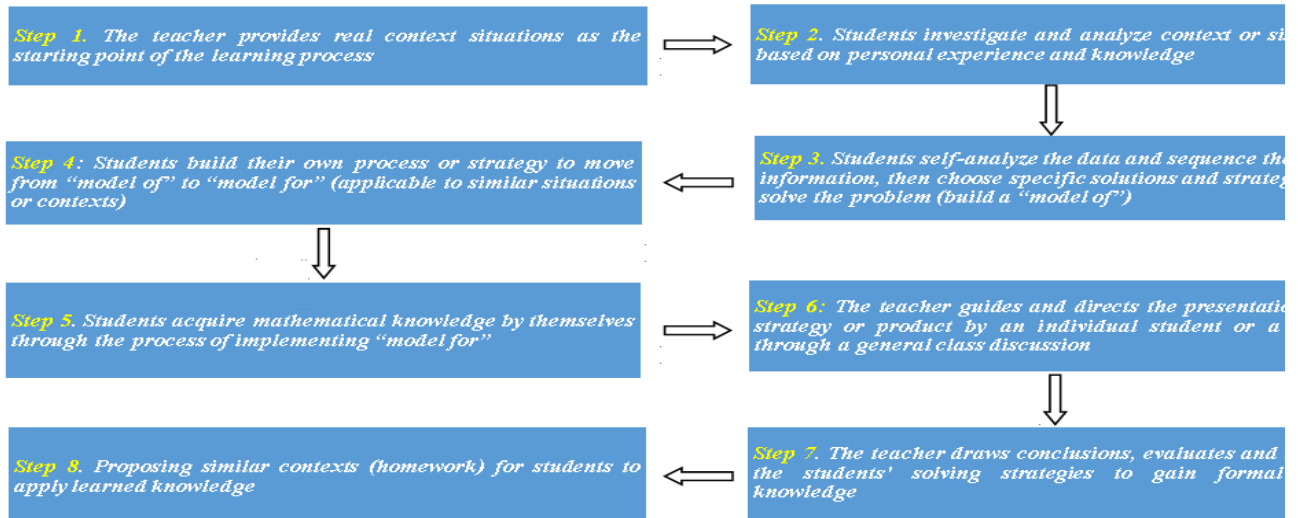


Figure 5. RME-based teaching model

RESEARCH METHODOLOGY

General Background

RME emphasizes that learning math means that students experience a different understanding of contextual solutions at an informal level, through creating different levels, to better understand how related concepts and strategies. Modeling is an important bridge to bridging the gap between informal mathematics and formal mathematics. To make this function, models must change from a “model of” (for a particular situation) to a model that is applicable to all other types of situations but is equivalent (“model for”); in particular, the process includes the following levels (Zulkardi, 2010):

1. The situational level, where domain-specific, situational knowledge, and strategies are used within context of situation;
2. A referential level or the level “model of”, where models and strategies refer to the situation described in the problem;
3. A general level or level “model for”, where a mathematical focus on strategies dominates over reference to context; and
4. The level of formal mathematics, where one works with conventional procedures and notations.

How to Design Realistic Mathematics Education Lesson?

Based on the characteristics of RME, principles of teaching and learning, we propose steps to design lessons base on RME, including six steps, as follows (Figure 4):

1. **Step 1:** Look for contexts or situations that contain mathematical content to be taught;
2. **Step 2:** Choosing the reasonable context or situation as the starting point of the learning process;
3. **Step 3:** Design learning activities to assist students in creating specific products or models;
4. **Step 4:** Design learning activities to assist students in creating general products or models (general models);
5. **Step 5:** The teacher guides the students to receive the knowledge they need to acquire; and
6. **Step 6:** Choose similar contexts or situations for students to practice and consolidate knowledge.

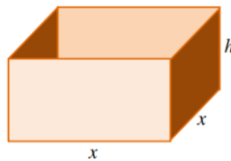
Model of RME-Based Teaching

From the fundamental points of RME, we develop the RME-based teaching, which consists of eight steps, as follows (Figure 5):

1. **Step 1.** The teacher provides real context situations as the starting point of the learning process;
2. **Step 2.** The student investigates and analyzes the context or situation based on personal experience and knowledge;
3. **Step 3.** Students self-analyze the data and sequence the information, then choose specific solutions and strategies to solve the problem (build a “model of”);

Table 1. Role of steps in RME-based teaching model

Reality situations	Mathematization activity		Interaction of the teaching process	Model application
	Horizontal mathematization	Vertical mathematization		
Step 1 & Step 2	Step 3	Step 4 & Step 5	Step 6 & Step 7	Step 8

**Figure 6.** Problem of designing an open box**Table 2.** Questions (for students) used in the teaching process

No	Questions	Objects
Q1	How is the surrounding surface area of a rectangular box calculated?	Suggestions for students to take the first step: -Establish relationships between data;
Q2	How is the relationship between height & size of the bottom shown?	-Give a specific constraint between bottom edge & height of box
Q3	The volume of the cube is determined by what formula?	Set up a model, give a specific formula for the problem of calculating the volume of a rectangular box
Q4	Can we give the formula for the volume of a rectangular box in terms of an algebraic expression of a function of one variable?	Developing functional thinking, mathematizing practical situations

Table 3. Questions (for students) used in the group activity process

No	Questions	Objects
Q5	Is finding the size of the cube for the largest volume of the rectangular box related to any mathematical knowledge that you have been learning or not? If so, please indicate that mathematical knowledge.	-Move from solving practical requirements to solving mathematical problems -Identify strategies and specific solutions
Q6	What is the size of rectangular box so that the volume is the largest?	-Solve mathematical problems -Convert mathematical results to contextual results
Q7	From solving the above situation, can you come up with a general model for the mathematization of practical situations related to optimization factors?	Directing students to build a general model ("model for")

4. **Step 4.** Students develop their own process or strategy to move from a "specific model" to a "model for" (applicable to similar situations or contexts), (this step may require teacher guidance or suggestion);
5. **Step 5.** Students acquire mathematical knowledge by themselves through the process of implementing "model for";
6. **Step 6.** The teacher guides and directs the presentation of a strategy or product by an individual student or a group through a general class discussion;
7. **Step 7.** The teacher draws conclusions, evaluates and adjusts the students' solving strategies to gain formal mathematical knowledge; and
8. **Step 8.** Proposing similar contexts (homework) for students to apply what they have learned.

Table 1 shows the role of the steps in RME-based teaching model.

Research Procedures

Selected mathematical content for experimental teaching: Application of derivatives in calculus program 12.

Classes used for experimental teaching: 12 A8 Nong Cong II High School, Nong Cong, Thanh Hoa, Vietnam.

Teaching method used for experiment: Using model of RME-based learning with a real-life situation, as follows: A manufacturer wants to design an open box whose base is square and area surface is 48 area units as shown in **Figure 6**. Find the dimensions of the box to create produce a box of maximum volume?

Data Analysis

The research data was analyzed by descriptive statistics and from the students' answers recorded on the study sheets. We used qualitative analysis to make comments and conclusions. Data analysis was performed using Excel, the results were summarized in charts and tables.

RESULTS

Table 2 shows the questions for students used in the teaching process. Similarly, **Table 3** depicts the questions for students used in the group activity process.

Table 4. Results of students' answers on the worksheets

Questions	Answer results of students in class 12A8		
	Right	Incorect	No answer
Q1. How is the surrounding surface area of a rectangular box calculated?	17 51.51%	10 30.3%	6 18.19%
Q2. How is the relationship between the height and the size of the bottom shown?	20 60.6%	8 24.24%	5 15.16%
Q3. How is the volume of a rectangular box determined?	25 75.76%	4 12.12%	4 12.12%
Q4. Can we give the formula for the volume of a rectangular box as an algebraic expression of a function of one variable?	29 87.88%	2 6.06%	2 6.06%

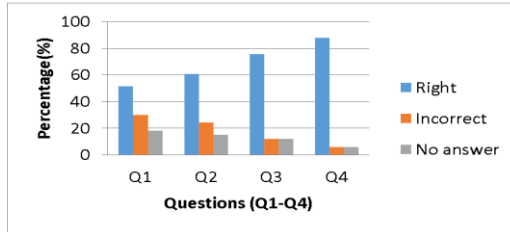


Figure 7. Chart of student answer results

Table 5. Students' products in group activities

No	Discussion questions	Products of the groups
Q5	Is finding the size of the cube for the largest volume of the rectangular box related to any mathematical knowledge that you have been learning or not? If so, please indicate that mathematical knowledge.	Figure 8, Figure 9, & Figure 10
Q6	What is the size of rectangular box so that the volume is the largest?	Figure 8, Figure 9, & Figure 10
Q7	From solving the above situation, can you come up with a general model for the mathematization of practical situations related to optimization factors?	Figure 13

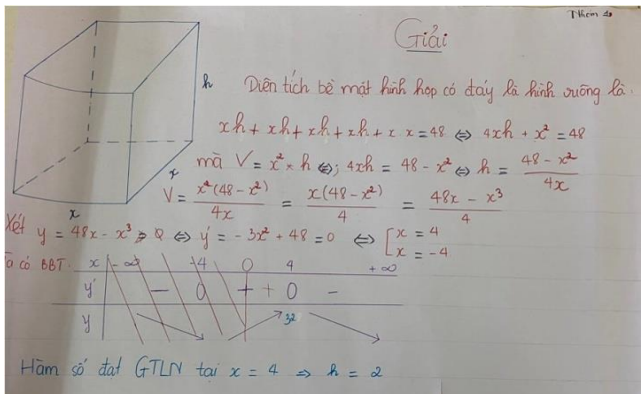


Figure 8. Products of group 1

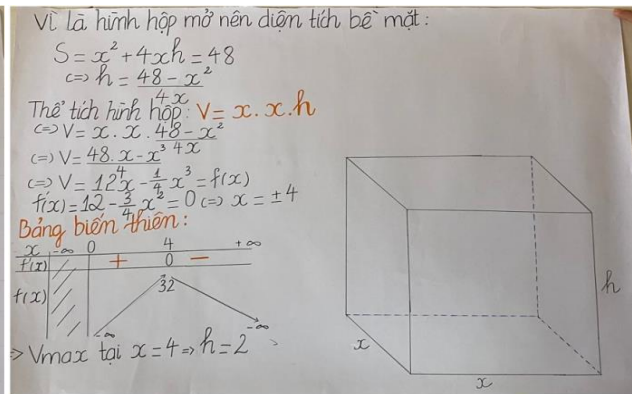


Figure 9. Products of group 2

The results of the students' responses are shown in **Table 4**. **Figure 7** depicts the student answer results using chart. **Table 5** shows the students' products in group activities. The results and products of the groups are shown through the actual pictures in the experimental classroom in **Figure 8**, **Figure 9**, and **Figure 10**.

DISCUSSION

Statistics of students' study sheets (see **Table 4**), we found that with the first question, some weak students and average students had difficulty because most of these students did not remember the formula for calculating the surrounding area of a rectangular box. the results are, as follows: For question number 1, only 17 students answered correctly (accounting for 51.51%), 10 students answered incorrectly (accounting for 30.95%) and six students did not have any response (accounting for about 21.43%). With question number 2, after receiving the teacher's suggestion, the results were better but not significant. There were 20 students giving the correct answer (accounting for 60.6%), eight students answered incorrect answer (24.24%), and five students had no answer (15.16%). With questions 3 and 4, thanks to the help of teachers, the number of students giving correct answers increased (75.76% and 87.88%, respectively) and the number of students giving correct answers increased and incorrect answers decreased significantly (14.29% and 4.76%, respectively). And the number of students who did not give an answer also decreased, from 15.16% to 6.06%. This result shows that students were more active and active in thinking and giving answers.

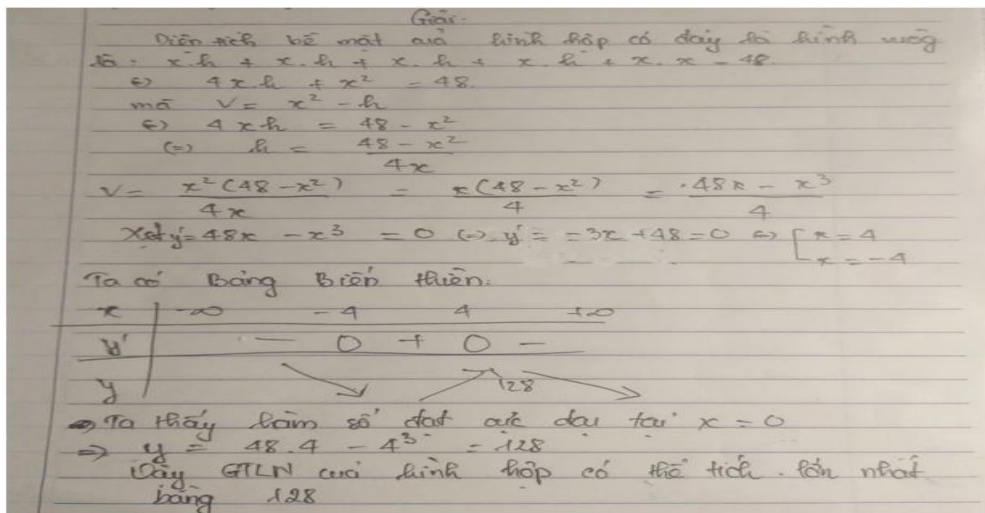


Figure 10. Products of group 3



Figure 11. Presentation of group 1



Figure 12. Presentation of group 2

In the discussion and working sessions of the groups, through observing the classroom, we see that the students work very seriously and responsibly. Each individual is very excited and completes their task well. They are very willing to support the group leader to represent their group to present in front of the class. Besides, the teacher also suggested that the students prepare to ask some questions to the other group when their group presents. The products of the groups are carefully presented on an A0 sheet of paper, which proves that the students have prepared well, have made a lot of effort, and tried to complete the assigned tasks (see Figure 8, Figure 9, and Figure 10).

Under the cooperation and serious teamwork, all three groups came up with a strategy to solve the situation for their group. Thereby showing the positive aspects of the designed learning activities. Through the guidance and control of the teacher, these results were presented to the whole class by representatives of groups 1 and 2 (see Figure 11 and Figure 12). This activity aims to give students the opportunity to present, give their personal opinions and debates, from which they will receive reasonable adjustments in the problem-solving strategy from the teacher. This is also a good opportunity for them to absorb knowledge and understand the lesson more deeply.

In addition to giving a specific model (“model of”) for the initial situation, they also came up with a general model (“model for”), (see Figure 13) that can be applied for a class of similar situations. This activity thoroughly grasps the level principle in teaching according to RME theory, by building a strategy and a specific model for the initial situation, students gradually form logical thinking and reasoning to It is possible to build a general model for dealing with similar situations. This is very important in forming and developing mathematical modeling competence for students when solving practical problems.

CONCLUSIONS AND RECOMMENDATIONS

In the research that we have done, the process of designing math lessons using RME method and RME-based teaching model has been proven quite successfully. From experimental teaching, we have found that in order for RME-based teaching to be effective, teachers must be well prepared and require a large investment of time. First, it is important for teachers to be careful in choosing teaching situations. The selected context must be reasonable, both close to reality and able to create the mathematical knowledge that students need to learn. Next, teachers need to prepare a system of guiding questions to help students build “model of” and “model for” step by step. Finally, teachers need to formally introduce the mathematical knowledge that students acquire after the course of learning activities.

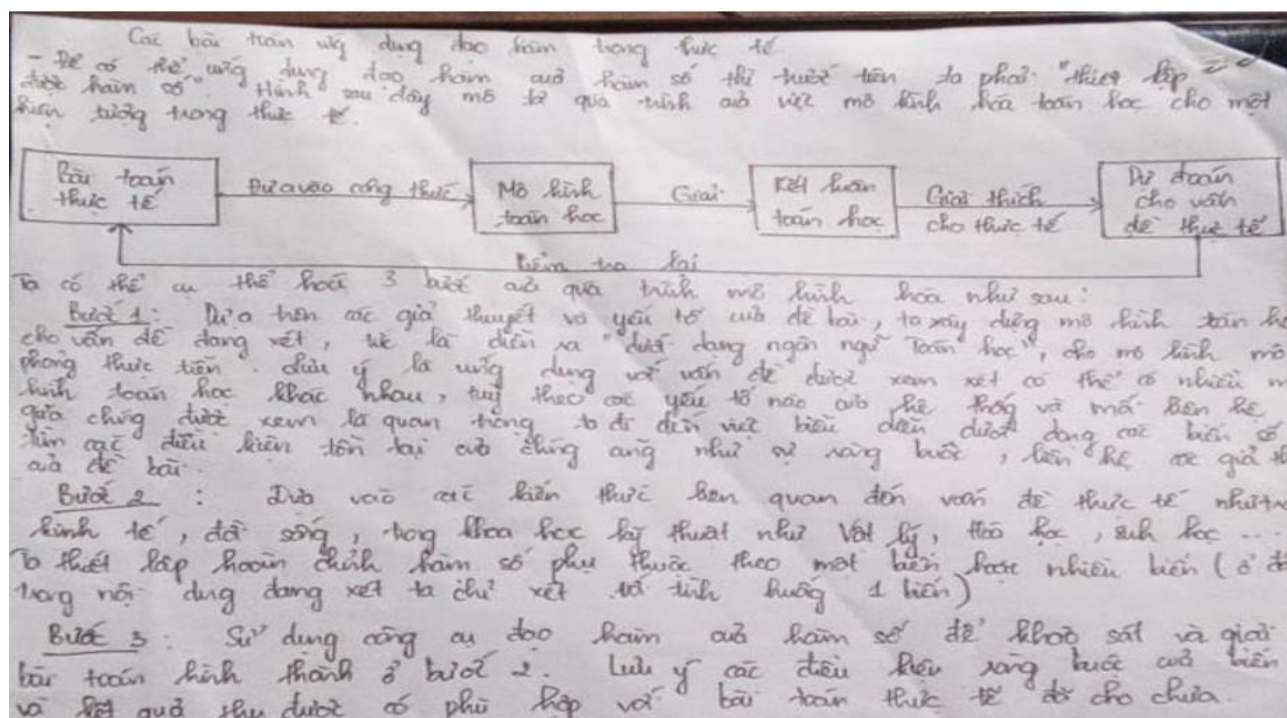


Figure 13. The final product of the groups

Through the situations designed in the article, we see that when faced with real-life situations, students may not be familiar with it at first because they feel it is quite foreign to exercises with purely mathematical content. This makes them curious and see that it is necessary to explore, so when they are given specific tasks, most are very enthusiastic, attentive, trying to do well. At first, doing group activities can be noisy and lack of concentration, but under the control of the teacher, the class is taken very seriously and the children participate actively and enthusiastically. Participating in group activities is an opportunity for students to practice and develop teamwork skills, cooperation capacity, and presenting results in front of the class will help them be more confident in expressing their views in front of a group, through which their presentation skills will improve and develop.

Although our study was a case study, with positive results, we believe that RME has positive effects on students' attitudes, problem solving, interest in learning, or other aspects related to learning mathematics. Thus, based on RME, teaching can completely meet the goal of improving the practicality of teaching mathematics in the coming period of Vietnam.

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