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History of mathematical concepts and students' cognitive understanding of mathematics: Effect of pedagogical content knowledge

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ABSTRACT

This study aimed to explore the relationship between teachers' pedagogical content knowledge (PCK), students' interest and motivation in the history of mathematical (HoM) concepts, and students' cognitive understanding (SCU) of mathematics. A five-level Likert scale questionnaire was used to collect quantitative data from 232 junior high school mathematics teachers. It was found that, on average, students have higher levels of interest, motivation, and cognitive understanding when they are taught utilizing HoM concepts as a pedagogy. The correlation analysis reveals that there is a positive relationship between each of the independent variables and the dependent variable. The regression analysis also shows that teachers' PCK and students' interest were strong predictors of SCU, while students' motivation was a weaker predictor. Hence, it can be said that teachers' PCK and students' interest play crucial roles in promoting SCU of using HoM concepts. Further studies are recommended to explore how historical context impacts students' confidence, enjoyment, and aspirations in mathematics-related fields, which would contribute to understanding the broader implications of incorporating history in mathematics instruction.

Keywords: history of mathematical concepts, pedagogical content knowledge, motivation, cognitive understanding, interest

INTRODUCTION

Mathematics is the branch of the science of structure, order, and relations as a result of counting, measurement, and shapes (Devlin, 2003; Purohit, 2016). Mathematics has been the anchor in the physical sciences, technology, and most importantly, life sciences.

For all these decades, sparse studies have been on the history of mathematical (HoM) concepts for mathematics education (Fauvel & Van Maanen, 2000b). Otte (2007) postulates that the integrating HoM concepts into teaching mathematics helps learners better understand the concepts being taught. Other aspects of the concepts like the theories, proofs, and so on, are easily understood and followed when learners are first introduced to HoM concept (Fauvel & Van Maanen, 2000a). Having an idea of HoM concept serves as a motivation and clear the conception that mathematics is a formalized body of knowledge (Mendes, 2020) and makes the classroom interesting by being humorous and less stoic (McCabe et al., 2017).

From literature, the roles of HoM concepts can be categorized under two umbrellas: that is, the integration of HoM concepts into the teaching and learning of mathematics, which gives learners ideas about what mathematics is all about and secondly, it aids learners to better appreciate and understand the concepts, theories, and principles in mathematics (Jahnke et al., 2002). It is, therefore, important to direct resources toward examining HoM concepts in mathematics education.

Statement of the Problem

Going through the basic school, senior high, and college of education mathematics curricula, close to nothing is documented in the area of HoM concepts and deployment of same as a pedagogical tool in the teaching of mathematics. The focus of these curricula is equipping its learners with the content knowledge but not critically factoring in how to equip them. Basic school mathematics is aimed at equipping learners with basic mathematics that can help them to be fruitful in society; and also prepare them for higher studies and chosen vocations (CRDD, 2010; NaCCA, 2019). The college of education mathematics curriculum is to prepare pre-service teachers to be able to teach mathematics at the basic school levels. College mathematics is to serve two purposes: First, to learn the mathematical content and secondly, to learn how to teach this content at the basic school levels. The teaching failed to explore the significance of HoM concepts as a pedagogical tool (Ho, 2008; Panasuk & Horton, 2013). For instance, HoM is treated as a subtopic in a course, 'theories in learning numeracy for the colleges of education' but not linked in any way as a tool for the delivery of the subject' content. Therefore, it is obvious from the discussion that, pre-service teachers and by extension in-service teachers are not vested in HoM concepts as a pedagogical tool to deliver on their mandates as mathematics teachers.

It is, therefore, necessary to investigate teachers' pedagogical content knowledge (PCK) in HoM concepts and its effect on learners' cognitive understanding of mathematics. Through that, teachers' content knowledge and pedagogical knowledge will be measured.

Purpose of the Study

The purpose of this study was to explore the relationship of teachers' PCK, students' interest, and motivation in HoM concepts, and SCU of mathematics.

Objective of the Study

The objectives of the study are, as follows:

1. Determine relationship between teachers' PCK in HoM concepts and SCU of mathematical concepts.
2. Determine relationship between students' interest in learning mathematical concepts and their cognitive understanding of the mathematical concepts.
3. Determine relationship between students' level of motivation (SLM) in learning mathematical concepts and their cognitive understanding of the mathematical concepts.
4. Determine the extent to which teachers' PCK of HoM content, students' interest in learning mathematical concepts, and their level of motivation in learning the mathematical concepts influence their cognitive understanding of the mathematical concept.

Research Questions

The research questions driving this study are,, as follows:

1. What is the relationship between teachers' PCK in HoM concepts and SCU of mathematical concepts?
2. What is the relationship between students' interest in learning mathematical concepts and their cognitive understanding of the mathematical concepts?
3. What is the relationship between SLM in learning mathematical concepts and their cognitive understanding of the mathematical concepts?
4. To what extent do teachers' PCK of HoM content, students' interest in learning mathematical concepts and their level of motivation in learning the mathematical concepts affect their cognitive understanding of the mathematical concept?

Significance of the Study

The findings of this study will go a long way to benefit stakeholders like students, teachers, curriculum developers, and researchers. This research can form the basis for further studies in the area of teachers' PCK in using HoM concepts in teaching mathematics. This will improve SCU and hence their performance in mathematics. It will inform teachers of the use of HoM concepts as a means of pedagogy to teach mathematics. Lastly, the findings of this study will be a wake-up call for the curriculum developers at the colleges of education of the importance of HoM concepts in teaching the concepts hence, a possible reform in the mathematics curriculum at the colleges and basic school levels accordingly.

Theoretical Framework

The theoretical framework underpinning this study is Bandura's social cognitivist theory of learning, which was born out of the criticisms against the Behaviorist theory of learning that individuals can learn successfully from others' actions. The theory postulates that people can learn indirectly from the success or failure stories of others. Also, these theorists are of the view that cognition is very key when it comes to learning (Ormrod, 2017; Slavin, 2016). That is, cognition such as awareness, attention, expectations and retention play an important role in the process of learning.

Therefore, when teachers share HoM concepts (success and failures of the proponents in proofing the concepts) with students, it would play a major role as a confidence booster in their learning efforts of the concepts. When these histories are shared with students, they become aware of things that happened in the past. Learners then become interested, which will make them pay attention in class better. Hence, triggering their motivation and would want them to know more about the concepts being taught. Thereby, developing high expectations about the topic or concept, which makes them to more questions and or search on the concept. This will result in easy assimilation and accommodation (retention) of the new concept.

Teachers' Pedagogical Content Knowledge in Using HoM Concepts to Teach Mathematics

Having knowledge about a concept is different from using it to teach. Many teachers may have very high knowledge of mathematics concepts but have difficulty in how to teach them for learners to understand (Hiebert & Grouws, 2007). Teachers' pedagogical and content knowledge needed to be appreciably high enough to be able to execute teaching duties. PCK of the teacher could be viewed as a means of planning and presenting a concept in a comprehensive way to learners (Darling-Hammond et al., 2005). These types of knowledge can be acquired through continues teaching of a particular topic or subject. Rowan et al. (2002) observe the knowledge for teaching to include teaching procedures, such as effective lesson planning, classroom management, and practices (implementation). Boaler and Greeno (2000) postulate that students who are taught by teachers with strong PCK are capable of demonstrating good problem-solving skills to their students. And are equally, able to effectively engage their students in the classroom. In view of this, Ho (2008) investigated the preparedness and adaptability of Singaporean mathematics teachers in using HoM in their classroom instructions. It turned out that most of the teachers lacked the capacity to use HoM as a pedagogical tool to deliver their mathematical concepts as expected.

The use of HoM as a strategy for mathematics lesson delivery was frustrating to teachers and "might be distracting the students from gaining basic mastery of mathematical skills and problem-solving" (Ho, 2008, 22). Ho reported that the teachers were even resistant to the use of HoM because they lacked the necessary expertise and training and feared that danger looms if this methodology is enforced. Since mathematics teachers are not explicitly trained to teach history, they lack the expertise and the acumen to combine HoM and the instructional methods to teach it (Panasuk & Horton, 2013). In a similar study, Butuner (2018a) noted that more than half of the 32 participating teachers failed to have been using HoM because "they consider themselves incompetent" in using it. Further, the teachers enumerated a number of reasons why they could not use it such as lack of time, maladjustment between HoM and the exams students were to take, and lack of adequate knowledge and skills in history and resources. It is obvious that most teachers had difficulty in using HoM and even failed to use it, and it is without a doubt that teachers' PCK is limited in the use of HoM. Those who reported to have used it, did so by employing the life stories about the personalities of mathematicians, such as their contributions to human development and their dates of birth; and they found it beneficial.

Against all this backdrop of teachers' lack of instructional methodology in using HoM, students almost always express their satisfaction and likeness of having experienced HoM in their class (Doz, 2021; Panasuk & Horton, 2013). In the same manner, some teachers are very enthusiastic about it and its use in the classroom (Azman & Maat, 2021). If teachers know how to integrate HoM in the teaching of mathematics, it will be beneficial to them and their students, even though some teachers' self-efficacy belief is low towards the use of HoM (Kusumawati & Fachrudin, 2019). If students found it interesting and enjoyable to encounter HoM in their learning, it will do all good to find a place for HoM in the mathematics curriculum of Ghana as well.

As much as the content knowledge of teachers is important, so is their PCK in teaching mathematics. There has been a number of studies on teachers' PCK in mathematics topics like geometry (Martinovic & Manizade, 2017), algebra (Guler & Celik, 2018), and others. These studies have been widened to include the integration of ICT in teaching mathematics, that is, technological PCK. Closed to no studies have been conducted in HoM as a pedagogy in the teaching of mathematical concepts in Ghana. Hence, the need is for this study.

Teachers' Knowledge Level in HoM Concepts

That HoM concepts should be incorporated into the teaching of mathematics is not new (Dejić & Mihajlović, 2014). And the first point of call on the integration of HoM concepts in the teaching and learning of mathematics is the teacher. This suggests that the teacher needs to have knowledge of the content as well as HoM concepts so they can integrate it well into their teaching. Some studies have demonstrated the teachers' knowledge level in HoM and other areas of interest.

In his studies, Goodwin (2007) found the relationship that exists between the mental picture teachers have about mathematics and their level of knowledge in HoM. The 193 high school teachers were tested using the achievement test, which was in the form of marching. It was found that the teachers have a low level of knowledge of HoM concepts. This finding showed that hardly do teachers incorporate the art of mathematical history in their lesson delivery. A similar study was carried out on whether teachers integrate HoM concepts in their lesson presentation, and the possible challenges they encounter as they do so. Of the 1,000 teachers who participated, 90.0% of them indicated non-usage of HoM concepts in their mathematics instruction. However, they reported a number of challenges that were of interest to consider, such as time factor, not seeing history as the same as the mathematical concepts, history not being part of the standard test, low resources to support the teaching of the history, students' low interest in the history as they will not be assessed based on it and mathematics students general dislike for history (Ho, 2008). These seemingly plausible excuses tended to portray mathematics and mathematics teaching as though mathematics were on some island, where teachers only visit to pick ideas from to students, making teachers the sole authors of mathematical knowledge. Mathematics teachers, therefore, need to humanize their classrooms (Fried, 2001) to liven them up for what mathematics can be.

The difficulty presented in asking mathematics teachers to use HoM concepts to teach mathematics is that teachers are at a loss as to whether to teach mathematics history or to use mathematical historical antecedents to teach the subject. Thereby using it to explain, justify, enhance, and encourage the participation of students in the subjects, termed as "radical accommodation" (Fried, 2001) of the teaching of HoM concepts in mathematics lesson delivery. This confusion is clearly demonstrated by teachers of both basic and high schools when their mental image of HoM concepts was investigated. These were teachers of 10 or more years of experience and have not been using HoM as a pedagogy. The reason is that students will not be examined on it, and it requires extra time to teach it but admitted using HoM as an introduction to lesson delivery and giving it out as a project (Smestad, 2008).

Incorporating HoM concepts in teaching mathematics, one needs to possess that transformational knowledge the of discipline that is included in pedagogical dimensions of HoM concepts (Mendes, 2020). Those who do not possess it will fail to appreciate and use it. Thus, Alpaslan (2011) examined the knowledge level of pre-service teachers in HoM concepts and found out that the mean of their knowledge possessed was only 0.44, which was interpreted as low. Again, in-service and pre-service mathematics

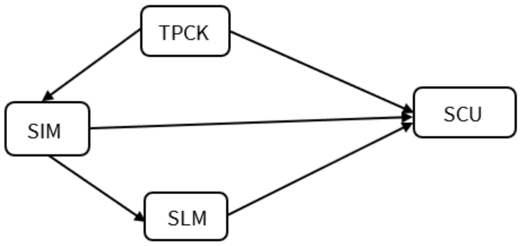

Figure 1. Conceptual framework establishing the relations between criterion & independent variables (TPCK: Teachers' pedagogical content knowledge in mathematical concepts; SIM: Students' interest level in mathematics; SLM: Students' level of motivation in mathematics; & SCU: Student's cognitive understanding) (Source: Field study, 2023)

teachers' knowledge level in HoM concepts was measured and found to perform poorly, especially in the area of the circumstances surrounding how the roots of our number system were developed (Gazit, 2013). Also, Panasuk and Horton (2012) investigated whether mathematics teachers used HoM in their mathematics classroom delivery and their reasons for doing so. The survey showed that 204 respondents do not use HoM concepts in their teaching, for reasons such as limited knowledge about HoM concepts and how to integrate it into teaching, lack of resources, and time factors. The 133 who indicated they use it, gave reasons like: it improves learners' interest and attitudes in mathematics, makes learners enjoy the mathematics concepts and learners are able to establish the relationship between mathematics concepts. In a related study, pre-service teachers express their illuminated understanding of finding solutions to quadratic equations and Clark (2012) concluded that "new mathematics teachers may experience significant shifts in their attitudes, beliefs, and mathematical knowledge for teaching" (p. 81), if exposed to didactical integration of HoM concepts to teaching mathematics.

Developing Interest & Motivation in HoM Concepts

A successful shift in the pedagogical approach to presenting mathematics to learners will lead to increased interest in the subject (Doz, 2021) since students find the introduction of HoM concepts in their lesson fun, enjoyable, and very motivating (Butuner & Baki, 2020), hence having a good understanding of the concepts taught. Doz (2021), therefore concluded that serious consideration should be given to incorporating HoM concepts in the mathematics curriculum, and teachers integrating the same in their teaching of mathematics. This is important because most mathematics teachers have positive attitudes and beliefs toward the use of HoM concepts as their students showed much interest and motivation leading to an understanding of the concepts (Azman & Maat, 2021).

Therefore, using HoM concepts in teaching the concepts can be motivating to students due to the interest that it generates in them and hence improving their academic achievement (Illiyas & Charles, 2017). Teachers have reported the interest and hence motivation that using HoM concepts has generated in their learners (Butuner, 2018b). This was possible because HoM concepts was found to catch students' attention during the lesson. In the same manner, Lim and Chapman (2015) observed that students' achievement levels increased even after one year of using HoM concepts to teach a group of students. This shows relatively permanent learning suggesting cognitive learning from the students.

From the literature, it is observed that if both the in-service and pre-service teachers are armed with education of the history of the various mathematical concepts, they would be able to impact positively on their ability to integrate this pedagogy in their classroom delivery. However, it is important to notice that the knowledge measured are all on the history of the concept developers but not on the circumstances leading to the development of the concepts themselves.

Conceptual Framework

Studies have shown that HoM concepts can be used in teaching mathematical concepts (Clark, 2012; Dejić & Mihajlović, 2014; Fried, 2001; Jankvist, 2009; Liu, 2003). When the knowledge of HoM concepts is used in teaching mathematics concepts, it increases students' interest in the mathematics concepts being taught (Farmaki & Paschos, 2007; Taimina, 2004). This interest can lead to intrinsic motivation (Hidi & Harackiewicz, 2000). This makes students reflect on the topic, which then improves their cognitive understanding of the topic. However, these variables can individually influence the student's cognitive understanding of the concepts of mathematics as depicted in **Figure 1**.

METHODOLOGY

Research Paradigm

According to Shanks and Bekmamedova (2018), experimentation, observation, and reason based on experience should form the basis for understanding human behavior and should be the only acceptable means to expand knowledge and understanding. The aim of this paradigm is to explain occurrences and make measurable predictions about them. As a result, the paradigm underpinning this study is that of positivism. This is because this study aims at examining the impact of teaching mathematics concepts through the use of HoM concepts on SCU. The dependent variable in the study is SCU and the independent variables are teachers' PCK of HoM concepts and the motivation and interest of students.

Table 1. Descriptive analysis of constructs (Field Survey, 2023)

Constructs	Mean	Standard deviation
Teachers' pedagogical content knowledge of the history of mathematical concepts (PCK)	2.11	1.140
Students' interest in learning mathematical concepts (SI)	2.89	1.301
Students' level of motivation in learning the mathematical concepts (SM)	3.09	1.129
Students' cognitive understanding of mathematical concepts (CU)	3.33	1.291

Research Approach

The approach for the study was quantitative. This was because the findings so made were used to make deductions based on the realities on the ground. The data were analyzed quantitatively to determine the total effect of teachers' PCK, motivation, and interest in HoM concepts on SCU.

Research Design

The design for the study was a survey. This was to help identify and describe the characteristics that played out by using HoM concepts as a tool for teaching mathematics. The research was conducted within a time frame of five weeks, taking into consideration the period within which the teachers were available.

Data Collection Instruments/Procedure

A five leveled adapted questionnaire was used to collect data from participants. It was in five sections. The first section collected data on participants' background information. The second section gathered information on participants' knowledge level in HoM concepts. The third section was on PCK of in-service teachers in mathematics. The fourth measured their PCK in relation to HoM concepts. The last section gathered data on SCU as they learn mathematical concepts through HoM concepts. The instrument is adapted from Butuner (2018a), Landicho (2021), and Pintrich et al. (1991).

Data Analysis

Exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) were run on the data collected from the five-level Likert scale. EFA was used to determine the validity of the test instruments, which accounted for four-factor loadings allowing for further analysis. CFA was used to determine the model fit of the data. Studies have shown that these analyses were more advantageous compared to the others (Lahey et al., 2012) for this type of study. To determine the relationships and the effects of the independent variables on the dependent variable, correlation, and regression analyses were used.

Validity a& Reliability of Instrument

Content and construct validity were conducted on the instrument since it was an adapted instrument. Since the instrument was adapted, it was given to experts in the field of mathematics education and the languages department to examine and make their inputs. This feedback was used to modify the instrument. The instrument was piloted, and reliability tested using Cronbach's alpha (CA). The reliability of the instrument was found to be 0.948. The reliability for each of the constructs, teacher's PCK, students' interest in mathematics, SLM, and SCU were found to be 0.829, 0.871, 0.880 and 0.892, respectively. However, when EFA was conducted, some of the items failed to load properly and therefore were removed accordingly. The reliability of 10 loaded items was found to be 0.841. All CAs were found to be above 0.7, hence, the instrument was reliable according to Howard (n. d).

Population

The accessible population for the study was all the teachers in the northern and Eastern Regions of Ghana who are undertaking further studies in a degree in distance basic education. These participants were expected to comprise 553 in-service teachers with different teaching background experiences.

Sample & Sampling Techniques

A sample number of 232 participated in the study. They were selected based on Miller and Brewer's (2003) choice of sample for a quantitative study. The number of females and males were 96 and 136, respectively. The largest age group was 26-30 years of 44.8%, while the least age group was 41-45 years of 2.6%. Their years of experience ranged between 1 to 24, with the largest age group being between one-five and six-10 of 37.1% accordingly. The participants in the study were accessed from designated centers of learning.

Data Analysis

Descriptive statistics

The five-level Likert scale instrument was used to collect the data. The points were scaled from one to five, where strongly disagreed was scaled as one to strongly agreed as five. The constructs measured included the independent variables of 'teachers' PCK of HoM concepts' (PCK), 'students' interest in learning mathematical concepts' (SIM), and 'SLM in learning the mathematical concepts' (SM), and the dependent variable 'students cognitive understanding of the mathematical concepts' (CU). The highest mean is 3.33. **Table 1** gives a summary of the descriptive statistics. From **Table 1**, it was observed that on average, students' reported level of interest, motivation, and cognitive understanding in learning mathematical concepts were higher than the teachers' reported level of PCK. Comparing the standard deviations, the range of 0.271 can be said not to be a very great level of variability in the responses within each construct, which can be considered good.

Table 2. Exploratory factor analysis (Field Survey, 2023)

	Components			
	1	2	3	4
PCK6	.887			
PCK7	.664			
PCK8	.849			
CU2		.599		
CU5		.916		
CU7		.723		
SI6			.886	
SI7			.767	
SM3				.618
SM5				.863

Table 3. Confirmatory factor analysis fit indices (Field Survey, 2023)

Model fit index	Acceptable threshold	Reference	Model fit index from this study
Chi-square test (χ^2)	Non-significant & p-value	Kline (2011)	χ^2=188.885, df=29, & p=0.000
Comparative fit index (CFI)	≥0.95 (very good)	Hu and Bentler (1999)	0.827
Tucker-Lewis index (TLI)	≥0.95 (very good)	Hu and Bentler (1999)	0.731
Root mean square error of approximation (RMSEA)	≤0.06 (very good)	Steiger (2007)	0.154
Normal fit index (NFI)	≥0.95 (very good)	Sarmento and Costa (2019)	0.805

Exploratory factor analysis

According to Suhr (2005), EFA deals with identifying the number of latent variables as well as the underlying factors. EFA is purposed to determine the factors that represent the measured constructs (Hu & Li, 2015). The results of EFA are shown in **Table 2**. It was observed from **Table 2** that all the items in each of the construct loaded above 0.5. On average, PCK, students' interest, and SCU loaded positively stronger while students' motivation loaded positively moderately. Generally, **Table 2**, suggested a relationship between the various components of the study. In all, PCK may be well related to SCU, their interest in the mathematical concepts, and their level of motivation to learn the mathematical concepts.

Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy for the data was 0.739, which was greater than 0.5 and falls within the acceptable zone. That is, a KMO of 0.739 indicated that the sample size of the data was adequate to proceed to CFA with χ^2=950.00 (df=45, p=.000<0.05). These results suggested that the correlation matrix was significantly different from the identity matrix, hence supporting the use of factor analysis to identify underlying factors in the data. The cumulative variance obtained for the four components was 76.3% with a determinant of 0.015, which was greater than 0.000. This suggested that the four components explained a significant amount of the variation in the data set and the factor analysis will produce stable results and therefore, CFA was performed on the data.

Confirmatory factor analysis

CFA was used to determine how well the data set fits into the study model. **Table 3** gives a summary of ranges of fit indices and their interpretations and how well the current data fitted in.

From **Table 3**, it appears that the default model has a good fit for the data, as indicated by the other indices. With a CMIN/df ratio of 6.513 for the default model, which is less than 10, which indicates a good fit, CFI of 0.827, which is below the threshold of 0.95 suggest a poor goodness of fit. Although RMSEA value of 0.154 is within the moderate range, ECVI value (1.129) of the default model is less than ECVI value (4.361) for the independent model (see **Figure 2** for path diagram). That is, the saturated model had perfect fit indices and the independent model had poor fit indices. Thus, the default model has acceptable fit indices but could be parsimonious while the saturated model overfits the data. The fact that the saturated model overfits the data, means that it fits the data too well and does not account for the random variability in the data. This can lead to poor generalizability and overly optimistic estimates of the model parameters. Also, the independent model does not adequately explain the relationships in the data.

To this effect, conclusions drawn from these results may be biased, and therefore the study used correlation to determine the relationship between the individual independent variables and the dependent variable. And regression analysis was used to establish the effect of the independent variables on the dependent variable.

Relationship between variables

Table 4 gives an overview of the averages and the dispersions of the variables in the study.

From **Table 4**, it is observed that the deviations were relatively close suggesting a small variability from the means implying no extreme variation in any of the variables. A further analysis was done to determine the relationship that may exist between the independent variables and the dependent variables.

Table 5 shows the relationship that exist between each of the independent variables (teachers' PCK, students' interest, and students' motivation) and the dependent variable (SCU).

From **Table 5**, it appears that there is a positive correlation between each of the independent variables and the dependent variable. That is, there is a positive correlation between the teachers' PCK of mathematical concepts and that of SCU (r=0.709,

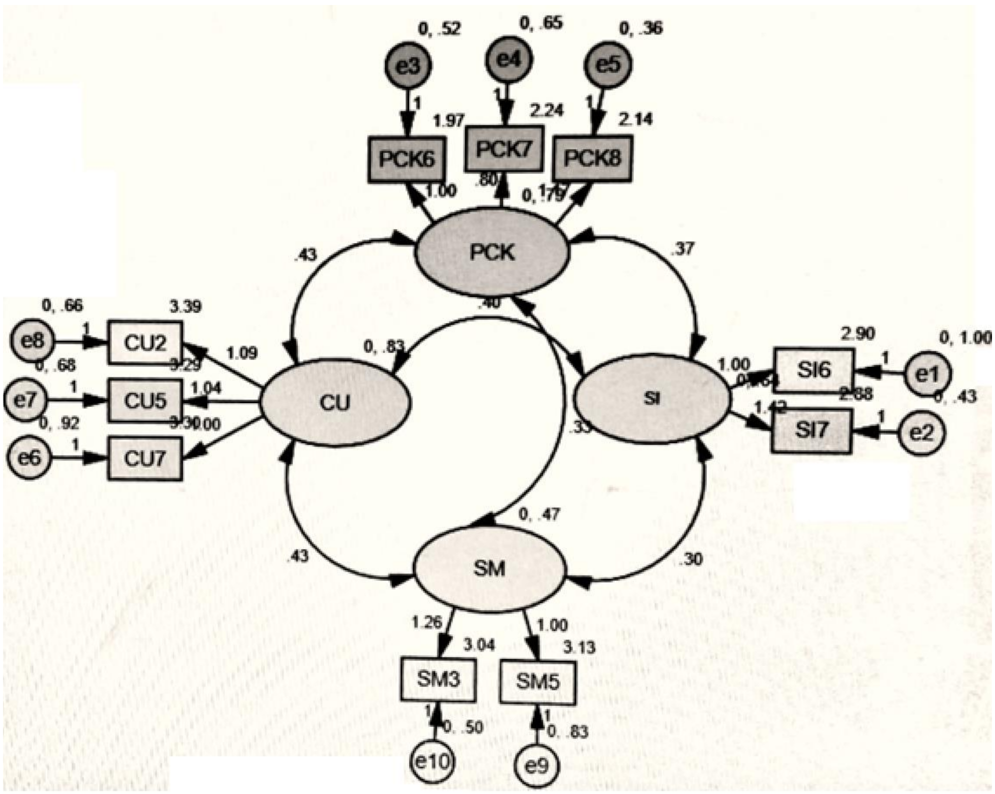

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Figure 2. Path diagram of model fit2018b (Source: Field study, 2023)

Table 4. Descriptive statistics (Field Survey, 2023)

Constructs	Mean	Standard deviation	n
CU	3.3906	1.00389	232
PCK	2.5724	.77141	232
SI	3.0770	.90546	232
SM	2.9810	.93734	232

Table 5. Correlation coefficients (Field Survey, 2023)

Variables		PCK	SI	SM
Pearson correlation	CU	.709	.738	.621
Sig. (1-tailed)	CU	.000	.000	.000

$p<0.001$), students interested in learning mathematical concepts and their cognitive understanding ($r=0.738$, $p<0.001$), and student level of motivation to learn mathematical concepts and their cognitive understanding ($r=0.621$, $p<0.001$). All these correlations are statistically significant, and it was possible that these relations among the variables did not occur due to chance. Therefore, a further analysis was done to determine the causal effect of the independent variables on the dependent variable.

Cause & effect relationship

The value for the multiple correlation coefficient (R) for the model (0.797) shows that there exists a positive relationship between the independent variables and the dependent variable. A coefficient of determinant of 0.636 shows that about 63.6% of the variance of the dependent variable (cognitive understanding) can be explained by the independent variables (teachers' PCK, students' interest, and students' motivation) in the model.

To determine the predictive power of the regression model, an adjusted R square value was calculated and the value for the model was found to be 0.631. This was an indication that the model fits the data set well and also had a good predictive ability. The standard error of the estimate was also found to be 0.60999. This indicated that the model has a moderate level of accuracy in predicting SCU of mathematical concepts. The regression model was statistically significant, as evidenced by the F-statistic of 132.556 ($p<0.001$). This indicates that the independent variables in the model significantly predict the variance in the dependent variable. **Table 6** gives a snapshot of the analysis of the significance of the individual independent variables on the dependent variable From **Table 6**, a unit increase in teachers' PCK will result in a 0.376 unit increase in SCU of mathematical concepts when the other two independent variables are held constant. The variation is statistically significant ($p<0.05$). A beta value of 0.451 for students' interest in learning mathematics concepts shows that a unit increase in SCU would result in a 0.451 increment in the dependent variable. This increase is equally statistically significant ($p<0.05$). It was expected that there will be a 0.058 increase in

Table 6. Regression model (Field Survey, 2023)

Model	Unstandardized coefficients		Standardized coefficients	t	Sig.	95.0% confidence interval for B	
	B	Standard error	Beta			Lower bound	Upper bound
(Constant)	.408	.156		2.614	.010	.100	.715
PCK	.489	.074	.376	6.579	.000	.342	.635
SI	.500	.066	.451	7.547	.000	.370	.631
SM	.062	.065	.058	.968	.334	-.065	.190

SCU as a result of a unit increase in SLM, but this was not statistically significant ($p>0.05$). Thus, there may be a weak relationship between SCU and their level of motivation to learn mathematics concepts through history. It is also observed that when all the independent variables are held constant, there was a 0.408 increase in the dependent variable and this increment was not due to chance ($p<0.05$). Therefore, the regression model can be stated, as follows:

$$CU = 0.408 + 0.489(PCK) + 0.500(SI) + 0.062(SM). \quad (1)$$

Generally, it can be said that teachers' PCK and students' interest are strong predictors of SCU of mathematical concepts, whereas students' motivation was a weaker predictor of the dependent variable.

DISCUSSION

Delivering mathematics concepts to students can be very frustrating to both teachers and students alike if teachers failed to use conceptually convincing approaches to teaching it and therefore can make students perceive it as a given idea instead of a created concept. In light of this, it was believed that having ideas on HoM concepts can liberate the mind of mathematics as a fixed body of knowledge (Mendes, 2020) to make mathematics teaching and learning interesting and motivating (McCabe et al., 2017). This study, therefore, considered the teachers' use of HoM concepts and SCU of mathematics in the context of the teachers' PCK, students' interest, and motivation as to how these influenced SCU of mathematics.

The results of the findings showed that even though the majority of the teachers, just as teachers in the study of Butuner (2018b), did not have much knowledge in HoM concepts but had a substantially strong relation of their PCK (PCK) with the student's cognitive understanding for HoM concepts. Because they showed their enthusiasm of readiness (Azman & Maat, 2021) to use HoM concepts in teaching the concepts. This could be the reason for the strong relationship between the teachers' PCK and its influence on SCU of it. Teachers' PCK in practice, is all the efforts and skills available for the teacher to use before, during and immediately after the presentation. In this wise, these participants exhibited the characteristics of Goodwin's (2007) study participants also. This might be accounting for the strong relationship between PCK and SCU. The result of this strong relationship is established in the findings of Boaler and Greeno (2000) that students taught by teachers with strong PCK and effectively engaging their students in the classroom, are capable of demonstrating good problem-solving skills.

Unlike the teachers in the study of Ho (2008) who believed students will have low interest in HoM concepts, the participants in this study showed that students' interest was positively correlated with their cognitive understanding if HoM concepts were used to teach the concepts. This finding agreed with the findings of Doz (2021) and Panasuk and Horton (2013) who had their students claimed that they almost always expressed their satisfaction and likeness of having to experience HoM concepts in their class. Such feelings could explain the positive relationship between the students' interest and their cognitive understanding of mathematical concepts using HoM concepts. Having interest to positively correlate with SCU for the use of HoM concepts for teaching the concepts was the strongest among the three variables of PCK, students' interest and motivation. Accordingly, Renninger et al. (2014) observed that students' interest in what they are learning plays a significant role in their cognitive engagement and the learning process.

In much the same way that the participants in Butuner's (2018a) study reported their students having interest and motivation generated towards using HoM concepts, so have the teachers in the current study demonstrated that the motivation of their students had a relationship with their cognitive understanding for using HoM concepts to teach the concepts. This was shown in the positive correlation of motivation with SCU. Even though motivation showed the least of strengths correlating with SCU were HoM concepts to teach the concepts used, it was worth recognizing that Lim and Chapman (2015) noticed that this can always lead to students' achievement even after a long while. This result therefore aligned with Illiyas and Charles (2017) finding that there was a moderate to strong relationship between SLM and their mathematics achievement in general. Most importantly, it helps in building a positive attitude towards the subject.

It was important to determine from the study what effect did the teachers' PCK, students' interest, and motivation had on their cognitive understanding when HoM concepts was used in teaching the concepts. The findings from the study showed that these variables altogether, accounted for 63.6% of the variance in the criterion variable. It is significant to understand that if so much of this was accounted for by the three variables, then knowledge of HoM concepts to teach the concepts was very critical to transforming the delivery of mathematics to students in the manner as to humanize the classroom (Fried, 2001; McCabe et al., 2017).

The findings indicated that all the variables had some degree of influence in determining SCU of the mathematical concepts if HoM concepts were used to teach it because it would have been fun to students (Butuner & Baki, 2020). However, as shown in the strength and direction of their relationship of accounting for SCU, so have they shown their effects on accounting for their influences. Nonetheless, motivation had shown forth strong enough as it was not significant in determining SCU of using HoM concepts in teaching it. As was stated by Mendes (2020) there possibly could be other reasons that could explain SCU for using HoM concepts as a strategic tool for lesson delivery. Therefore, teachers need to be abreast of their professional acumen to use it to the benefit of students and teachers themselves.

Practical Implications of the Study

The practical implications of this study are that teachers need to have a strong knowledge of HoM concepts to effectively teach mathematics to their students. They also need to have the enthusiasm and readiness to use HoM concepts to teach the concepts to their students. Additionally, teachers should recognize the importance of generating interest and motivation in their students to learn mathematical concepts by using HoM concepts. The study indicated that using HoM concepts in teaching can humanize the classroom and transform the delivery of mathematics to students.

Therefore, teachers need to be trained in the use of HoM concepts as a strategic tool for lesson delivery. This training will help teachers to understand how to use history to make mathematics teaching and learning interesting and motivating. It will also equip them with the necessary skills to generate interest and motivation in their students. Ultimately, this will lead to improved SCU of mathematical concepts.

CONCLUSIONS

In conclusion, the study highlights the significance of incorporating HoM concepts in teaching to make the subject interesting, motivating, and enhancing cognitive understanding for students. The findings revealed that the teachers' PCK, students' interest, and motivation have a considerable influence on the cognitive understanding of students when using HoM concepts to teach. The study underscores the importance of teachers having a thorough knowledge of HoM concepts to humanize the classroom and transform the delivery of mathematics to students. Moreover, the study emphasized the need for teachers to adopt conceptually convincing approaches to teaching mathematics to avoid students perceiving it as a given idea instead of a created concept. Overall, the study provided valuable insights into the use of HoM concepts in teaching and its impact on SCU.

Further Studies

Further studies on this topic could explore the effectiveness of different approaches to teaching HoM concepts. For example, a study could compare the effectiveness of using written texts versus multimedia resources in teaching HoM concepts and assess the impact of these approaches on SCU, interest, and motivation. This is because some students are more visual learners and others may prefer hands-on activities.

Again, future research could investigate the impact of incorporating HoM concepts into the curriculum at different grade levels. Understanding how the use of historical context in mathematics instruction impacts students at various developmental stages could help educators determine when and how to introduce this type of instruction.

Finally, future studies could explore the impact of HoM concepts on students' attitudes toward mathematics as a subject. This could include investigating how the use of historical context impacts students' confidence in their mathematical abilities, their enjoyment of mathematics, and their future aspirations in mathematics-related fields.

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