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Emerging perspective of a mathematics teacher educator on connections between advanced and school mathematics

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ABSTRACT

Contemporary research on mathematics teacher education reveals attention by researchers on mathematics connections (MC). Nevertheless, researchers have not investigated experiential views of mathematics teacher educators (MTEs) around MC. This article addresses the scenario by relating findings of an intrinsic case study involving a purposively chosen MTE. MTE responded to interview questions regarding experiential viewpoints on connections between advanced and school mathematics. Narrative data analysis suggested that MTE prefers special mathematics education courses (MECs). Envisaged MEC are supposed to align with school mathematics but comprised of subject matter, which is neither advanced mathematics nor school mathematics. Such courses coupled with school-based mentorship are expected to acquaint mathematics student teachers with MC. MTE could not provide subject matter-based connections except for pedagogic ones. Study of advanced mathematics was encouraged to foster understanding of MC. These findings motivate further research concerning how advanced mathematics ought to be connected to school mathematics concepts.

Keywords: mathematics teacher educator, mathematics connections, advanced mathematics, emerging perspective, school-based mentorship

INTRODUCTION

There is considerable recognition by mathematics education scholars, affirmed among others through development of teacher knowledge for teaching frameworks, that special knowledge is required for effective teaching of mathematics (Ball et al., 2008; Rowland et al., 2005; Shulman, 1986, 1987). Alongside that acknowledgement, previous research shows that mathematics student teachers do demonstrate deficiencies in conceptual knowledge of school mathematics (Malambo, 2020, 2021; Malambo et al., 2018, 2019). Student teachers' superficial understanding of school mathematics is exhibited even when student teachers will have studied advanced mathematics courses at university. Moreover, this occurrence persists despite a view that the study of advanced mathematics enhances experience, which is relevant for development of problem-solving abilities (Yan et al., 2021). Consequently, recommendations have been made in the past regarding the need to identify common concepts between advanced university mathematics and school mathematics (Malambo, 2015). Identification of common concepts is perceived as a pre-requisite to development of unique mathematics education courses (MECs) for mathematics student teachers. Similarly, there has been a suggestion for devising of special mathematics content, which can provide a link between advanced mathematics and school mathematics (Dreher et al., 2018).

Furthermore, there seems to be a trend in mathematics education research around the idea of connections between mathematics taught in university and the mathematics taught in schools (De los Ángeles et al., 2022; Murray et al., 2017; Rina & Roza, 2010; Suominen, 2018; Ticknor, 2012). Some studies involved mathematicians (Yan et al., 2021), while others engaged mathematics teacher educators (MTEs) who intimated that they lacked the experience of having taught mathematics at school level (De los Ángeles et al., 2022). Rina and Roza (2010) promoted a requirement for teachers' understanding of connections in mathematics subject matter. The two authors encouraged identification of "explicit content-based connections between advanced mathematical knowledge (AMK) and mathematics taught in secondary school" (Rina & Roza, 2010, p. 280). They asserted that instructional design in teacher education can be enhanced when there is understanding of the connections and an extensive collection of examples of such connections. However, Rina and Roza's (2010) study demonstrated that their participants could not provide specific mathematics examples to confirm that advanced mathematics knowledge aids the teaching of secondary mathematics. Although the participants in that study claimed that AMK was generally useful in the context of teaching, most of them could not give accounts of specific examples of occasions when they used or when they could use their AMK. Thus, a gap between advanced mathematics and secondary school mathematics was observed.

Murray et al. (2017) emphasizes that advanced mathematics and secondary mathematics have connections, which directly influence teaching. These scholars believe that a teacher's knowledge of mathematical connections between AMK and secondary mathematics can influence his or her pedagogical choices in a classroom. Similarly, Ticknor (2012) recommends that in addition to understanding of advanced mathematics, teachers of mathematics should acquire ability to connect that advanced mathematics to the mathematics taught in schools. An obligation is placed by Ticknor on mathematics teacher trainers to specify to student teachers the connections that exist between advanced mathematics and the mathematics, which is taught in secondary schools. Suominen (2018) investigated connections that exist between abstract algebra and the algebra taught at school level. In that regard, connections as they relate to mathematical processes, artifact for learning, and characteristics of mathematics were highlighted. Other connections discussed in research literature are content connections, disciplinary practice connections, classroom teaching connections, and modeled instruction connections (Wasserman, 2018).

De los Ángeles et al. (2022) identifies mathematical connections between the mathematics taught during teacher training and the mathematics, which prospective teachers are supposed to teach in school. Based on analyses of semi-structured interviews, the researchers indicate that teacher educators do establish content connections, modeled instruction connections, and disciplinary practice connections and professional practice connections. Nevertheless, De los Ángeles et al.'s (2022) study reveals that the sampled teacher educators never approached the issue of connections in a systematic and planned way. While the teacher educators seemed to understand the value of identifying and specifying connections as they trained mathematics teachers, they claimed that "they do not have time, that it is something that should be left to the person who is in training and will be acquired in professional practice, or that they are unaware of the connections because they have not worked in lower secondary education." (Ángeles et al., 2022, p. 9).

Dreher et al. (2018) acknowledge that there is a discrepancy between school and academic mathematics. These authors posit that the field of mathematics education has no clear answer concerning the kind of profession-specific content knowledge, which prospective secondary mathematics teachers need to study during training. A construct called School-related Content Knowledge (SRCK), which relates to a profession specific content knowledge for teaching secondary mathematics was therefore introduced (Dreher et al., 2018). This construct is about interrelations that exist between academic and school mathematics and is distinguishable from specialized content knowledge (SCK).

Despite the efforts of researchers as discussed in the preceding sections, it is important to note that mathematics curricula are not standardized across countries and teacher education programs (Tatto & Senk, 2011). Belbase (2019) gives an example of the USA were there are no centralized mathematics curricula, but instead states and school districts are allowed to develop their own standards. The issue of decentralization of curricula is also a feature in teacher education in the USA. Incidentally, teacher education programs are developed by each specialized department in institutions of learning. Decentralized teacher education curricula and the increasing concern of mathematics education scholars in the issue of connections justify investigations of MTEs' understanding about mathematics connections (MC). However, the current investigation concerning MC is motivated mainly by research findings (Murray et al., 2017; Ticknor, 2012). The idea is to understand a single mathematics teacher trainer's emerging perspective regarding the kind of connections, which exist between advanced mathematics and school mathematics including the connections' influence on teaching. This is imperative because MTEs' views impact on the learning of their student teachers as is the case with mathematics teachers and their pupils. Hopefully the lessons gained from the experiences of a practicing MTE will provide a basis for hypotheses formulation and encourage further research around the idea of MC in different contexts. Besides, the lessons may inform mathematics teacher education curricula developments in other contexts. The current study was guided by the following research question: What perspective emerges from the experiential views of an MTE concerning connections between advanced mathematics and school mathematics?

THEORETICAL CONSIDERATIONS

A focus on the views of an MTE was motivated by the researcher's theoretical understanding that views of participants provide mediums for understanding reality (Nieuwenhuis, 2014a). The aspect of MC was grounded on the mathematical knowledge for teaching (MKT) framework (Ball et al., 2008). MKT builds on Shulman's (1986) ideas regarding necessary teacher knowledge. MKT framework consists of the following six categories: common content knowledge (CCK), horizon content knowledge (HCK), specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). The category relevant to the current article is HCK.

Ball and colleagues initially acknowledged that the construct HCK was not yet fully established. However, other researchers have provided amplification to the construct of HCK (Bair & Rich, 2011; Jakobsen et al., 2013). Bair and Rich (2011) asserted that HCK refers to "how mathematical topics are related over the span of mathematics included in the curriculum, across grade levels" (p. 294). Jakobsen et al. (2013) conceptualized HCK as an orientation to and familiarity with the discipline that contribute to the teaching of a school subject at hand. Additionally, they contended that HCK includes knowledge that could make teachers to operate with knowledge of connections to topics, which students may or may not meet in future. These interpretations of HCK render it relevant to mathematics teacher educators especially in the context of their perspectives regarding how mathematics topics are related. Knowledge of mathematical relationships can enable MTEs to give explanations relating to how mathematics ideas contribute to mathematical goals including how mathematics ideas should be taught. Since the present study investigated the views of a MTE regarding connections between school and advanced mathematics, which may include relationships among mathematics concepts, the choice of HCK as a theoretical basis is justifiable.

METHODOLOGY

This is a qualitative intrinsic case study (Merriam, 2009; Nieuwenhuis, 2014b). An intrinsic case study allowed for a focus on the experiential views of a particular case itself-MTE. In this regard, I was not inclined to seek understanding of any abstract construct or to develop a theory. Furthermore, the purpose of the study was not to compare perspectives of MTEs, but to establish a perspective that emerges from the experiential views of a particular MTE. Consequently, I involved one PhD holding MTE chosen purposively and who was teaching mathematics student teachers in a USA university. A focus on one MTE facilitated acquisition of detailed information. Arguably, a sample of size one was still appropriate for an intrinsic qualitative case study for which depth of understanding of phenomenon and not generalization of findings is sought (Merriam, 2009). In qualitative research, meanings are situated in contexts. Thus, different people have different perspectives and contexts, and hence the world is perceived to have different meanings, which are not necessarily more valid or true than others (Gay et al., 2011).

Equipped with a bachelor's degree in mathematics, the sampled MTE had previously taught mathematics at secondary school level. At university, she had taught advanced mathematics after acquiring a master's degree in mathematics and later earned a PhD in mathematics education. At the time of this study, MTE had been teaching for at least three years a course that focused on MC to mathematics student teachers. Characteristics of MTE such as school mathematics teaching experience and teaching a university course designed for MC are a departure from those of some respondents in previous studies. Accordingly, I thought that the respondent was well placed to be a source of enriching information on MC.

A semi-structured interview schedule was developed for the study and thereafter a face-to-face interview was conducted with MTE. Generically, the interview questions characterized by probing and prompting focused on

(1) the kind of knowledge, which was emphasized in MEC on MC taught by MTE,
(2) meaning of MC from the standpoint of MTE,
(3) opportunities provided to mathematics student teachers for acquisition of in-depth understanding of MC,
(4) subject matter upon which MC were and should be taught to student teachers,
(5) the teacher educator's understanding of the construct 'advanced mathematics' in comparison with that of 'school mathematics',
(6) perceived gaps between advanced mathematics and school mathematics,
(7) how gaps between advanced and school mathematics were and should be addressed, and
(8) whether advanced mathematics should be studied by mathematics student teachers.

The interview was audio recorded and then transcribed for analysis. Data were analyzed qualitatively, specifically using narrative analysis. The idea was to obtain a pattern from the narrative of MTE concerning the connections between advanced and school mathematics. It was, therefore, not the intention of the study to focus on specific MC, but to establish an emerging perspective based on the experiential views of MTE. In this vein, transcript exploration and subsequent interpretations were conducted to establish an emerging perspective. Ordinarily, I subscribe to McMillan and Schumacher's (2006) stance that there are no plainly prescribed procedures to follow when analysing data. Notwithstanding, when analysing data of this study I employed ideas espoused by Merriam (2009) as I deemed them suitable for a qualitative study of the current kind. According to Merriam (2009), the process of data analysis is about making meaning and it involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read. With this view, consolidation and reduction were conducted concomitantly as the researcher read the transcript several times and subsequently interpretations of MTE's responses to specific questions were made. This process included a search for patterns of meanings and relating of interview extracts to corresponding interview thematic ideas. Furthermore, MTE's views were compared with and interpreted using notions generated from the works of Dreher et al. (2018), Murray et al. (2017), Rina and Roza (2010), Suominen (2018), and Ticknor (2012). It is these processes described above, which culminated into establishment of an emerging perspective.

RESULTS

In this section, I report results, which were derived from the semi-structured interview, which was conducted with MTE. To enhance anonymity, a pseudonym, Dingani, is utilized. Since this article is intended to provide findings based on the experiential views of MTE, results are presented in a narrative form. Relevant excerpts from the interview transcript will be provided to confirm the narratives both before and after every commentary. I now provide the results hereafter.

Dingani explained how the topics in the course she was teaching were linked to each other and gave generic descriptions of relevant tasks, which required fostering in each topic. She emphasized the need to tackle common mistakes, which students commit when solving questions on topics as well as the necessity to allow for different methods when resolving questions on topics. According to her, if student teachers get abreast with such aspects during training, it is likely easy than not for them when they become teachers to effectively guide their learners. To bring out significant experiential views, Dingani was requested to elucidate the knowledge emphasized in the university MEC she was teaching. She first elaborated that the course was less focused on mathematics content as student teachers would have previously studied relevant content in courses like college algebra, pre-calculus, and calculus 1:

> **Interviewer (I):** What kind of knowledge is emphasized in the mathematics course that you teach mathematics student teachers?
>
> **Dingani (D):** This course [name of course] focusses only on linkages or connections between what the student teachers have already done and the mathematics that they are going to teach. It is mostly focused on showing student teachers how to use the knowledge that they have, to connect that to teaching in the classroom. So, anticipating student misconceptions, making connections between different mathematics concepts, and being able to show different things in multiple different ways, like showing visually using technology and embracing lots of different types of solutions.

This excerpt suggests that Dingani's course was built on the assumption that mathematics student teachers would have acquired sufficient understanding of pre-requisite mathematics content knowledge. That assumption was the basis upon which student teachers were taught how to implement previously acquired content knowledge in mathematics classrooms. Dingani clarified that the course also focused on how to use technology to represent mathematics concepts in different ways and how to identify learners' misconceptions. Her rationalizations were elaborated in the extract below:

> **D:** What we found in the past is that the pre-service teachers' biggest difficulty is connecting content knowledge to actual teaching in the classroom and so this course is designed to be like one of the bridges that we have between content and teaching because a lot of times student teachers will have the mathematics knowledge, but they do not know how to express it and they do not know how to teach it. They do not know how to address student misconceptions, so it is like a bridge between the mathematics and the education knowledge. It is different from the [other] mathematics courses in the university in that this mathematics is focused on helping them to be able to unpack the mathematics for teaching purposes.

The respondent justified the teaching of MC to mathematics student teachers based on historical findings, which suggest that student teachers do demonstrate challenges to link content knowledge to classroom teaching. Dingani also pointed out that student teachers do demonstrate challenges to address mathematics misconceptions. These realities then motivated the development of a course she was teaching with an objective of attending to the identified difficulties of student teachers. Overall, the course was intended to facilitate student teachers' acquisition of appropriate pedagogies to enable them to unpack and teach the subject matter taught at school level. MTE was requested to provide her understanding of the idea of connections:

> **I:** I want us to consider this idea of connections. What is your understanding of connections that exist between school mathematics and advanced university mathematics?
>
> **D:** In the university a lot of times mathematics lecturers are not teaching the students how to teach. They just teach the 'how'. So, they teach you how to solve a mathematics problem [and] they teach you how to get the answer that you need. However, in a situation, where you are trying to train teachers to teach mathematics, you need to show them why things work so that, that way they can explain better to their students. Because if you just teach computations like here is how to solve this problem, the students are not going to have a deep understanding of the mathematics concepts.

Dingani's conception of the idea of connections was about student teachers' capacity to explain the 'why' of mathematics concepts. She claimed that 'connections' had something to do with provision of explanations relating to why 'things' work and likened connections to the issue of acquisition of 'deep understanding'. Another aspect on connections underscored by Dingani centered on the connectivity of mathematics topics. The ensuing excerpt affirms her perspective that mathematics topics, which are at times taught as though they are separate units are connected to each other:

> **I:** What other kinds of mathematical connections do you focus on as you train teachers of mathematics apart from the 'why'?
>
> **D:** Showing student teachers how different concepts are connected to each other. So, the connections from the perspective of sequencing, that this concept as you learn it is related to this one, and therefore you need to understand this concept before that one.

Dingani posited that the aspect regarding how mathematics concepts are connected to each other is a core nature of 'connections.' The claim here was that there are 'connections' that demonstrate how mathematics concepts are related to each other. Those connections relate to why certain mathematics topics should be taught before others. Thus, 'sequencing' characterized by ordering of mathematics topics or concepts was suggested as another kind of connections. When prompted to do so, Dingani was not able to readily illustrate on paper at least an example of content-based connections between advanced and school mathematics. An experiential view that student teachers do experience challenges to sequence subject matter was given by Dingani in the following excerpt:

> **D:** I think the difficulty is more in the nature of the student teachers' ability to adjust than in the subject matter. I see them sometimes when they are starting to teach without a sequencing guide. They will start teaching something and then will say oh, but I need to know this. Until they have a grasp of all the connections and how [subject matter] is all related, it is difficult for them. So, if they do not really understand how everything is all related and that this topic needs this topic or things like that, then they have a hard time building a structure.

Having explained the nature of connections emphasized in the course, MTE was interviewed concerning the nature of the mathematics subject matter, which student teachers were being taught to unpack. The conversation progressed as follows:

> **I:** The mathematics that student teachers are unpacking, is it university mathematics or school mathematics?
>
> **D:** It is closer to the school mathematics, but it is not school mathematics, and it is not university mathematics. It is just in between.

MTE clarified that the connections taught involved subject matter, which is aligned to school mathematics and that the course development thereof was based on topics taught in school mathematics. However, the respondent concurrently asserted that the subject matter is neither university nor school mathematics in nature but lies in between. By implication, the mathematics, which was being unpacked is a 'special kind of mathematics'. Dingani's understanding of the construct 'advanced mathematics' in comparison with that of 'school mathematics' was then sought:

> **I:** What is your understanding of this construct 'advanced mathematics' in comparison with 'school mathematics'?
>
> **D:** I think advanced mathematics is more on the theoretical side. In school mathematics here in the USA we do not do a lot of proofs of theorems or anything like that. It is mostly computational, but mathematics courses that go beyond that [school mathematics] help student teachers to understand why things work in mathematics the way that they work. That gives a much better insight to how you can help students make connections between different concepts, which they might not make on their own because a lot of times they see things like algebra as being separate from geometry and you know students do not really make the connections very easily.

Dingani believed that advanced mathematics is the kind of mathematics, which provides ample opportunities for proofs to theorems and an understanding of 'why things work the way they do'. She claimed that comparatively speaking, such opportunities are not prominent in school mathematics. When probed to clearly state her position whether advanced mathematics and school mathematics are different, she expressed views reflected in the following extract:

> **I:** Do you take the view that there is a difference between advanced mathematics and school mathematics?
>
> **D:** Yeah [yes]. The further that you get [in] mathematics, the more abstract you get and that is when you start working with the proofs behind the theorems. Instead of learning how to do things, you start learning why things work the way they do, and it might not be this way everywhere in the world. But here [USA] we do not focus on the why as much in school mathematics. It is more about the how in school mathematics while in advanced mathematics we do focus on the why.

Dingani could not give comprehensive differences between 'advanced mathematics' and 'school mathematics' other than emphasizing that advanced mathematics somehow empowers students to understand the 'why' aspect of mathematics. She was quick to mention that at school level learners are rarely taught the 'why' of mathematics. To acquire insight regarding her understanding of the construct 'advanced mathematics', Dingani was asked to explain why it is necessary for student teachers to study a mathematics course on connections in addition to 'advanced mathematics':

> **I:** What is the necessity of teaching the connections course if advanced mathematics includes the components that you are teaching in the connections course?
>
> **D:** When I talk about advanced mathematics, I'm talking about mathematics even a little bit further than what our teachers will teach like master's level mathematics, which is not teacher focused.

It became apparent that what the respondent referred to as advanced mathematics included the kind of mathematics, which is not teaching focused. This is despite the nature of that mathematics to also provide opportunities for understanding why particular aspects work as they do. Dingani argued that there is mathematics, which may not be necessarily relevant for purposes of teaching at school level. According to her, such a reality justifies the need to have mathematics courses in which student teachers are taught MC. Dingani was asked to indicate whether it is necessary for mathematics student teachers to study advanced mathematics:

> **I:** Should prospective mathematics teachers be taught the advanced mathematics?
>
> **D:** I think student teachers need to take mathematics courses that go beyond the level that they are to teach. They, however, do not necessarily need to get into the master's level mathematics, but should go past the level that they plan on teaching. That way they can acquire an overall view of how subject matter is connected.

The respondent did not explicitly state whether student teachers should study the mathematics for mathematicians. However, she contended that student teachers are supposed to study mathematics that is 'higher' than the one they would be teaching for purposes of appreciating the connectivity of subject matter. Dingani declared that there are 'gaps' between advanced mathematics and school mathematics. Her understanding of the phrase 'gap' suggested that the phrase is synonymous with the word 'difference'. She was therefore requested to provide examples of 'gaps':

> **I:** Would you give me two examples of the gaps that exist between advanced mathematics and school mathematics?

D: Yeah, so for example in lower grades mathematics especially in geometry classes, they do very little work with proofs or with logic as they cover it usually in a day or two, but then they just move on, and it is mostly computational problems that they go to after that. Whereas at the university level, if you were to take a geometry course it is going to be mostly focused on proofs and showing the geometric works and why they are true. But in the lower levels they just apply the theorems they do not learn why they are true.

Dingani's example of a gap between advanced and school mathematics was anecdotal. She only referred to activities that do not substantively feature in school mathematics such as provision of proofs as to what constitutes a gap. Moreover, she did not clarify whether the absence of proofs was due to teachers' decisions or because of the way school mathematics syllabi were designed. Dingani elaborated the context of her viewpoint in the excerpt below:

D: At the lower levels [school level] a lot of times we teach algebra and geometry as two very different subjects and so students do not get to see the connections between algebra and geometry, whereas at the university level you are going to see a lot more of those connections.

The excerpt above suggests a different dimension of what Dingani considered to be a 'gap' between advanced mathematics and school mathematics. It appears that perceiving categories of mathematics as separate from each other constitute a 'gap' in Dingani's view. Another angle of Dingani's viewpoint relates to the emphasis placed by teachers and not the nature of the mathematics taught at school level. In response to a question concerning how the course she was teaching addressed mathematics gaps, Dingani stated the following:

D: The pre-service teachers take university mathematics courses like college algebra, pre-calculus, calculus in which they learn mathematical knowledge. In my course, we take the stuff that you learned in college algebra [for example] and now we are going to connect that to what the students at school level are learning and how you can help them further their connections and help them make those leaps.

Dingani gave a generic explanation regarding how the issue of connections was being addressed in the course she was teaching. Her explanation suggested that the mathematics subject matter studied earlier by student teachers and in other mathematics courses was normally linked to what is taught in school mathematics. The purpose of that practice was to facilitate student teachers' comprehension of the connections between university and school mathematics. The respondent was prompted to specify how 'gaps' were being addressed in her lectures:

I: Let us focus on your lectures. How do you address the gaps?

D: Uh, one of the examples is during our lectures we have a lot of discussion questions, where things as simple as when you are working with inequalities, and you multiply or divide by negative [number] the inequality sign flips. That is something that all the pre-service teachers know, and they do it. They know how to solve such problems. However, when you ask them why they do that a lot of them are stunned. So, in the course that I teach [name of the course], we focus on those types of questions.

Apparently, a strategy, which was being used to address 'gaps' during lectures involved promotion of discussions among student teachers and the teacher trainer. The discussions were based on mathematical questions, which were prepared by Dingani. She gave a scenario involving inequalities and the flipping of the inequality sign after multiplying or dividing by a negative number as an example of discussion points in her lectures. This prompted the researcher to request for a demonstration based on the cited scenario. While Dingani attempted to illustrate why the inequality sign flips after multiplying or dividing by a negative number, her explanation was superficial. At the instigation of the researcher, an alternative example in the context of addressing gaps between advanced mathematics and school mathematics was elicited:

I: Is there another way in which the gaps could be addressed?

D: I think one of the ways that the gaps could get addressed is having the pre-service teachers work so closely with the schoolteachers because a lot of times the schoolteachers have been doing this for so many years. They have built up such a wealth of knowledge that they are able to share about common student misconceptions, connections to make between different mathematical topics, how to prepare students for topics that are coming in their future. So, working closely with the school faculty is one way.

Dingani could not, in specific terms, describe an alternative way in which specific 'gaps' between advanced and school mathematics can be addressed. She explained only in a generic sense that 'gaps' can be attended to by giving opportunities to student teachers to work closely with schoolteachers. Her implicit view was that facilitation of mentorship by mathematics schoolteachers of mathematics student teachers could assist in resolving the issue of 'gaps'. Dingani was requested to give examples of opportunities provided to student teachers for acquisition of in-depth understanding of school mathematics:

I: I would like for you Dingani to describe some of the opportunities, which you provide for your mathematics student teachers to acquire in-depth understanding of the school mathematics subject matter.

D: We do a lot of discussion questions as a group because they have already taken college algebra. So, there is no real reason to re-teach the algebra content. Instead, what we focus on is explaining why certain things work the way that they do. We do a lot of group discussions so that they can learn from each other's thinking.

It was noted that the same strategy employed to address perceived 'gaps' was conceived as a viable option for facilitation of student teachers' understanding of school mathematics. Dingani's dominant strategy in that regard involved promotion and undertaking of class discussions among student teachers. She accorded opportunities to student teachers to actively engage in the school mathematics concepts, which they would be teaching when employed as teachers. Thus, student teachers were provided mathematics questions on which class discussions would then ensue. The teacher educator presumed the use of discussions as a helpful strategy to facilitate opportunities for student teachers to learn from each other.

DISCUSSION

The discussion provided henceforth hinges on issues that emerged from narrative analysis of the interview transcript. The context is that of the experiential views that were articulated by a practicing MTE concerning connections between advanced mathematics and school mathematics. Derived data suggests that MTE held a fundamental opinion that among the courses, which mathematics student teachers must study is a mathematics education application course. The sole purpose of such a course was conceptualized as that of providing for demonstration of MC between previously studied 'university mathematics' and the mathematics taught at school level. A viewpoint was championed that student teachers do face difficulties to connect content knowledge to classroom teaching. Furthermore, a historical experience, which typified mathematics student teachers' challenge to explain the 'why' of mathematics as well as to appropriately sequence subject matter was recounted. This view corroborates findings of previous research in respect of mathematics student teachers' inability to provide justifications for their mathematical reasoning (Malambo, 2020, 2021). These realities motivate a view promoted by other researchers that student teachers should be accorded opportunities to study MC during their training (Rina & Roza, 2010). However, MTE posited that connections are supposed to be based on unique subject matter, which is neither advanced mathematics nor school mathematics in nature, but which is nonetheless consistently aligned with school mathematics. This view appears to have been an invitation to MTEs to develop special mathematics content for mathematics teacher education.

Another matter consequent from the study concerns the mathematics teacher trainer's conception of what constitutes MC, and this conception seemed to have been restricted to two kinds. First, MC were understood to be what enabled the provision of explanations of why certain mathematics concepts are as they are and why they work as they do. A claim was made that failure to understand the 'why' of mathematics amounts to lack of in-depth understanding of mathematics concepts involved. Secondly, MC were perceived to center around sequencing of mathematics subject matter taught. It was contended that student teachers ought to be empowered to appropriately sequence subject matter so that they could in turn help their learners to understand how mathematics concepts are connected. Based on these ideas, the respondent was deliberate and intentional in orchestrating pre-planned mathematics activities that were believed to have potential to enhance student teachers' understanding of the 'why' of mathematics. MTE prepared mathematics questions and activities upon which classroom discussions concerning the 'why' of mathematics concepts were spearheaded among the student teachers. This strategy by MTE is consistent with findings of recent studies through which it has been confirmed that active engagement of mathematics learners improves their achievement (Malambo et al., 2023; Zakaria & Syamaun, 2017).

The knowledge of the 'why' of mathematics, and identification with resolution of anticipated learners' mathematics misconceptions were conceived to be of a kind that can augment student teachers' 'deep understanding' of mathematics concepts. This viewpoint somehow corroborates that of another scholar who discusses similar ideas, but under the heading of relational understanding (Skemp, 2006). In addition, MTE's conception about MC appears to have shaped her choices of pedagogical strategies such as fostering classroom discussions when teaching connections. This corresponds with Murray et al.'s (2017) view that knowledge of connections influences pedagogical choices. Further, the teacher trainer's views seem to demonstrate her inclination to pedagogical content knowledge, which is a blend of content and pedagogy (Shulman, 1986, 1987). Strategies of the teacher educator are slightly divergent to those in De los Ángeles et al. (2022), where the teacher educators are said not to have been systematic concerning the issue of connections. However, the strategies are consistent with De los Ángeles et al. (2022) and Murray et al. (2017) in suggesting that connections between advanced mathematics and secondary mathematics do directly influence teaching. Murray et al. (2017) illuminated this perception through use of examples based on algebra.

MTE in the present study believed that advanced mathematics is different from school mathematics and that there are gaps between the two types of mathematics. She seemed to agree with the view of Dreher et al. (2018) who contend that there is a difference between school and academic mathematics. Her postulated notion is that advanced mathematics usually provides for proofs to theorems and the understanding of why particular mathematics concepts work the way they do. A claim was made that availability of proofs and provision of the 'why' of mathematics in advanced mathematics are what distinguish advanced mathematics from school mathematics. It was also claimed that the absence in regularity of activities such as provision of proofs and explanations of the 'why' in school mathematics is among what creates a 'gap' between advanced and school mathematics. Regardless, knowledge of advanced mathematics was deemed to be inherently capable of empowering student teachers to understand MC for purposes of teaching school mathematics.

Additionally, MTE was of the view that mathematics student teachers ought to study advanced mathematics with an objective of facilitating their comprehension of MC and ultimately understanding school mathematics. Thus, the respondent advocated as in Ticknor (2012) for the need to connect advanced mathematics to school mathematics. However, the teacher trainer's understanding of the specific subject matter connections between advanced and school mathematics seemed fuzzy. Besides, and while the significance of mathematics student teachers studying content-based MC was espoused, the data suggests that MTE could not readily provide examples of mathematics subject matter-based connections. She could not provide explicit subject matter examples to demonstrate MC between advanced and school mathematics. What stood out in her discourse were pedagogic descriptions of what consists of MC and how to address them during lectures. Furthermore, no subject matter specific examples were cited by MTE to confirm how knowledge of advanced mathematics concepts can facilitate comprehension of specific school mathematics concepts. This scenario is like the findings of the study by Rina and Roza (2010) and speaks to the continued need for research that should specify how advanced, and school mathematics subject matter should be linked.

CONCLUSIONS

This study has brought to light an emerging perspective of a practicing MTE regarding MC that exist between advanced and school mathematics content. An explicit case has been made in this article of the necessity to educate mathematics student teachers on connections between advanced and school mathematics. Fundamentally, MTE elucidated experiential viewpoints about MC, which are inclined to pedagogy. While examples of subject matter connections could not be provided by the respondent, the viability of teacher education courses whose sole purpose is to address MC was pronounced. Thus, a case for development of special mathematics content higher, but aligned to school mathematics was established for mathematics teacher education. This is in view of the gaps experienced by MTE between mathematics content studied by prospective teachers in university and the subject matter taught at school level. The emerging perspective aligns with recommendations by other researchers for special mathematics content to connect advanced and school mathematics (Dreher et al., 2018, Malambo, 2015).

The findings of this study suggest that advanced mathematics should be studied by mathematics student teachers to the extent that it assists them to make connections with the mathematics taught at school level. This view corresponds with the essence of HCK for which relationships among mathematics topics and concepts are critical (Bair & Rich, 2011; Ball et al., 2008). The idea is that advanced mathematics ought to capacitate mathematics student teachers to identify anticipated learners' misconceptions, represent concepts in different ways, sequence subject matter taught among others. Furthermore, advanced mathematics ought to empower mathematics teachers with ability to assist their learners to understand why certain concepts work. Some findings here corroborate those of Yan et al. (2021) who posit that the value of advanced mathematics is in its potential to facilitate connections, for example, across mathematical domains. Further research in different contexts is necessary to specifically inform MTEs' efforts to train mathematics student teachers how to use advanced mathematics to teach school mathematics. Such studies may equip mathematics teachers to understand what learners require to learn, which is consistent with the recommendation by the National Council of Teachers of Mathematics (NCTM, 2000). Hopefully too, such research endeavors will contribute to the quality of mathematics teacher education through facilitation of mathematics student teachers' comprehension of school mathematics.

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