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Exploring the relationship between computational estimation and problem-solving in year 2 and year 4 children

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ABSTRACT

The present study attempted to examine the relationship between computational estimation and problem-solving in young children. For this purpose, quantitative research based on a cross-sectional design was conducted with 94 year 2 and year 4 children, who were presented with a computational estimation task (CET) and a problem-solving task (PST). In the CET, participants were asked to produce an estimate for a computational result, whereas in the PST they were presented with mathematical problems from different mathematical topics that they had to solve with accuracy. The analysis of the results revealed high performance from participants in both age groups on both tasks. Moreover, a significant positive correlation was found between success in the CET and performance in the PST: children who accomplished successful computational estimations tended to have a high success rate in problem solving. This finding was true for both age groups and indicates the strong relationship between these two abilities from an early age. Educational implications of the study are discussed in relation to mathematics teaching and learning.

Keywords: computational estimation, primary school mathematics, problem-solving

INTRODUCTION

Too often in everyday life we make estimations to provide answers that allow us to choose the right decisions without making complex or precise calculations. Unfortunately, estimation is not always adequately addressed in school; its teaching is often overlooked (Andrews et al., 2022), mainly because of the considerable interest given to fluent and precise written computations (Tsao, 2009) as well as because of teachers' skepticism about recognizing its value and finding ways to teach it (Alajmi, 2009; Anestakis & Desli, 2014). However, in recent years, research in estimation tends to expand rapidly, as its usefulness and importance in our daily living have been appreciated (Siegler & Booth, 2005) and its correlation with later mathematical competence has been established (Daker & Lyons, 2018; Link et al., 2014; Sasanguie et al., 2013; Schneider et al., 2018), particularly when using numbers in novel situations (Holloway & Ansari, 2009). Although there is still a long way to go for the enhanced presence of estimates in the curricular documents (Sayers et al., 2020), the recommendations for its teaching increase.

This increased research interest may be explained by the fact that estimation allows us to move beyond the mechanistic application of mathematical procedures and techniques and focus on applying mathematical knowledge in flexible and efficient ways. In fact, as Luwel and Verschafel (2008) argue, estimation is a "complex problem-solving activity" (p. 320) leading to an approximate judgment or rough calculation that requires various computational skills and mathematical procedures. The view of Usiskin (1986) that the applications of estimation are aligned with the '*ideals of mathematics*', namely, "clarity in thinking and discourse, facility in dealing with problems, and consistency in the application of procedures" (p. 2), justifies the feeling that mathematics is not exclusively associated with accuracy and, therefore, estimation is part of it, highlighting a way of using it. Mathematics, in this light, involves a distinct way of thinking rather than the applications of a set of rules, often disconnected from each other.

Four are the more often referred types of estimation in the mathematics education literature (Hogan & Brezinski, 2003; Siegler & Booth, 2005): computational estimation (estimating a computational result without paper and pencil), numerosity or quantity estimation (estimating the number of objects in a set without counting), measurement estimation (estimating a measurement without measurement tools) and number line estimation (estimating the position of a number on a number line). This paper fits within the growing interest regarding computational estimation and reports the findings from a quantitative study that investigated the relationship between computational estimation and problem-solving in young children.

Although we already know about the existence of strong relationships between mental computation and mathematical reasoning (Gürbüz & Erdem, 2016; Kasmer & Kim, 2011) or even between computational estimation and problem-solving (Desli & Lioliou, 2020), our knowledge is so far mainly focused on older children and adults. Investigation into young children's performance on both computational estimation and problem-solving tasks (PSTs) is warranted as educational findings in older age groups are not necessarily applicable to younger children, and the experiences and skills of younger children are not necessarily the same as those held by older individuals. In addition, the study aligns with the aims of the curricula for primary mathematics in many countries around the world, which include the recognition of the importance of making reasonable computational estimations, and the appreciation of learning computational estimation as a significant factor in facilitating students' number sense and mathematical reasoning (McIntosh, 2005).

LITERATURE REVIEW

Computational Estimation

Computational estimation refers to determining an approximate but satisfactory solution to arithmetic computations with the use of mental mathematics (Dowker, 1997; LeFevre et al., 1993). It is considered an essential life skill (Ganor-Stern, 2016; Sekeris et al., 2019), frequently employed in situations that answer the question "about how much is …?" For example, to decide regarding the products to buy according to their price or to compute the rough total amount to be paid (e.g., dividing a bill of 144 euros among 5 people or adding 4.57 + 3.48 + 1.93), an estimate generated in our head for the answer without or before performing the precise calculations is sufficient as it allows us to get a clearer understanding of the situation. In other words, an estimate of the outcome of an arithmetic calculation is provided either when performing a calculation is too complex to be done accurately or when an exact answer is not required due to the context of the situation. This issue is very important because it determines the purpose of computational estimation. As it requires less time and effort than an exact calculation, computational estimation can therefore be used in situations where there are time or attention constraints.

In addition to the practical usefulness of computational estimation, its role as a sanity check on the reasonableness of an answer is also recognized (Deslis & Desli, 2023; Dowker, 2003; Ganor-Stern, 2016; LeFevre et al., 1993; Yang, 2019). For example, adding 28 + 16 is expected to give a sum close to 45 or 50 (given that the numbers are rounded as 30 + 15 or 30 + 20, respectively). In the case where the exact written calculation is followed, if a sum of 314 is found (possibly due to an error in the execution of the written algorithm), then the previous estimate can serve as a means of checking the reasonableness of the result, thus confirming the success of the result, or leading to its refinement. Since it is easy to make a simple mistake when working out a computation by hand, an estimation of the expected answer is a useful tool in identifying potential errors in calculations. As Reys et al. (2012) point out, exposing children to situations that offer opportunities for estimation from an early age will assist them to include estimation into their mathematical methods and have knowledge of what to expect.

Researchers (e.g., LeFevre et al., 1993; Reys et al., 1982), in describing the computational estimation process, have provided models that clarify how procedural knowledge and conceptual knowledge are used to produce an estimate. In these models, to produce an approximate answer for an arithmetic problem, numbers are first modified into approximate numbers and then used in mental computations, with three key procedures identified: reformulation, i.e., changing numerical data to produce a more mentally manageable form (e.g., through rounding), while maintaining the structure of the problem intact; translation, i.e., changing the mathematical structure of the problem to a more mentally manageable form (e.g., changing addition to multiplication); and compensation, i.e., adjusting an estimate to account for changes resulting from reformulation or translation. Thus, computational estimates may begin with an approximation of the result that can be obtained from an operation and conclude with the evaluation of the estimation as to whether it is close enough to the starting approximation. On these grounds, computational estimation involves the coordination of various types of mathematical knowledge in flexible ways, as it requires going beyond the rote application of procedures (Siegler & Booth, 2005). In fact, it builds upon and extends the understanding of the number system and its principles, such as the place-value principle (Sowder, 1992), and requires advanced number sense (Alajmi, 2009). Thus, computational estimation ability encompasses high-level abilities and reflects conceptual understanding when mentally processing numbers during calculations.

Case and Sowder (1990) were among the first to argue that children are considered able to estimate only when they reach a more advanced stage of cognitive growth, at which they achieve the coordination of converting an exact answer into an approximate number and processing numbers mentally. In their neo-Piagetian analysis of estimation of multi-digit sums, they proposed that estimating sums involves two essentially different tasks, approximating and mentally computing. Thus, children typically are not able to coordinate these two tasks until they enter the vectorial stage, around 11 or 12 years of age. Nevertheless, this perspective has been challenged in recent years by research indicating that younger children are capable of carrying out this dual task. In fact, subsequent researchers have observed age-related changes in the computational estimation performance of both young and older children (e.g., Dowker, 2003; Ganor-Stern, 2016; Gilmore, 2015; McCrink & Spelke, 2010; Sekeris et al., 2020), as well as differences in adaptability regarding strategy choices (LeFevre et al., 1993; Lemaire & Brun, 2016). In addition to examining the characteristics of individuals, such as differences in working memory updating (Hammerstein et al., 2021) and confidence levels in estimation success (Yang & Sianturi, 2020), the researchers also investigated specific features of the arithmetic task involved in a computational estimation. These included factors such as number size and the arithmetic operations involved, which were found to influence estimation performance. Sekeris et al. (2019) provide a systematic literature review on computational estimation that describes its development as well as the age-related changes observed from kindergarten to the end of primary school.

Computational Estimation and Mathematical Abilities

Computational estimation highlights an alternative approach to understanding and manipulating numbers, thereby facilitating the development of novel concepts associated with numerical comprehension. Therefore, its beneficial effects and its correlation with various mathematical abilities, such as mental calculation and number sense, have underscored its significance and influence on mathematical knowledge (Dowker, 2003; Sowder, 1992).

In particular, strong positive associations have been identified between the ability to estimate and numerical and computational skills (e.g., Booth & Siegler, 2008; LeFevre et al., 1993; Sekeris et al., 2021), including place value understanding. When a complex arithmetical task is needed or when a rough computational result is the objective, there is a significant reliance on computational estimation, which is considered as a perceptual process that enables an approximate interpretation of the situation. This interpretation is evidently less accurate than the exact calculation and requires a form of translation between quantitative representations to occur: this ability to translate a complex arithmetical task into an estimated outcome reflects our emphasis on "the meaning of the numbers and the operations" (van de Walle, & Lovin, 2006, p. 125). It seems that successful estimations necessitate experience with numbers and counting, which in turn enable estimators to approximate and verify answers to arithmetic tasks, as well as to gain familiarity with large magnitudes that exceed their current ability. Hence, the ability for computational estimation relies heavily on the development of number sense.

The errors observed in both children and adults concerning place value, which are most prevalent in estimates of results of multiplication and division problems, confirm the significant relationship between estimation and the understanding of numbers and operations. For example, Ganor-Stern and Siegler (2004) cited in Ganor-Stern (2016) report that children, especially the youngest, often make place-value errors, such as incorrectly stating that multiplying 85 by 43 yields an estimate of 320. In fact, their research found that 52% of 6^{th} graders, 28% of 8^{th} graders, and only 9% of adults made such errors in multiplying two-digit numbers. While such findings undoubtedly demonstrate a developmental shift in computational estimation accuracy, they also reveal deficits in conceptual and procedural knowledge. It is also possible that place-value errors stem from weaknesses in working memory. Case and Sowder (1990) and Seethaler and Fuchs (2006) suggest that individuals who excel in computational estimation typically exhibit greater working memory capacity compared to those with deficient computational skills.

The fact that we make estimates when we truly need them highlights the broader significance of estimation in mathematics and problem-solving. Reys et al. (1982) had long recognized that computational estimation is significantly linked to problem-solving, primarily in terms of computational speed. Adolescents and adults who participated in their research and demonstrated a high capacity for quick and efficient mental computation were also found to be proficient estimators, exhibiting flexibility in employing computational estimation strategies. Other studies have found that competence in computational estimation serves as a robust indicator of mathematics achievement, problem-solving abilities, and reasoning skills, particularly across grades six through eight (Foegen & Deno, 2001). While these studies are dated, we acknowledge that their findings remain important alongside new data. For example, in a recent study Desli and Lioliou (2020) found a highly positive correlation between the success of year 6 children and adults in computational estimation tasks (CETs) and their performance in PSTs. The participants who demonstrated a high success rate in estimations were also those who showed a high success rate in problem-solving. This finding confirmed their initial hypothesis that the solver not only modifies and adapts numbers in the problem but also reflects on the type of adjustments before arriving at a reasonable estimation. Similar findings have been reported regarding the relationship between mental computation and success in PSTs (Gürbüz & Erdem, 2016; Kasmer & Kim, 2011; Kindrat & Osana, 2018). In general, these connections might imply that the development of mental computation and computational estimation fosters a deeper understanding of number properties and number sense (Sowder, 1992), thereby enhancing mathematical reasoning. It is highly valuable to delve deeper into computational estimation, given its potential for advancing knowledge and skills in problem-solving. Nevertheless, additional research is needed to substantiate this hypothesis.

The Current Study

In this paper, we generally aimed to further unravel the relationship between computational estimation and problem-solving. We hypothesized that success in CETs would be highly correlated with success in PSTs, whereas less successful estimators would be less able on the cognitive demands of the problems (partly because participants can rely on procedural knowledge rather than the conceptual knowledge required in both estimations and problems).

We focused on slightly younger primary school children attending year 2 and year 4 to complement previous studies that employed a similar research approach in older children or adults (Desli & Lioliou, 2020) and to get a better understanding of the relationship between computational estimation performance and other basic numerical and mathematical abilities in children. Although this relationship has mostly been investigated with a focus on mathematical reasoning using a variety of measures, the present study examined computational estimation while using a measure of problem-solving achievement that could be completed by young children at the beginning and middle of primary school for direct comparison purposes.

On these grounds, the main aim of the present study was to explore the relationship between computational estimation and problem-solving, as drawn in the following research questions:

1. How do year 2 and year 4 children perform computational estimations?
2. What is children's problem-solving performance?
3. Is there a relationship of computational estimation performance to problem-solving success?
4. Does this relationship differ by age?

METHODOLOGY

Participants

A cross-sectional sample of 94 primary school children (47 year 2 children [24 boys, mean age = 7.78 years, standard deviation (SD) = 0.36] and 47 year 4 children [22 boys, mean age = 9.85 years, SD = 0.42]) was assessed on computational estimation and PSTs. Children were randomly recruited from five public primary schools located in different cities of Greece in an attempt to achieve greater geographical representativeness as well as variation in their socioeconomic and academic performance characteristics. The study utilized convenience sampling to select the schools, taking into consideration to include both urban and rural classes. Responses from five children with learning difficulties (e.g., attention deficits, dyscalculia, dyslexia) were removed from the data, as they would result in data distortion. All children participated voluntarily and were included in the sample only after their parents and teachers provided a signed informed consent form. It is worth noting that the students had not received specialized instruction in either computational estimation or problem-solving, but only what their classroom mathematics curriculum provided. However, the words ‘about’ and ‘approximately’ used in the CET may have been familiar in other social settings.

Data Collection–Research Instruments

Two tasks, one for computational estimation (CET) and one for problem-solving (PST), were designed and administered to all participants in one of two counterbalanced orders. The total number of trials was 16, equally coming from both tasks.

In CET, participants were asked to perform computational estimations mentally without using any means (paper and pencil) and without making accurate calculations. This task (**Appendix A**) consisted of eight trials, two for each arithmetic operation (addition, subtraction, multiplication, division), in order to test whether success in computational estimation is affected by the type of operation. More specifically, for each trial, participants were first given an arithmetic operation and three possible answers and then asked to indicate the answer they thought was closest to the actual result of the arithmetic operation. The three response options given to participants were designed based on the exact result of the arithmetic operation to ensure that a response option would include an estimate that was very close to the exact result, one that was less close, and one that was far from the exact result. For example, in the trial to estimate the result of the arithmetic operation “49 + 56”, the following answers were given, as options: (a) about 10, (b) about 80, and (c) about 100, the last of which is very close to the actual result. In contrast, the first two response options are more or less far from the exact result of the arithmetic operation, respectively. The estimates that had the smallest deviation among the three response options from the exact computational result were considered successful. The total number of successful estimations was used as an indicator of participants’ computational estimation ability. One-digit and two-digit numbers were chosen, while the addition and subtraction items were all with carrying (e.g., 28 + 39, 61 - 27). The Cronbach’s alpha coefficient of internal consistency for this task was .87.

The PST consisted of a total of eight trials (**Appendix B**) and required participants to solve word mathematical problems accurately, all coming from five specific mathematical topics: arithmetical operations (applying addition, subtraction), geometry (calculating the perimeter), statistics (reading a table), measurement (calculating time, weight) and patterns (finding number, image/shape patterns). These problems demand consideration of the reality of the situation described and many of them cannot be solved by using straightforward arithmetic operations. Furthermore, in half of the trials the problems could be solved in more-than-one ways, leading though to one solution (e.g., “Helen is 8 years old and her brother, Nikos, is 14 years old. When Helen is 10 years old, how old will Nikos be?”). All problems referred to real-world situations and their difficulty corresponded to the cognitive level and the curriculum requirements. Similarly to the CET, in the PST each trial presented a problem and three response options, of which only one was exact and therefore correct. The total number of correct responses were used as an indicator of performance on mathematical word problem-solving. The internal consistency of the task was high (Cronbach’s alpha = .92).

To avoid order effects in participants’ responses (e.g., fatigue), participants were randomly and equally divided into two different groups (group A and group B), depending on the order in which the tasks were presented. More specifically, participants in group A were first presented with the CET and then the PST, while in group B the tasks were presented in reverse order. This ensured that the potential poor or high performance of participants on a certain task would not be a result of the order in which the tasks were presented. However, the order of the trials in each task was the same for all participants. Finally, from a methodological point of view, it would have been preferable to include more trials in each task, but the attention span of young primary school children is limited compared to older children.

Procedure

Data were collected in the spring of 2023 in a session of approximately one hour (45 minutes) during school hours. Both tasks were administered groupwise and participants were asked to complete them individually. Children were assured that their scores in the tasks would not count in terms of their mathematics course grade. Participation in the study was completely voluntary and anonymous.

Prior to presenting the tasks, examples were given for each task to avoid misunderstanding the task requirements. The instructions and the presentation of the tasks were read aloud. All participants were then given trials on paper on which they could display their answers (by circling one of the three response options). No feedback was provided to the children regarding the correctness of their responses for either practice or experimental trials in both tasks. Participants were also told that there are no time constraints, and they can work at their own pace.

Table 1. Descriptive statistics for CET and PST

Task	Trials	Mean	Standard deviation	Minimum	Maximum
CET–Task 1	Addition	1.64	.565	0	2
	Subtraction	1.52	.668	0	2
	Multiplication	1.41	.646	0	2
	Division	1.16	.752	0	2
	Total in CET	5.73	1.718	1	8
PST–Task 2	Arithmetical operations	1.64	.602	0	2
	Geometry	.62	.489	0	1
	Statistics	.72	.450	0	1
	Measurement	1.72	.537	0	2
	Pattern	1.74	.527	0	2
	Total in PST	6.44	1.644	2	8

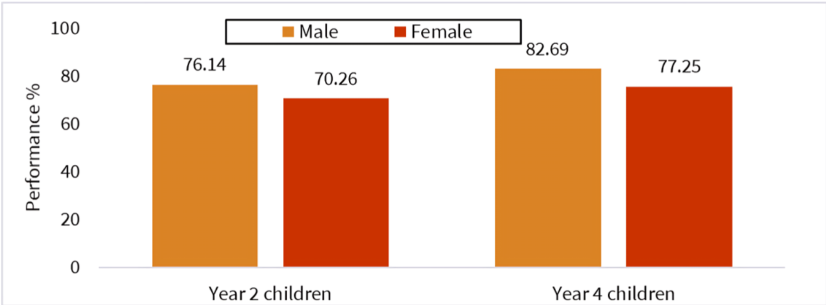

Figure 1. Percentage of total correct responses by age and gender (Source: Authors' own elaboration)

Data Coding

For the statistical analysis presented in this paper, 1 point was attributed if a participant gave a correct response, whereas unsuccessful responses were assigned 0 points. The points were summed to obtain a total score, which could reach a possible of 16 points. The data were analyzed using statistical package for the social sciences (SPSS 29). Since normally distributed data were observed, descriptive statistics (means, standard deviations) and comparisons with parametric tests (analyses of variance, Pearson's correlation analyses) were performed.

RESULTS

Main Results

In **Table 1**, an overview is given of the mean scores with standard deviations and minima and maxima for the tasks used in this study. It is evident that there was a good variance in performance on each task, with no evidence of floor or ceiling effects. Below we first explore performance on each task in more detail before proceeding to consider the relationship between computational estimation and problem-solving.

The overall performance of participants in the total of trials was remarkably good, with success rates exceeding 76% for both age groups. However, as depicted in **Figure 1**, significant age differences were observed between the two age groups (t [92] = -1.398, $p < .01$), with success rates close to 73% for second graders and 79% for fourth graders. No statistically significant gender differences were found in participants' performance (t [92] = 2.468, $p = .097$) with boys and girls offering similar percentages of successful responses. This finding was also revealed when the analysis was carried out separately for each age group (t [45] = 1.902, $p = .119$ and t [45] = 1.694, $p = .530$, for year 2 and year 4 children, respectively). Last, the order of task presentation did not affect children's performance (t [92] = 2.689, $p = .237$).

Rates of Correct Responses in Computational Estimation Task

Success rates for year 2 and year 4 children in the CET exceeded 68% and 75%, respectively. A mixed analysis of variance was conducted in order to detect the effects of age (year 2 and year 4 children) and gender (males and females) as the between-subjects factors, and type of arithmetic operation (additive [addition and subtraction] and multiplicative [multiplication and division] trials) as the within-subjects factor. The main term of the type of arithmetic operation was significant (F [1.90] = 1.772, $p < .001$), with all participants having better rates of correct responses when making addition and subtraction estimates than multiplication and division estimates. There was a significant main effect of age (F [1.90] = -1.646, $p < .01$), indicating that the fourth graders performed significantly better than the second graders. Neither significant gender effect (F [1.90] = .804, $p = .473$) was found, nor the interaction between age and type of arithmetic operation (F [1.90] = 1.042, $p = .723$) was significant, showing that differences in participants' performance among additive trials and multiplicative trials were not affected by age and gender: Both age groups performed significantly better when computing estimations with additive operations rather than with multiplicative operations.

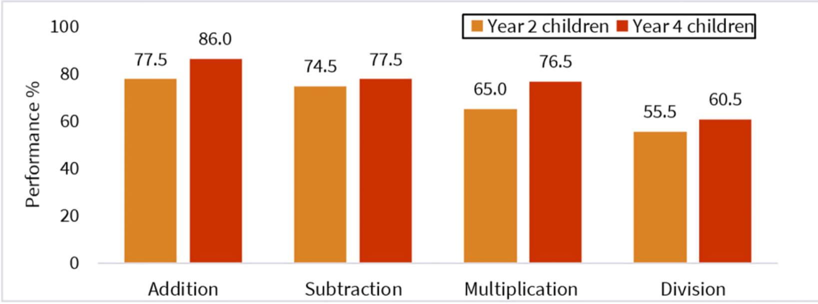

Figure 2. Percentage of correct responses by type of arithmetic operation and age in CET (Source: Authors' own elaboration)

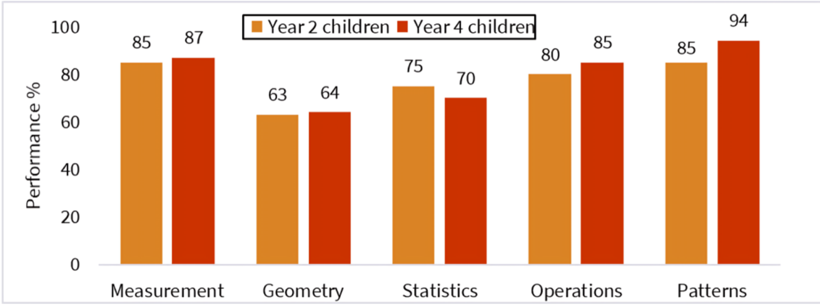

Figure 3. Percentage of correct responses by type of mathematical topic and age in PST (Source: Authors' own elaboration)

When further analyses were carried out for each arithmetic operation and for each age group separately, it was found that there were significant differences in the performance of participants between addition and multiplication trials ($t [46] = 2.885$, $p < .01$ and $t [46] = 1.771$, $p < .05$ for year 2 and year 4 children, respectively) and between addition and division trials ($t [46] = 3.301$, $p < .001$ and $t [46] = 5.636$, $p < .001$ for year 2 and year 4 children, respectively). However, no significant differences were found in the performance of participants between addition and subtraction trials ($t [46] = .503$, $p = .309$ and $t [46] = 1.594$, $p = .059$ for year 2 and year 4 children, respectively) nor between subtraction and multiplication trials ($t [46] = 1.543$, $p = .065$ and $t [46] = .158$, $p = .437$ for year 2 and year 4 children, respectively). In other words, all participants performed best on the addition trials (77.5% and 86% for children in year 2 and year 4, respectively). However, year 2 children had difficulties in the multiplication and division trials (65% and 55.5%, respectively), while the division trials were the ones in which year 4 children performed the lowest (60.5%). **Figure 2** shows the interaction between age and type of arithmetical operation.

Moreover, it is worth noting that even if participants did not choose the answer with the smallest deviation from the actual result of the arithmetic operation, they tended not to choose the answer with the largest deviation. For example, in the addition "49 + 56" few participants ($N = 10$) chose the answer "about 80" instead of "about 100", and none chose the answer with the very large deviation from the exact result ("about 10"). However, this was not the case in the more difficult trials, such as the division "64 ÷ 15", where children were almost evenly split across the three response options (22 children chose "about 10" as the correct answer compared to 37 and 35 children who chose "about 4" and "about 6", respectively).

Rates of Correct Responses in Problem-Solving Task

Preliminary statistics analyzed success in participants' responses in trials of PST and indicated accuracy of 77% or better even for the youngest age group.

The effect of age and type of mathematical topic on children's problem-solving performance was examined by a 2 (grades) × 5 (mathematical topics) Analysis of Variance with repeated measures on the type of mathematical topic. The results failed to yield a significant difference of age ($F [1.90] = .751$, $p = .184$) in overall problem-solving scores, with children in both age groups performing similarly. However, a statistically significant main effect of the type of mathematical topic was found ($F [1.90] = 5.836$, $p < .01$), indicating that the participants performed significantly worse in the geometry and statistics trials (about 67%) than in the arithmetical operations, measurement and pattern items (about 82%, 86% and 87%, respectively). More specifically, performance in the measurement problems was similar to the arithmetical operations problems ($t [93] = 1.209$, $p = .115$) and to the pattern problems ($t [93] = -.293$, $p = .385$). Children's performance was also similar between the arithmetical operations problems and the pattern problems ($t [93] = -1.452$, $p = .085$). In addition, their performance on the geometry problem was significantly better than in the statistics problem ($t [93] = -1.849$, $p < .05$), however, these were the two types of problems in which children had significantly the poorest performance than any other type of problem ($p < .001$).

The interaction type of mathematical topic by age was not significant ($F [1.90] = 2.264$, $p = .129$). Further analyses showed that both year 2 and year 4 children performed lower in geometry problems (63.8% and 59,6%, respectively) compared to their rates of success in operations, measurement and pattern problems ($t [46] = -1.478$, $p < .05$, $t [46] = 2.920$, $p < .01$ and $t [46] = -1.592$, $p < .05$, for year 2 children, respectively/$t [46] = -3.953$, $p < .001$, $t [46] = 3.806$, $p < .001$ and $t [46] = -5.511$, $p < .001$, for year 4 children, respectively). Participants' performance in the five types of mathematical topics by age is presented in **Figure 3**.

Table 2. Pearson's correlations between CET and PST by age

Task	Age group	Overall achievement	CET	PST
CET–Task 1	Year 2 children	.891**		.543**
	Year 4 children	.815**		.374**
	Total	.867**		.481**
PST–Task 2	Year 2 children	.865**	.543**	
	Year 4 children	.842**	.374**	
	Total	.854**	.481**	

Note. **p < .01

Computational Estimation and Problem-Solving

Finally, Pearson's correlations were used to explore the relationship between children's computational estimation accuracy and their problem-solving ability, as assessed in the two tasks used. A significant positive correlation was revealed (**Table 2**) between computational estimation scores and problem-solving scores (r = .481, p < .01), reflecting that participants with a high level of computational estimation also reveal a high level of problem-solving. When this relationship was examined at each grade level, there was the same significant correlation at the 2nd and 4th grade (r = .543, p < .01 and r = .374, p < .01 for year 2 and year 4 children, respectively), confirming the initial strong correlation. Additional statistical analyses showed that better problem-solving abilities were strongly related to significantly fewer and less variable computational estimation errors when children's performance was controlled for each arithmetic operation trial (r = .350, p < .01, r = .226, p < .05, r = .208, p < .05 and r = .455, p < .01, respectively for addition, subtraction, multiplication and division trials in CET). This finding indicates that when successful estimates were made on each of the four arithmetic operation trials, problem-solving scores increased and vice versa.

Participants' computational estimation scores as well as problem-solving scores were also correlated with overall achievement (r = .867, p < .01 and r = .854, p < .01, respectively), meaning that participants who scored highly in each measure tended to obtain high total rates of success (**Table 2**). These positive correlations were also confirmed with regards both to year 2 children (r = .891, p < .01 and r = .865, p < .01, respectively for CET and PST) and year 4 children (r = .815, p < .01 and r = .842, p < .01, respectively for CET and PST).

DISCUSSION

The present quantitative study aimed to investigate the relationship between computational estimation ability and problem-solving ability in a sample of year 2 and year 4 children. Building upon previous research that has identified a strong relationship between these abilities in older children and adults, our interest lay in determining whether such a relationship exists in younger participants. This inquiry is motivated by the recognition of the importance of problem-solving within the mathematics curriculum and by the interest in examining the accuracy of computational estimations and their potential in young children. Two main findings were revealed.

Firstly, children from an early age demonstrate significant computational estimation ability when presented with simple arithmetical operations involving one- and two-digit numbers requiring estimation. The high percentage of correct performance (over 75%) shown by the 7- and 9-year-old children indicates their advanced cognitive abilities in both producing estimates and evaluating their reasonableness. Especially concerning the latter point, it is interesting to note that children's estimates closely aligned with the correct answer. Specifically, when presented with three answer options, they often chose the option with the smallest deviation from the actual result of the arithmetic operation. Such highly demanding abilities, previously recognized in older age groups (e.g., Case & Sowder, 1990; Siegler & Booth, 2005) but increasingly observed in younger age groups (Ejersbo, 2016; Sekeris et al., 2019, 2020), are likely developed through the frequent necessity for computational estimations in familiar everyday situations. This finding suggests that, despite estimations recently becoming an integral part of primary school mathematics curriculum in Greece and other countries and recognizing computational estimation as a determinant of later arithmetic competence (Holloway & Ansari, 2009), introducing children to computational estimation could commence as early as the beginning of primary school.

Secondly, young children's successful computational estimation performance was highly correlated with their achievement in mathematical problem-solving. In essence, participants with a high level of computational estimation also demonstrated a high level of problem-solving ability. This finding regarding the strong relationship between computational estimation and problem-solving corroborates earlier research, which suggested that solving mathematical problems involves not just manipulating numbers but also reflecting on the nature of those manipulations before arriving at a reasonable estimate (McIntosh, 2005). On this basis, Kasmer and Kim (2011), Gürbüz and Erdem (2016), and Kindrat and Osana (2018) subsequently identified that mental computation and mathematical reasoning are interconnected and should be collectively acknowledged, given the recognized prominent role of mental mathematics in the mathematics classroom. Moreover, the existence of a relationship between computational estimation and problem-solving in young children, akin to that observed in older children and adults by Desli and Lioliou (2020), may also imply a developmental pattern among young students, as there was an increase in computational estimation accuracy with age. The increase in performance was consistent across participants, particularly evident in their improved performance on addition and subtraction estimations compared to multiplication and division estimations. Similarly, problem-solving trials involving Geometry and Statistics were found to be the most challenging for both age groups, while all children excelled in pattern trials. These findings align with cognitive development theories, indicating that problem-solving abilities vary among individuals and progressively become more complex and sophisticated with each advanced stage. It is possible that, consistent with these theories, the relationship between problem-solving and computational estimation could be strengthened through the development of instructional strategies that facilitate solvers' progression to higher cognitive developmental stages, reflecting more complex problem-solving demands. Therefore, recognizing the cognitive aspects of computational estimations is crucial for gaining a deeper understanding of mathematical concepts and for more effectively fostering problem-solving skills right from the beginning of mathematics learning.

The data from this study, confirming that young children who provide successful computational estimations tend to be competent on problem-solving processes and emerge as successful problem solvers, suggests implications for curriculum refinement as well as for improved teaching practices in this domain, especially in countries like Greece that are currently revising their mathematics textbooks or curricula. The curriculum for children in their first years of formal schooling could more effectively integrate computational estimation, addressing topics that are challenging and require emphasis, such as place-value and number properties. Introducing children to computational estimation situations early on, and encouraging mathematical reasoning, may indeed have a significant impact on their success as problem solvers. One approach to achieve this is by incorporating a variety of estimation tasks into mathematics textbooks and instructional materials, while emphasizing their connection to everyday situations. In addition, teachers can consider ways in which computational estimation learning experiences can be embedded in their practice.

Limitations and Future Directions

There are several potential limitations in interpreting the findings, which can guide future research directions. One limitation is that the findings in this paper are based on a small sample of young children from similar schools, which may restrict the generalizations drawn from the data. However, the data provided rich information about young children's computational estimations as well as their problem-solving skills. Another limitation was that, even though young children had not typically encountered computational estimation experiences before the study, it is likely that they had engaged in similar computational estimation content or learning during their classes. It would be beneficial to repeat the study before too much school-based teaching input occurs.

Since computational estimation reasoning may strong relate to children's problem-solving abilities, it is possible that understanding estimates may contribute to a child's ability to comprehend the relationships between numbers and develop effective problem-solving strategies. As supported by Sekeris et al. (2019), such findings merely reveal the relationship, and further research is required to gain a better knowledge of the role of computational estimation in problem-solving. To this end, it is of utmost importance to understand children's strategies in computational estimation situations as they are likely to have an impact on problem-solving learning environments.

By expanding the scope of this study, the cognitive demands of other types of estimation situations, such as measurement estimation or quantity estimation, can be investigated in relation to problem-solving skills. By fully understanding the demands of estimation, teaching and learning practices can be modified to better support young students in problem-solving and, more broadly, in their mathematics learning. Comparisons can also be made by applying the available data to students in the same age group across different countries, thus enabling the determination of whether socio-cultural factors or variations in the mathematics curricula have an effect on these relationships.

CONCLUSION

A strong positive correlation was identified between children's accuracy in computational estimation and their problem-solving ability: those who excelled in computational estimations tended to demonstrate high proficiency in problem-solving as well. This finding held true for both second and fourth graders, highlighting a robust relationship between these two abilities from an early age.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Table A1. CET

Items			
A. 49 + 56 =	B. 28 + 39 =	C. 61 - 27 =	D. 88 - 22 =
a) approximately 10	a) approximately 50	a) approximately 30	a) approximately 20
b) approximately 80	b) approximately 70	b) approximately 50	b) approximately 60
c) approximately 100	c) approximately 100	c) approximately 90	c) approximately 80
E. 19 × 3 =	F. 33 × 2 =	G. 82 : 10 =	H. 64 : 15 =
a) approximately 30	a) approximately 60	a) approximately 4	a) approximately 4
b) approximately 40	b) approximately 80	b) approximately 8	b) approximately 6
c) approximately 60	c) approximately 100	c) approximately 10	c) approximately 10

APPENDIX B

Table B1. PST

Items

A. Helen is 8 years old and her brother, Nikos, is 14 years old. When Helen is 10 years old, how old will Nikos be?
a) 12 b) 16 c) 20

B. Kostas weighs 48 kilograms. His bag weighs 4 kilograms. How many kilograms did the scale show when Kostas stepped on it while holding his bag?
a) 42 b) 46 c) 52

C. Eugenia is making a square card and wants to decorate it all around with ribbon. If one side of the card measures 15 centimeters, how much ribbon will Eugenia need for her card?
a) 30 b) 50 c) 60

D. At school, the children were asked where they would like to go on a field trip. These were their answers. How many children in total voted for the two most popular destinations?

Cinema	34
Theatre	18
Sports stadium	27

a) 35 b) 61 c) 79

E. A football team has 30 children. After a week, 11 children left the team, and 2 new children joined. How many children are in the team now?
a) 17 b) 21 c) 43

F. Margaret has 50 markers. She gives each of her two brothers 10 markers. How many markers does she have left?
a) 20 b) 30 c) 40

G. Which number comes after the following number pattern? 3, 10, 17, 24?
a) 30 b) 31 c) 35

H. Which shape comes after the following pattern of shapes? ● ● ▲ ■ ●?
a) ● b) ▲ c) ■