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MODESTUM

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From friction to air resistance

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ABSTRACT

Friction occurs early in our lives. We rub our hands and stamp our feet to get them warm. Another effect of friction is the use of winter tires at cars, new bicycle tires, or special shoes for icy surface. This article discusses how to present the way from friction to air resistance through GeoGebra applets. The approach has been analyzed using the theory of technology as amplifier and reorganizer, theory of variation, and the notion of internal and external representations. The dragging feature of GeoGebra enables students to experience variation and make conjectures. Furthermore, students' static internal representations will be enhanced to dynamic external representations, which can play an important role in students' learning. Three aspects of fidelity have been discussed with regard to the use of GeoGebra based applets and how such applets need to be critically designed and assessed in order to support students' learning.

Keywords: air resistance, applets, friction, free fall, GeoGebra

INTRODUCTION

In everyday life, friction is seldom seen as involving forces and not seen as being present if things are not moving (as for a child sitting still halfway down a slide). Hence, friction is an unclear influence in everyday experiences of motion since people are often not aware that friction includes force.

Friction is probably caused by interactions between tiny parts on surfaces when they rub against each other. The surfaces twist and use force on each other making it difficult for the surfaces to slip over each other. The force on the moving surface is in the opposite direction and that force resists the motion.

If we consider the surface of a car tire designed to maximize the friction between the tire and road surface in different weather conditions. If the friction is reduced because of snow or ice on the road, the tire slips and becomes unusable to help steer, drive or break the car. The friction becomes too small.

Friction can slow things down and stop stationary things from moving. In a frictionless world, new objects would be sliding, shoes and caps would be difficult to keep on and it would be difficult for people to steer cars to get moving or change direction. Some tasks we do easily because of friction and some tasks are perhaps impossible without frictions such as walking, surfing, or skiing.

The idea that surfaces are uneven if we look close enough (think microscopic) provides a useful model for explaining cause and effect of friction. The idea of surfaces causing friction against each other is why applying oil to the surfaces can reduce friction and allow them to move more freely.

There are many examples where too little friction can present safety issues, such as wet or icy roads, wax on surfboards, smooth shoes, rubber gloves and handle grips and where too much friction means that moving objects do not work very well such as moving parts in bicycles, rollerblades, skateboards, door locks, and gate hinges. We often need more friction for some parts of the world and less friction for other parts of the world.

In physics, friction is often introduced as a force. If a minor ride in the slide in **Figure 1**, we can attach forces to him or her. As of copy right issues we insert a schematic image of a minor in **Figure 2**.

Any moving object in a slid has a gravity force acting on the object. Otherwise, the child would not come down. There will also be a normal force from the slide pressing up to the child. There will most likely also be some friction force holding down the velocity in the slide. Often, we seek for the gravity center of an object to attach our forces there. This is difficult with a schematic view of a child. I allowed the forces to go from the left point of the image.

Figure 1. Empty slide

Figure 2. Slide with a schematic image in it

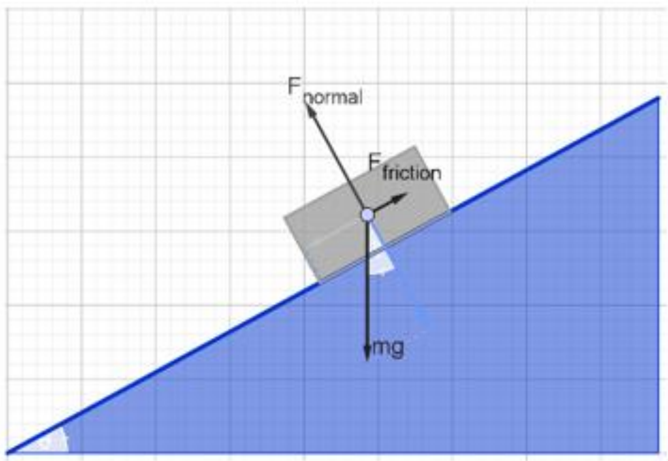

Figure 3. Box on an inclined plane

In this case it is easy to imagine friction as a force holding down the velocity. Let us now consider this situation from a physics standpoint. Let us study a box in an incline plane. To start with, we have a box and while it is on the inclined plane there are three forces action on the box. We have gravity, we have a normal force, and we have friction (**Figure 3**).

As we can see, we have the arrows for the normal force and the gravity force pointing in different directions. Furthermore we can let the gravitational force be divided into one *x*-part and one *y*-part of the gravitation. One way to do that is by a trigonometric spit as in **Figure 4**.

Here we have divided the vector mg into two components $mg \cdot sin(\theta)$ and $mg \cdot cos(\theta)$.

The *x*-split is downward parallel to plane.

The *y*-split is downward perpendicular to plane.

The F_{normal} is equal to the mg·cos(ϴ). What causes the box to slide along the plane is the *x*-component of mg=$mg \cdot sin(\theta)$.

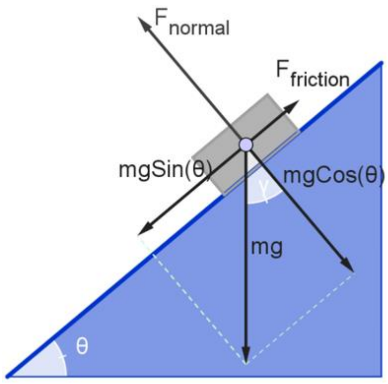

Figure 4. Box with split forces on an inclined plane in GeoGebra

In order for the box to slide, the $mg \cdot sin(\theta)$ must be at least equal to $F_{friction}$. We know that $F_{friction}$=$\mu \cdot F_{normal}$, but F_{normal}=$mg \cdot cos(\theta)$. $F_{friction}$=$mg \cdot sin(\theta)$; thus, $mg \cdot sin(\theta)$=$\mu \cdot\ mg \cdot cos(\theta)$, from where we get that $\mu = \tan{(\theta)}$.

The friction between two bodies is never the same when resting and moving. The force required to make a body start to slide is greater than the force required to end its sliding. Therefore, we distinguish between static friction and movement friction. Static friction occurs when two bodies do not move in relation to each other. The coefficient of friction in static friction μ is calculated by $\mu = \frac{F_{friction}}{F_{normal}}$.

Movement friction occurs when two bodies move relative to each other. A body begins to move if the force F_{move} is greater than the friction force $F_{friction}$.

The friction coefficient μ can be said to be measured on how rough different surfaces can be against each other. The coefficient of friction for wood-to-wood, is not equal to the coefficient of friction between steel and ice. In practice however, there are no completely frictionless conditions. When talking about air friction, we do not call it friction but resistance, air resistance.

THEORETICAL FRAMEWORK

DGE as Amplifier and Reorganizer

There are two metaphors frequently used about the use of technology in education, namely that of technology being an ‘amplifier’ and a ‘reorganizer’ of mental activity (Pea, 1985, 1987). A DGE also plays the role of and ‘amplifier’ and a ‘reorganizer’ in enhancing students’ physical thinking. The term ‘amplifier’ refers to the fact that the technology performs tedious computations (which are time consuming to do by hand) quickly and accurately e.g., generating a table of values or a set of graphs. Thus, students can focus on making observations and developing insight rather than be bogged down by manual procedures. In this mode, the tool does not change student’s thinking but rather facilitates their explorations. On the other hand, when technology is used as a reorganizer, it has the power to extend students’ thinking by giving them access to higher level processes. For example, a DGE supports looking for patterns, identifying invariances or making and testing conjectures. However, in a paper-pencil environment students spend a significant amount of time on drawing and measuring objects only. The theoretical frameworks related to Pea’s theory of technology as amplifier and reorganizer (Pea, 1985, 1987), will be used to analyze the teaching sequence of free fall and the physical objects relation to the GeoGebra applets.

Internal and external representations

There is a close connection between the way we develop our understanding of mathematical or physical objects and our internal concepts related to them. Learning takes place from the early years as we notice *external representations* of objects and use them to construct our own *internal representations* of the same objects. The term prototype or mental image is often associated with the formation of these internal representations.

Internal representations can be formed from concrete objects or from verbal cues by an adult or a peer. Showing a slide picture frame may help a child develop an internal representation of the concept of ‘slide’. Similarly, describing a slide as having ‘a slope’ is a verbal clue which may lead to forming an internal representation. Thus, to form a holistic internal representation of a slide, one needs to experience it in different forms. Dienes (1963), in his variability principles propounded the benefits of presenting different representations of physical objects:

> The *perceptual variability principle* states that to abstract a physical concept effectively one must meet it in a number of different situations to perceive its purely structural properties.
>
> The *physical variability principle* states that as every physical concept involves variables, all these physical variables need to be varied if the full generality of the physical concept is to be achieved.

Internal representations comprise of ideas or mental images, which we can refer to mentally. They help us communicate ideas about physical objects and concepts, even abstract ones. In physics, initially it is hard to imagine or visualize the concept of velocity. Later we learn to represent velocity as the slope of a graph over displacement versus time.

Vygotsky (1978) referred to internal and external representations as external tools and internal signs, and wrote about the mental transformation between the external and internal. According to Falcade et al. (2007, p. 321),

> But the link between tools (externally oriented) and signs (internally oriented) goes beyond pure analogy in their functioning and rests on the real tie that can be recognized between particular tools and particular signs. One could say that externally oriented tools may be transformed into internally oriented tools.

While attempting to learn about a physical situation, we elicit both external and internal reactions. Our external reactions are related to exploring and solving the problem, drawing diagrams to communicate our ideas, and providing verbal explanations. All these are in turn related to our mental images and internal representations.

The cognitive process of transitioning from an external to an internal representation of the verbal material is called building a *verbal representational connection* (or verbal encoding), (Mayer & Sims, 1994). This entails describing our thoughts to another person using a visual explanation such as a sketch. For example, we could say "an object in free fall only depends on the gravitational force *g*" and perhaps sketch a situation on paper. Such an explanation will make it possible for another person to construct a mental representation of the visually presented system.

Other than the *verbal representational connection*, the cognitive process of going from an external to an internal representation of visual information is called building a *visual representational connection* (or visual encoding). An example could be that we throw stones of a cliff down into water and visually experience the idea of free fall. These stones are not as perfect as the objects we have in our mind and they do experience air resistance but nevertheless we call them objects in free fall.

Finally, we need to construct referential connections between the two mental representations (verbal and visual), that is, the mapping of a *structural relation* between the two representations. For example, building referential connections involves noting that a verbal statement such as "an object in free fall" is analogous to a static image or an animation. Vygotsky (1978) referred to this process as internalization as described by Falcade et al. (2007, p. 321),

> In summary, a Vygotskian perspective may explain the contribution of tool mediated action to concept formation: signs generated in relation to the use of a tool, through the complex process of internalization accomplished after social interchange, may shape new meanings. In this respect, a specific tool may function as a semiotic mediator. At first, externally oriented, a tool is used in action to accomplish a specific task, then, within semiotic activities under the guidance of an expert (for instance, the teacher), the articulation of new signs, generated by (derived from) actions with the tool, may foster an internalization process producing a new psychological tool. This new tool is internally oriented, completely transformed, but still maintains some aspects of its origin.

Thus, it would be apt to say that learning physics hinges on constructing internal representations of concepts and on developing referential connections between these representations. On one hand, our internal representations enable us to identify, recognize or interpret the different external representations encountered by us. External representations, on the other hand, are physically embodied observable configurations, interpreted by us as belonging to structured systems and their representing relationships. As we build connections between our internal representations, we learn to use them and more flexibly. Developing this flexibility between different forms of representation is an important aspect of learning physics.

Visualization and DGE: A Key Aspect in Developing Thinking

Visualization is a fundamental process in the understanding and construction of physical concepts. This is especially true in the case of functions. By allowing the user to drag and manipulate objects, a dynamic geometry environment (DGE) facilitates visualization and conjecture formation. It transforms the possibilities for representation and has a great impact on the conceptualization of objects and internalizing their meanings (Falcade et al., 2007; Moreno-Armella et al., 2007). The contribution of technology in teaching and learning of is perceived as strongly linked with dynamical interactive graphical representations (Laborde et al., 2006).

In the traditional classroom, a variety of representations, such as diagrams, drawings, and graphs are used for teaching physical concepts. The use of such multiple representations facilitates and enhance student's understanding of physical concepts. Traditionally, physics is taught and learned in a pencil and paper environment (using formulas and drawing for constructions) and textbooks provide iconic illustrations. A conceptual understanding of physics however requires the development and flexibility of mental imagination. Textbooks with static diagrams are not able to highlight the dynamic nature of figures over physical situations.

In a DGE figures and shapes can be manipulated using the dragging feature and this provides a dynamic opportunity to the learning of physics. It allows the user to perform investigations and thus affords the possibility of a dynamic visual representation of concepts in a physical sense. Such investigatory activities are hard to experience in a static environment such as paper and pencil (González & Herbst, 2009).

A DGE can provide ample opportunity for exploring geometrical figures and constructions through dragging. According to Marrades and Gutiérrez (2000, p. 96),

> DGS helps teachers create learning environments where students can experiment, observe the permanence, or lack of permanence, of mathematical properties, and state or verify conjectures much more easily than in other computational environments or in the more traditional setting of paper and pencil. The main advantage of DGS learning environments over other (computational or non-computational) environments is that students can construct complex figures and can easily perform in real time a very wide range of transformations on those figures, so students have access to a variety of examples that can hardly be matched by non-computational or static computational environments.

Theory of Variation: Dragging as a Tool in a DGE

Forming mental images is a critical step to abstracting a physical concept. This entails experiencing the concept in several diverse forms. For example, to help students abstract the concept of a free fall, the teacher may present stories and perhaps drawings of free fall, hoping that the student will 'see' some common features, among all the figures, namely, the gravitation force. In a DGE, the student may drag experience how we could change the value for the gravitation force and through this continuous process, be able to 'see' the properties of the free fall, which remain invariant and those that vary. For example, something that falls from 1000 meters fall faster or slower if we decrease or increase the gravitational force. Thus the dragging tool is perhaps the most powerful feature of a DGE as it allows the user to abstract an idea by observing properties of figures, which remain invariant during the process of variation. Leung (2003) aptly describes this affordance of a DGE

> … when engaging in learning activities or reasoning, one often tries to comprehend abstract concepts by some kind of "mental animation", i.e. mentally visualizing variations of conceptual objects in hope of "seeing" patterns of variation or invariant properties.

> … one of DGE's power is to equip us with the ability to retain (keep fixed) a background configuration while we can selectively bring to the fore (via dragging) those parts of the whole configuration that interested us in a learning episode.

Over the years many researchers have studied the role of dragging in DGE focusing on how it can be instrumental in helping students construct figures using their properties, explore physical problems, formulate conjectures, and even proofs. In particular, Arzarello et al. (2002) identified seven dragging modalities (wandering, guided, bound, dummy locus, line, linked, and drag test) while trying to analyze conjecture-making episodes by students working on a problem.

Marton and Booth (1997) proposed four inter-related functions of variation, which they also referred to as patterns of variation. These are

> *Contrast*: ''… in order to experience something, a person must experience something else to compare it with.''

> *Generalization*: ''… in order to fully understand what ''a graph'' is, we must also experience varying appearances of ''a graph''…''

> *Separation*: ''In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant.''

> *Fusion*: ''If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously.''

They advocated that variation and simultaneity play an important role in discernment of a concept. According to them, in order to discern a concept and to understand it completely, one must experience variations of it.

The three aspects of fidelity are mathematical, pedagogical, and cognitive fidelity of DGE based representations. While using technology for exploring mathematical concepts and problems, it is relevant to assess its pedagogical, mathematical, and cognitive fidelity. Zbiek et al. (2007, p. 1173) describe mathematical fidelity as

> "faithfulness of the tool in reflecting the mathematical properties, conventions, and behaviors (as would be understood or expected by the mathematical community)."

Let us consider the function $f(x)=(x^2-1)/(x-1)$. If a graphics calculator graphs this function, it may produce the linear equation $y=x+1$. This is inaccurate since the function $f(x)$ not is defined at $x=1$ and the correct graph of $f(x)$ should have a point break at $x=1$. Thus, the mathematical fidelity of the tool is compromised in relation to graphing of the function.

Zbeik et al. (2007, p. 1173) describe cognitive fidelity as

> "the faithfulness of the tool in reflecting the learner's thought processes or strategic choices while engaged in mathematical activity."

A tool has cognitive fidelity if the produced external representations match the user's internal representations and enhance their conceptual understanding. If appropriately used, a DGE has good cognitive fidelity.

The third kind of fidelity is that of pedagogical fidelity, which, according to Zbeik et al. (2007, p. 1187), is

> "the extent to which teachers (as well as students) believe that a tool allows students to act with a physics concept in ways that correspond to the nature of learning physic that underlies a teacher's practice."

Table 1. Height per second after a free fall from 1,000 meters

Time	Height	Time	Height	Time	Height	Time	Height	Time	Height
0	1,000	3	955.9	6	823.4	9	602.7	12	293.7
1	995.1	4	921.5	7	759.7	10	509.5	13	171.1
2	980.4	5	877.4	8	686.1	11	406.5	14	38.6

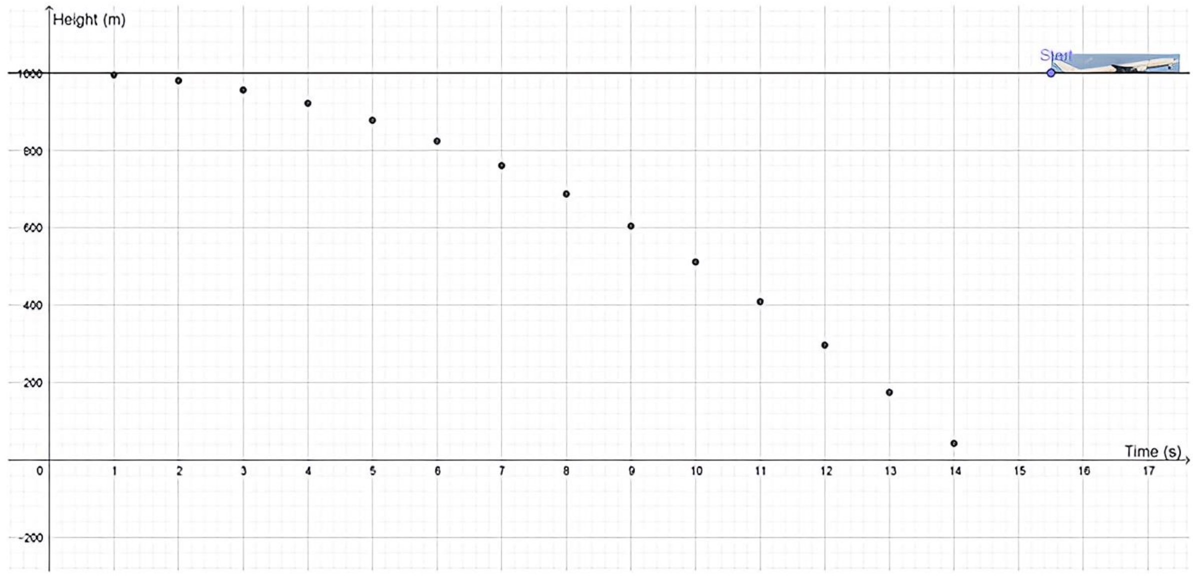

Figure 5. Free fall from a plane on the height of 1,000 meters

Pedagogical fidelity refers to the tool's ability to support students' explorations and learning. In a DGE, the dragging feature can afford this kind of fidelity. We can use sliders to vary the position of a box dropped from a plane and observe the change in the graphical representation of a function for the air resistance. The level and degree of the types of fidelity vary among technology tools and should be considered while selecting and evaluating appropriate tools for students' explorations. The aspects of fidelity must also be kept in mind while designing exploratory tasks for students.

The theoretical frameworks described above may be classified into two categories. Internal and external representations focus on understanding students' thinking and reasoning in a DGE environment, while Pea's (1985, 1987) theory of amplifier and reorganizer and the three aspects of fidelity are related to the design and positioning of DGE based geometrical applets for learning physics.

THE TRANSITION OF FRICTION INTO RESISTANCE

First, we note that friction is translated into air resistance when we are talking about air resistance and free fall. If we study a free fall from 1000 meters, we will have a distinct time frame of 0-14.29 seconds. Teachers and students need to accept that this value limits the coordinate system. Discuss the possible variation of this value. The gravitational force g is the usual gravity constant approximately equal to 9.8 m/s^2.

The formula is, as follows: $\text{distance}=\frac{1}{2} gt^2 \Leftrightarrow t^2=2 \cdot \text{dist} / g \Leftrightarrow t=\sqrt{(2 \cdot d/g)}=\sqrt{(2 \cdot 1000 / g)}\approx 14.2874$. We can now calculate our points by the formula $1{,}000-\frac{1}{2} gt^2$.

Secondly, we use a table for values (time, height) and thereby grounding for points in a coordinate system. We use one decimal (**Table 1**).

In relation to student's experiences, it might be good to mention that a table of points almost always correspond to and prepare the ground for the visualization of the values as points in a coordinate system. The coordinate system should be $0 \leq x \leq 15$ and $0 \leq y \leq 1{,}000$.

Thirdly, we go from external to internal representation by points in a coordinate system. The points show a descending situation (**Figure 5**).

The fourth step is to show values for every point and thereby amplify the internal representation as in **Figure 6**.

The fifth step is to show a graph through the points and thereby amplify the representation (**Figure 7**).

The sixth step is amplification by varying the gravitation force in graph with points and regression curve (**Figure 8**).

The seventh step is that there is always air resistance if something is falling close to earth. Since it is a complicated concept, many problems in physics add the sentence: Omit the air resistance. We will do the opposite.

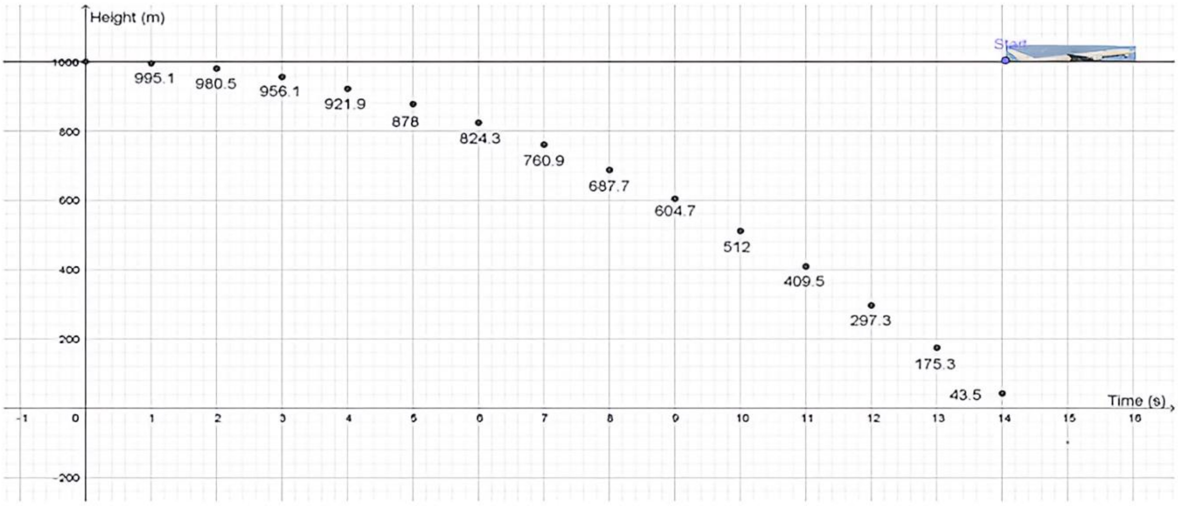

Figure 6. Free fall from a plane on the height of 1,000 meters with height numbers

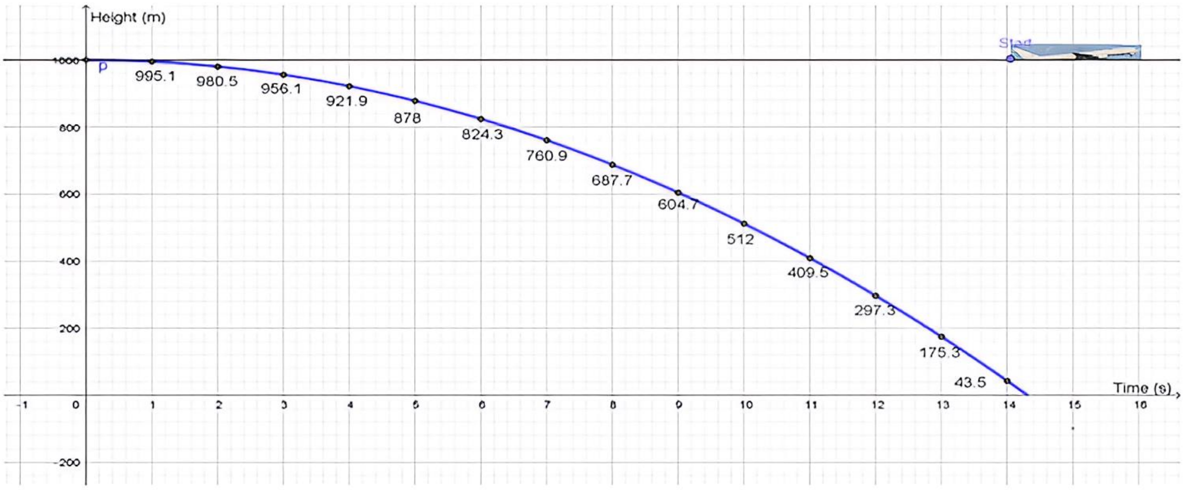

Figure 7. Free fall from a plane on the height of 1,000 meters with height numbers and a curve going through the points

Let us assume that an airplane is moving with a constant speed at the height of 1,000 meters above the Earth's surface. By some reason the plane drops a box. What will be the path of that box and where will the box be with respect to the plane? How can we describe the motion of the box?

By Newtonian laws the box will follow a parabolic path and always stay directly below the plane if we neglect air resistance. When the box falls, it undergoes a vertical acceleration, a change in its vertical velocity. This vertical acceleration is attributed to the downward force of gravity which acts upon the box. If the box's motion could be approximated as ideal motion (if the influence of air resistance could be assumed negligible), then there would be no horizontal acceleration. In the absence of horizontal forces, there would be a constant velocity in the horizontal direction. The horizontal motion of the box is the result of its own inertia. When dropped from the plane, the box already has1 a horizontal motion. The box will maintain this state of horizontal motion unless acted upon by a horizontal force. Newton's first law: An object in motion will continue in motion with the same speed and in the same direction.

An object in free fall above the surface of Earth is accelerated by a downward directed acceleration a=-g, where g is the usual gravity constant approximately equal to 9.8 m/s^2. All objects in free fall at the same location are falling with the same acceleration. If the falling object has an initial velocity equal to zero, the velocity down will increase for every second with 9.8 m/s (approximately about 35 km/h): v=- $g \cdot t$. The distance will increase quadratic for every second, d=-½ gt^2. The velocity will increase with $v=\sqrt{2gs}$.

If an object is dropped 1,000 meters up, will the box not have a totally free fall, since other forces, such as air resistance will affect the object. It will cause the downward acceleration to be smaller. Air resistance normally increase with the square of the fall velocity which cause that all falling objects affected by air resistance gradually reach a maximum velocity, sometimes called the limit velocity.

Figure 8. Variation of the gravity force for a free fall from a plane on the height of 1,000 meters (You find the GeoGebra file at https://www.geogebra.org/m/ygrqpw2f)

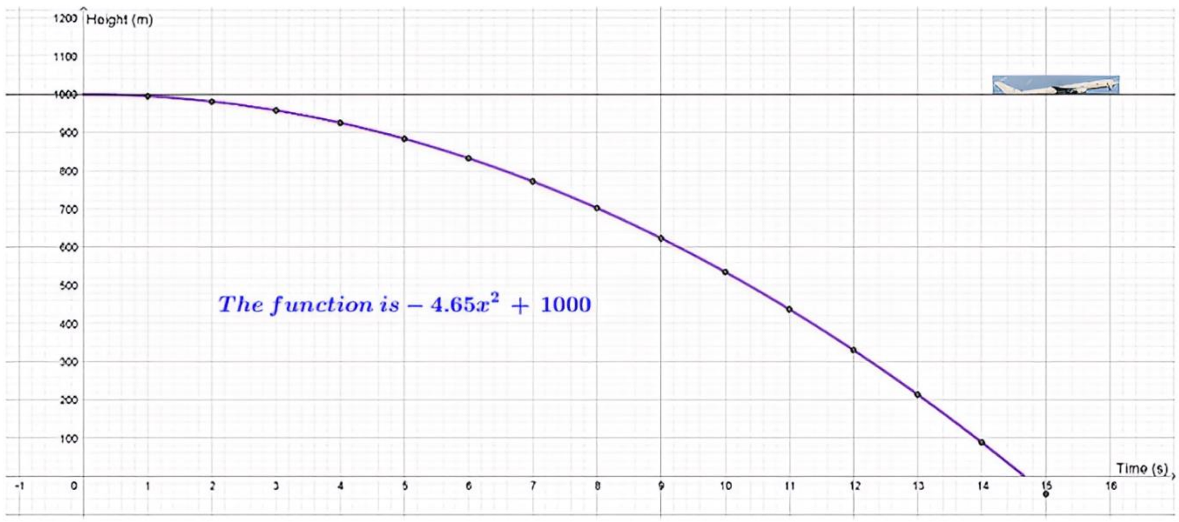

Figure 9. The plane and the box problem visualized by GeoGebra

Air resistance = $\frac{constant \cdot v^2}{2} = \frac{\rho \cdot C_D \cdot A \cdot v^2}{2}$ and $F = \frac{constant \cdot v^2}{2} = \frac{\rho \cdot C_D \cdot A \cdot v^2}{2}$, where *the constant* is a constant that collects the effects of density, drag, and area *(kg/m)*, *v* is the velocity of the moving object (m/s), ρ is the density of the air the object moves through *(kg/m³)*, C_D is the drag coefficient, includes hard-to-measure effects (no unit), and *A* is the area of the object the air presses on (m²).

The air resistance is possible to add or subtract from the velocity downward. The density of the air between 1 and 1,000 meters is about 1.29 kg/m^3, the area of the box is about 0.5 m^2, and the drag coefficient of the box C_D is about 1.0. What is the air resistance force on the box's velocity downward?

Solution: The box's air resistance formula is $F = \frac{\rho \cdot C_D \cdot A \cdot v^2}{2} = \frac{\frac{kg}{m^3} \cdot 0.5 \cdot m^2 \cdot \left(\frac{m}{s}\right)^2}{2} = m/s^2, F = \frac{1.29 \cdot 1 \cdot 0.5 \cdot v^2}{2} \rightarrow F = 0.3225 \cdot v^2$ N.

The force of gravity acting down is competing with the force of air resistance acting up. Therefore, we need to withdraw $0.3225 \cdot v^2$. The force of $0.3225 \cdot v^2$ is not large, since the box has an area of 0.5 m^2 and the weight of 1 kilogram, but it will nevertheless slow the box down. It is logical that the air resistance is growing with the velocity. The faster we are travelling, the more wind we meet.

The model we are using for air resistance is given as $Fd = ½ \rho v^2 CDA$. We want to see the velocity change as a function of distance. Since we know Newton's second law, we write: $F = m \cdot \frac{dv}{dt} = m \cdot \frac{dv}{dx} \cdot \frac{dx}{dt} = mv' v$. Here *v* is a function of distance. We write our differential equation: $mv' v = ½ \rho v^2 CDA$.

The negative sign is there since the force opposes the direction of motion. The force points backwards, and the box has a positive (forward) velocity. Simplifying, we get $v' = ½ \rho v CDA/m$. This is a differential equation we can solve: separate variables, i.e. $\frac{dv}{v} = -\frac{1}{2m}\rho C_D A \cdot v$ and by the chain rule we get $\frac{dv}{v} = -\frac{1}{2m}\rho C_D A \cdot dx$. We integrate both sides and get the solution $\int_{v(0)}^{v(x)} \frac{dv}{v} = -\frac{1}{2m}\rho C_D A \cdot \int_0^x dx$ or $v(x) = v(0) \cdot e^{-\frac{1}{2m}\rho C_D A x}$.

We can either use the formula or the differential equation. Since we want to change the conditions for the air resistance, we use the formula in GeoGebra. We start with the **Figure 9**.

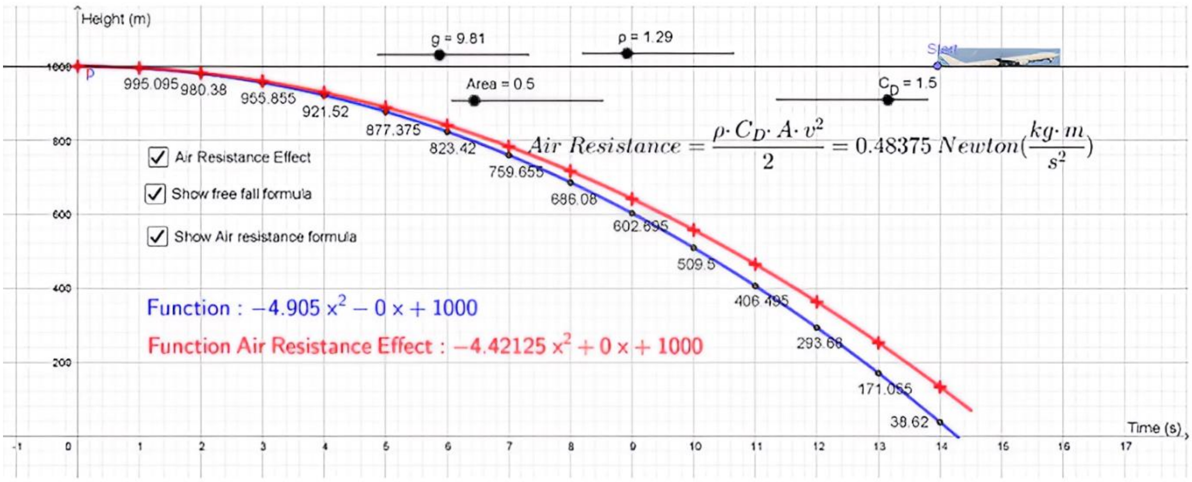

Figure 10. The free fall graph and the air resistance graph in the same figure (The GeoGebra file is found at https://www.geogebra.org/m/zqdcmct7)

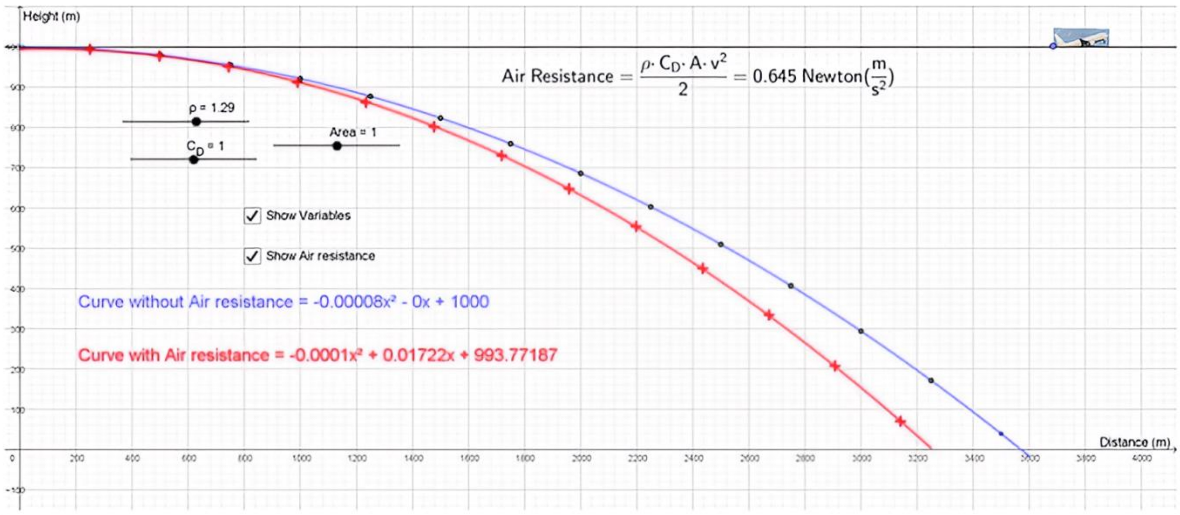

Figure 11. The free fall graph and the air resistance graph in the same height x distance figure (You find the GeoGebra file at https://www.geogebra.org/m/wb7hchzt)

The box drops with air resistance. Even if it is in a way impossible to watch something falling both in a free fall and at the same time falling with air resistance blocking the free fall, I have nevertheless graphed both the curves in the same graph. It is quite common that people do that although it is in a way impossible (**Figure 10**).

Plotting this in a distance versus time graph show that the air resistance values reach the ground after the free fall curve. A cognitive analysis of free fall versus time could be helped by metaphorically thinking. Imagine a race between two competitors that are competing for the shortest time in a race. Obviously, the free fall curve will have the shortest possible time and therefore the free fall curve wins and the curve with air resistance will come second.

In the GeoGebra construction you can vary the gravity and thereby affecting the free fall curve, you can also affect the area of the box, the ρ value and the C_D value. By doing this in a teaching sequence or for students doing this by themselves they will learn through amplification.

The situation in a height vs distance system. In the case of free fall, we used time as the horizontal axis so far. We may also use the x axis to be distance in the horizontal direction and the y axis to be height in the vertical direction. The acceleration of gravity is a=9.8 m/s^2. Set g=9.8 => $a=-g$ and $v_x=v_{0x}$, $v_y=-gt+v_{oy}$. The x component of the velocity is constant since no acceleration in that direction. For position: $x(t)=v_{0x}t+x_0$ and $y(t)=-\frac{1}{2}\cdot gt^2+v_{oy}t+y_0$.

The motion of an object in free fall has its x and y components independent of each other. If you change the motion in the x direction it does not affect the y motion. The motion in the x direction is that of an object going at a constant velocity. The motion in the y direction is the same equation as in one dimension for an object in free fall. By combining these two components, we get the overall motion (**Figure 11**).

COGNITIVE ANALYSIS

The GeoGebra applets for the child in the slide in **Figure 1** and **Figure 2** and the applet in **Figure 3** are there to form a ground for the mathematics and physics we use later on in the article. In this regard these applets may be considered to have a high level of mathematical and physical fidelity. The applets also had a degree of cognitive fidelity since they assist our thought processes while we engage in the physical thinking of friction. The applets, with its physical and cognitive fidelity, creates an environment that enables us to use them to make a conjecture about friction in a slide.

Finally, the applets also had a high level of pedagogical fidelity as they help us to further our exploration and learning. The applets encourage the participation of the readers. The fidelity of technological tools is an important criterion while evaluating their use in the classroom for physical exploration and learning. When visualizing free fall and air resistance in a distance vs. distance graph the air resistance curve is behind the free fall curve as opposite to when we visualized the same scenario in a distance vs. time graph. The coordinate system should be interpreted before analyzing the curves. When we have distance vs. distance coordinate system a possible metaphor is an athlete race over a specific height and the free fall wins the race with air resistance values coming second.

In the GeoGebra construction you can vary the area of the box, the ρ value and the C_D value. By doing this in a teaching sequence or for students doing this by themselves they will learn through the variation, and both amplify and reorganize their conceptual understanding. GeoGebra enabled applets may very well function as amplifiers and reorganizers in enabling students to explore the fundamental concepts related to friction and air resistance.

The essential characteristics (of the GeoGebra applets) that the position of a person or a box in space is a point, must teachers keep in mind while preparing tasks which are to function as amplifiers or reorganizers in enabling student's understanding of concepts of friction, and air resistance.

Teachers should use the applets to introduce and discuss the concepts of air resistance, thereby scaffolding students' explorations and help them develop the concepts.

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