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Integration of critical thinking and reasoning skills into lessons through block factor game for finding factors of a number

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ABSTRACT

Many researchers and philosophers recognize the importance of critical thinking and reasoning skills in the life of all students. However, they failed to address the question of how such skills may be incorporated into mathematics lessons or activities. While such skills are often included in mathematics curriculum, they are not always seen in the lesson. This paper presents the integrations of critical thinking and reasoning skills into a sample lesson through block factor game for teaching factors of a number to young learners to play and list factors of a given number. Scholars have agreed that making mathematics practical using concrete materials in the form of game makes it interesting and fun for learners. Two participants in primary 2 and primary 6 were involved in this study through qualitative research design of case study. Data collection was done through audio-and-videotaped. The students' responses to questions and tasks indicated conceptual understanding and acquisition of critical thinking and reasoning skills. Critical thinking and reasoning skills are incorporated into lesson through critical questions, and games or well constructed tasks in appropriate context. Also, the participants were able to list factors of a given number correctly and also determine the divisibility of a number by another number using block factor game.

Keywords: mathematical skills, mathematical thinking, mathematical reasoning, critical thinking and reasoning skills, factors of a number, brain factor game

INTRODUCTION

Reasoning and critical thinking skills are crucial for everyday life. People's experiences influence their thinking and mathematical thinking tends to depend on mathematical background (Borovcnik, 2016). Critical thinking and reasoning are indispensable skills of mathematics and help differentiate mathematics from other disciplines. What makes mathematics is its ability to equip people with critical thinking and reasoning skills (Ayal et al., 2016). This century demand responsible individuals who are critical, innovative and creative to identify and solve problems of the society and the world (Asigigan & Samur, 2021). Mathematics is just the discipline; critical thinking and reasoning skills give its meaning. Nevertheless, such skills are complex for students to comprehend because many teachers are unable to integrate these skills in their lessons or activities meaningfully (Sukirwan & Herman, 2018). Morsanyi et al. (2018) claimed that mathematics involves complex relationships that require reasoning which usually receives less attention. Therefore, there is a need to provide a context sample for inculcating critical thinking and reasoning skills into lessons for teachers to emulate. Because critical thinking and reasoning skills help keep our minds active always. Mathematics equips students to be logical, analytical, systematic, critical, creative, communicative, and with the ability to cooperate (Titin, 2014).

Many strategies we employ in solving problems in abstract algebra, advance number theory, complex analysis, and further statistics are things several years to come, we would find difficult to recall and use to solve problems that require them directly. However, the mathematical skills developed will forever remain with us. What we say; how we say it; and how we assess others depend on the extent to which we possess mathematical thinking and mathematical reasoning skills. For example, I was walking one Friday and my mobile phone rang. I picked the phone and noticed it was a friend of mine who was calling. I wanted to pick the call but the phone froze. In the second call, I attempted to pick the call again but I was unsuccessful. I walked to a nearby drinking bar and off the phone, so, that I could on it again. I never knew my friend was close to me before I could not see him. He packed his car and walked to me and wondered why I refused to pick up his calls when he realized I saw his calls coming and suddenly off my phone because he called the third time when the phone was off. How did he conclude that l was avoiding his calls? The fact that I had not picked his calls, is that enough to conclude that I was avoiding his calls? He drew his conclusion on the assumption that my mobile phone was functioning well at the time he called. He judged me wrongly because he did not consider the possibility of the phone freezing. The way we think affects the judgments or decisions we make and subsequently the way we see others.

Truth and certainty of truth are at the heart of mathematical thinking, and mathematical reasoning. Mathematical reasoning is the process of thinking to explain relationships on the premise of laws and properties that have been scientifically tested and accepted.

> "Mathematical reasoning encompasses quantitative reasoning as well as other forms of reasoning, such as spatial reasoning, which does not always involve the representation of quantities" (Nunes et al., 2015, p. 179).

On the other hand, mathematical thinking refers to techniques, concepts, and approaches that are employed directly or indirectly in mathematical problem-solving processes (Uyangor, 2019). Both mathematical reasoning and mathematical thinking are special because they are strongly connected to every concept regardless of the learners' level. Mathematical thinking is the ability of the individual to link the concepts in the problem before a decision or judgment is arrived at. This means before one guesses or predicts some thoughtful laws or principles that first precede the guess or estimation. The inclusion of critical thinking and reasoning skills in the curriculum is one thing, the integration of critical thinking and reasoning skills in lessons is another. Critical thinking and reasoning skills have been given the ultimate respect in every curriculum across countries (Goos & Kaya, 2020), nevertheless, its implementation in the classrooms at all levels tends to be difficult for several teachers, even though both critical thinking and reasoning skills equip students to assess information to determine its truthfulness or otherwise in life. In this century, there is a high increase in information which requires people to compare and contrast to ascertain the validity of the information. This indicates students' need for critical thinking skills and reasoning skills most to play their part in the economic development of their countries. Mathematical reasoning goes beyond solving real-world problems to include evaluating and making arguments to public policy debates and making choices in life (OECD, 2021a). The inability for one to think critically or reason would affect one's life negatively and that of the world. For example, how "driver A" thinks and reasons when driving on the road would not only affect him/her but also others. The skills of reasoning allow people to identify patterns, reconcile differences in situations, and provides justification for ones' actions or inactions. Also, mathematical thinking and reasoning prepare one to assess others' actions or inactions which is essential for decision-making. The decision we make has consequences on the quality of our lives. For example, in football, we are clear in our minds that Messi and Ronaldo are great players. But their greatness is most seen based on their choice of club and their teammates. Great players are not only great by the decision they make on the football field but also their decision on which club to play for based on the kind of system the club is noted for. A player who does not consider the system of play for the club and the players available in that club compared to his/her style of play is likely to underperform his talent or ability. This is because every department must be well connected for his performance to shine. Basra and Fauzi (2017) stated that reasoning skills enables students to master mathematical concepts and solve problems in daily life. So, students' ability to think or reason through things is necessary for their complete growth. Mathematical reasoning provides the intellectual acumen behind problem solving (OECD, 2021a, p. 4). Critical thinking and reasoning are germane skills which should be the focus of teaching mathematics as mathematics plays important role in decision-making. These skills empower learners to become problem solvers of the real world.

> "Mathematical reasoning plays an important role, both in solving problems and in conveying ideas when learning mathematics" (Ayal et al., 2016, p. 1).

Siswono (2008) recognizes the importance of open-ended problems in promoting mathematical creativity, conceptual understanding, and the development of mathematical skills in solving daily life problems. Mathematical reasoning enables people to adapt procedures or transformations to solve a problem (Nunes et al., 2015). Reasoning is a central concept in mathematics and play essential part in mathematical literacy (Kollosche, 2021). For learners to become good problem solvers, the cultivation of mathematical reasoning in learners from early grades is crucial. Kollosche (2021) outlined six styles of reasoning as the postulation style of reasoning; experimental style of reasoning; modelling style of reasoning; taxonomic style of reasoning; statistical style of reasoning; and genetic style of reasoning. Mathematical thinking is an important goal in the education of an individual to solve problems (Stacey, 2006). Mathematical thinking complements peoples' ability to function in engineering, law, technology, medicine, and industry. Kaufmann (2019) investigation of students' reasoning and strategy used in multiplication, found that students employed general principles of reasoning and different characteristics of multiplication in solving problems. Students' ability to use multiple strategies/reasons in solving problems is subject to teaching methods employed by the teacher.

One of the methods suggested by the curriculum designers and experts is the use of concrete materials to illustrate mathematical ideas (Ministry of Education, 2019a; Rusiman et al., 2017). The mathematics curriculum of Ghana requires teachers to use concrete materials in teaching at primary school while integrating critical thinking and reasoning skills into lessons (Ministry of Education, 2019b). Carbonneau et al. (2012) review examined studies on the relationship between teaching mathematics with concrete materials and instruction that focused on abstract mathematics symbol from preschool to college. Their review found statistically significant results in favor of the use of concrete manipulatives when compared with instruction that only used abstract mathematics symbols. However, the present of concrete materials alone does not present or organize the mathematical ideas in a way that is understandable to learners (Thompson, 1994). The actions taken by the teacher is what make the mathematical ideas visible and provoke students' thinking and reasoning.

Purpose and Research Questions

To our mind and heart, we know the beauty of mathematics involves the study of patterns and identifying relationships that exist between and justifying the same. In identifying the patterns and relationships I am certain in my mind that this rests on critical thinking and reasoning. It is on this bases that I contend that critical thinking and reasoning are fundamental skills in every mathematics lesson. Teachers' ability to integrate reasoning skills and critical thinking skills into mathematics activities or lessons are key to students' acquisition of mathematical skills. A growing body of research findings indicates that critical thinking and reasoning skills are crucial for students' intellectual growths but its inclusion in activities tends to be difficult. Therefore, the main purpose of this study is to serve as a guide on how to incorporate critical thinking and reasoning skills into lessons. Secondly, this study aims to examine the impact of the block factor games on students' reasoning ability to list factors of a number. Additionally, to determine if mathematical skills like reasoning, and critical thinking drive teaching strategies (techniques, questions, and assessment) students will be able to demonstrate conceptual repertoire. To achieve the purpose of the study the following research questions guided the study:

1. How can critical thinking and reasoning skills be incorporated into lessons?
2. What in students' responses and answers in block factor game for finding factors of a given number can be taken as indicators of mathematical thinking and reasoning?

This study is not only important for theoretical purpose but also crucial for classroom practices and to achieve the aim of mathematics instruction by cultivating reasoning and critical thinking skills in students which pave way for creativity and innovation. Understanding learners' way of reasoning can be a precursor for effective teaching and learning processes since practical examples can be provided to support or challenge their ways of reasoning about concepts (Bjorklund & Palmer, 2020). Teaching methods are important in students' development of mathematical thinking.

Teaching Methods

Society is dynamic as such teaching must reflect the dynamic nature of the world if learners are to make sense of what they learned. The learners' ability to apply what they learned has been the focus of instruction and research over the years (Noreen & Rana, 2019; von Glasersfeld, 1995). Two major approaches to teaching have been identified in classrooms and literature. These are traditional method and the learner-centered approach. Even though the learner-centered approach has been found to be effective, however, studies by Noreen and Rana (2019) concluded that the most part of educating in classrooms is generally traditional method. This method of teaching made it difficult for students to apply concept they learnt in new situations. As a result of this, various international companies and individuals have raised concerns about learners' inability to apply mathematical concepts efficiently after graduating from school. Because of this concern, international organizations (e.g., National Council of Teachers of Mathematics and the International Association for Statistical Education) have been calling for research on teaching and learning of mathematics in schools to improve learners' mastery of mathematical concepts. This leads to placing learners at the center of teaching and learning process because learner-centered approach has been found to contribute to students' understanding of mathematical concepts. The important role that constructivism philosophy played in learners' complete mastery of mathematical concepts and intellectual development has been the reason for many curriculums recommending its use in classrooms. The learning styles of students influence their preferred choices of mathematical teaching approaches (Johnson & Dasgupta, 2005). Similarly, Tularam and Machisella (2018) asserted that the learner-centered approach for teaching and learning in mathematics needs to be well understood so that an appropriate comparison with the traditional method of teaching is made. Theories of learning inform teaching methods. One such theory of learning is constructivist theory.

Theoretical Review

The constructivist theory includes socio-cultural constructivism (Vygotsky, 1978) and radical constructivism (von Glasersfeld, 1995) respectively propounded in 1934 and 1974. Vygotsky's (1978) work considers the reconstruction of knowledge in a social context. The author believes that learner's acquisition of knowledge takes time. This suggests that teachers should exercise patience for learners to explore concepts to acquire all the necessarily skills. Vygotsky (1978) proposed a "zone of proximal development" form of teaching and learning that allows learners to explore concepts for the acquisition of new ideas apart from what they have learned previously. Vygotsky (1978) advocated for the independent learning of the learner. The author further added that an experienced person should provide the learner with any assistance as at when he or she considers it fit. Vygotsky (1978) contends that the learners' intellectual life will be greatly developed when teachers succeeded in leading learners to reorganize their existing ideas with new ones in a meaningful way. This suggests that learners are to be at the center of teaching and learning where interacting with their colleagues and teachers serve as coaches.

The proponent of radical constructivism does not object to the attainment of 'truth' or objective knowledge (von Glasersfeld, 1995). Most experts in the field expect one's knowledge to verify with objective reality. It can be noted that learners only need to be guided to construct and reconstruct knowledge independently in a context that is relevant for learners' overall development that represents a real world. The independent construction of knowledge prepares learners to fit into the dynamic world. However, there are some misconceptions about radical constructivism. For example, Hardy and Taylor (1997) posited that

> "knowledge consists of those mental constructs that satisfy the constraints of objective reality" and alleged that radical constructivists are "asserting that knowledge does not 'match' the world's actual structure, but 'fits' within or slide between" (p. 1).

Radical constructivists call for 'construction of knowledge' that 'fits or matches' the world's actual structure. Constructivists are concerned about the way of establishing truth, not against the truth. In celebrated work of von Glasersfeld (1995), the author observed that

> “knowledge should represent a ‘real’ world that is thought of as ‘existing’ separate and independent of the knower, and this knowledge should be considered ‘true’ only if it correctly reflects that independent world” (p. 3).

The author further stressed that learning required self-regulation and reflection. Even though Glasersfeld’s (1995) works on radical constructivism received a lot of attention nevertheless it has received some criticism. Hardy and Taylor (1997) claim that radical constructivism:

> In its current cognitivist form may be insufficient to empower teachers to overcome objectivist cultural traditions. Teachers need to be empowered with rich understandings of philosophies of science and mathematics that endorse relativist epistemologies: for without such, they are unlikely to be prepared to reconstruct their pedagogical practices (p. 1).

Radical constructivists recognize the importance of assessment and reflection on instructions to improve learning. One can identify the strengths and areas that need improvement of his/her lesson after reflection. Based on this, one can meaningfully reconstruct a new way of teaching the concept. So, the claim that radical constructivism does not empower teachers to reconstruct pedagogical practices cannot be a reality and the same is rejected. Constructivism prepares students to demonstrate a conceptual repertoire when they are confronted with standard or difficult problems. Surprisingly, traditional ways of delivering explicit mathematics instruction are frequent in classrooms with combined difficult questions (Voskoglou, 2019). The traditional knowledge was based on epistemology that knowledge in all fields of human endeavor is based on the set of propositions jointly with procedures and laws for testing them. The epistemologists hold the view that the mathematical truth is certain and this has influenced instruction greatly. It must be noted that some things in mathematics will have to be memorized (e.g. the sine of an acute angle of a right-angled triangle is equal to the ratio of opposite over the hypotenuse), but it should not become our way of teaching. Tularam and Machisella (2018) explained the traditional method as

> “generally teacher-directed, where students are taught in a manner that is conducive to sitting and listening” (p. 29).

As Hokor (2020) wrote that because learners are listeners and observers they are unable to apply what they learned in new situations. Similarly, Lessani et al. (2017) investigated mathematics teaching approaches and their consequences on learners’ ability to solve challenging problems. Their findings portrayed the learner-centered approach to be more effective in students’ creativity and innovation in solving real-life problems than the traditional approach. The review here suggest that constructivism inspire learners to:

1. appreciate the importance of mathematics by connecting mathematical ideas with the real world;
2. develop creativity, curiosity, communication, and share ideas;
3. connect concepts acquired with new ones or ideas;
4. develop intellectual repertoire to solve real-life problems;
5. develop mathematical thinking and mathematical reasoning; and
6. reflect on their knowledge and learning.

This study relies on constructivist theory because it allows learners to construct the meaning of factors of a number through personal experience under the guidance of a facilitator. Also, this theory allows the facilitator to incorporate critical thinking and reasoning skills into lessons through questioning. The activities were based on their knowledge on grouping of objects. The section that follow consider the teaching of mathematics skills.

Teaching Mathematical Skills

In this study, the facilitator considers who to teach, what to teach, what the learner needs to acquire (skills), strategies for teaching the learner how to acquire the skills; and reflect on the lesson. The activities are logical, relevant, and direct students’ learning to acquire the reasoning and critical thinking skills. This strategy promotes conceptual knowledge and active learning among learners. The activities relate to students’ experience and foster students’ innovation, creativity, and thinking. Learners are actively involved in the lesson through critical questions asked by the teacher to lead learners to discover the ideal meaning of the concept and share their knowledge with the class. **Figure 1** was proposed based on the literature and personal experience by the researcher and used to develop the mathematical thinking and reasoning skills in students is presented next.

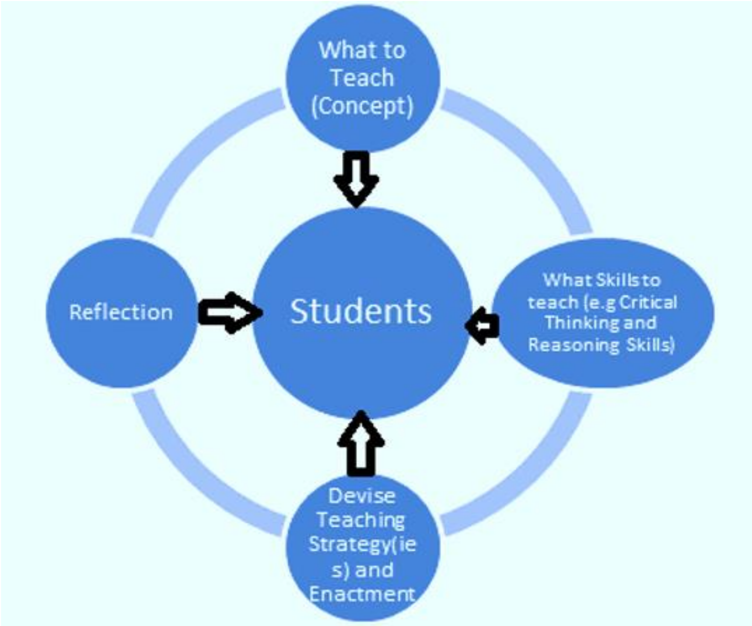

Figure 1. Mathematical skills teaching model

The students are at the center of focus in every stage of the framework. In what to teach stage, the teacher would have to refer to the curriculum which includes the strands for the learner. Also, the previous knowledge or experiences of the learner are to be taken into account. The next phase of **Figure 1** is to consider the skills the teacher expects the learners to acquire. A plan to teach without first considering mathematical skills is not only a waste of students' precious time but also the betrayer of the spirit and the letter of mathematics. Mathematical skills must be at the heart of teaching mathematics regardless of the funs students may have during mathematics lessons. At the skills stage, one needs to ponder over why teach the concepts which will lead to the teacher being able to identify all the skills the learner has to acquire. These skills inform instructional strategies (enactment stage). At this stage, the teacher should consider questions to ask, when, how, and the time to ask the questions to keep learners thinking and reflecting on their results and that of others.

Students' mathematical thinking and reasoning are carefully examined at this stage through questions or assessment which is an integral part of teaching. Skilful questioning does not only help monitor students' mathematical problem-solving abilities but also students' mathematical thinking and development of mathematical ideas and help estimate their mathematical growth (Mueller et al., 2014). At the final stage, the teacher has to reflect on what has been done, what worked, what didn't, and how it can be done differently in the future. This helps to improve teacher professionalism which would translate into students' better acquisition of mathematical skills.

METHOD AND MATERIALS

A qualitative research design of case study was used in this study. This research involves observing few participants (one or two or three) to gain an in-depth understanding of the issue under investigation (Stockemer, 2019). The qualitative study seeks to provide an accurate account of phenomena under investigation. This study seeks to uncover naturally on students' mathematical thinking and reasoning exhibited in teaching factors of a number (Sukirwan & Herman, 2018). Data were collected on the participants through video-and-audiotaped. The recording was done using a laptop. The lesson enactment was done with the prior permission of the parents since the participants were below 18 years. Both students were taught at time in one classroom for a three-hour lesson.

The facilitator is a researcher and teaches at the College of Education, responsible for training teachers to teach at junior high and primary schools. The lesson was planned and enacted by the facilitator. The facilitator has been teaching mathematics for the past six years.

A Practitioner Research Stance

A practitioner research stance was used to put myself as both the facilitator and researcher in the process of providing the teaching experiment and integrating critical thinking and reasoning skills into the lesson. My stance is vital for this study to ask critical questions linked to critical thinking and reasoning skills that direct students' learning. Borko et al. (2008), cited in Nguyen (2015), explained that the practitioner research is to

> "address questions and issues that arise from discrepancies between what the practitioner intends and what actually occurs" (p. 50).

In this case, to operationalized the critical thinking and reasoning skills in the lesson. One of the reasons for my stand as both the facilitator and researcher is to experience first-hand mathematical thinking and reasoning of the participants. Liu (2005) argued in support of teacher-researcher that it has the responsibility of interpreting what he or she sees the subjects doing, and attempting to perform the act of decentering by trying to understand the Mathematics of the learners. The next section dealt with the teaching experiment.

Design Experiment

To provide sample lesson on how to incorporate critical thinking and reasoning skills into lesson, the study designed the non-digital based-game by incorporating a mathematical skills of the basic mathematics curriculum. More specifically, the study adopts the mechanisms of block construction and grouping game, in which every student can use block to list factors of a number. Factors of a number are all whole numbers that divide another whole number without a remainder. The block factor game is the general term used to refer to all games that required the use of blocks to find the factors of a number. This game is made up of block game, and brain factor game.

Block game

In this game, learners are given a stick of a given length, say 15 units. There are several unit blocks for which learners are required to arrange along with the long stick of length 15 units. These unit blocks are arranged end to end. When the unit blocks are equal in length with the long stick of length 15 units, then the arrangement is complete. One main reason for arranging unit blocks along the long stick is for learners to deduce that 1 is a factor of every number and every number is a factor of itself. For example, the Learners are to form a group of threes from the 15 unit blocks and if they can form a group of threes with no remainder, then it follows that 3 is a factor of 15. If not, then 3 is not a factor of 15. If learners are able to form group of threes exactly with the 15 units' blocks, and noticed that the number of groups of threes is also a factor of 15 indicates mathematical thinking and reasoning. These procedures are repeated for half of all the numbers we want to find the factors of. For example, if one wants to find the factors of 12, one would start with the number 1, 2, … up to 6, which is half of the number 12. **Figure 2** is a photo of the arrangement of blocks along the long stick by the participants.

Figure 2. A sample of students' arrangement of unit blocks along a long stick

Brain factor game

This game is played between two people. In this game, learners are given some number of unit blocks equivalent to the number they are to find factors of. One will have to predict a number he/she considers as a factor of the given number, then the other member of the group will form groups of objects about the number predicted. If they can form groups such that each group has an equal number of objects, then the prediction is accurate and a point of one is recorded for that individual else zero is recorded. All the unit blocks will be put back, then the other person will also predict a different number. If it is possible to group objects such that each group has an equal number of objects, then a point of one is recorded for that person too else zero. If one is unable to predict a factor within one minute, then the opportunity is given to the next person. The one with the highest points at the end of the game is declared a winner called 'brain king'. 42 unit blocks were given to the learners. The game was played 8 times because we have eight positive factors of 42. Also, eight times allow each participant to predict four times in all. This game enables students to think carefully before predicting. This game help develops predictive skills in learners.

Participants

The researcher was approached by two friends in early 2021 to find a teacher who could teach their children mathematics to cover the lost hours during COVID-19 crises. While consulting the teachers, the researcher then seek permission from the parents for the involvement of their children. Two participants were involved in the study. This was done in line with COVID-19 guideline to avoid assembling more people to limit the spread of COVID-19. One student has not been officially exposed to factors of a number. The aim of this is to examine how student that has never been taught factors of a number thinks and reasons about factors of a number using the block factor game. The participants were 11-and-8-years-old. Both participants attend a private school in Hohoe. Both public and private schools use the same curriculum and write the same national standard examination (Ministry of Education, 2019a). The primary 2 curriculum recommends that students are expose to all the four basic operations (addition, multiplication, subtraction and division) in mathematics using concrete materials.

Nancy, who is 11 years old, is in primary 6. Nancy has been involved in the study because she has studied factors of numbers in stage 5 (Ministry of Education, 2019b). Nancy had been taught factors of numbers before but could not list factors of 10 or explain 'factors of a number' before the enactment of the lesson planned. She however has good knowledge about even numbers and odd numbers.

Awoe is a 8-year-old and in primary 2. The inclusion of *Awoe* was to determine whether a child can group objects as required, then that learner can lists factors of a number using block factor game. *Awoe* can count natural numbers and write them down correctly up to 100. Additionally, she can add two-digit numbers. She can group objects in groups equally and determine the number of objects in each case. However, she has not heard of "factors of a number" before the lesson.

Data Analysis

Content analysis (Columbia University, 2019; Luo, 2019) was done on the recorded data. The recording was played and transcribed for analysis to ascertain the impact of the block factor game on learners' ability to think, reason, and write down factors of a given number. In the analysis, the study highlights students' mathematical thinking and mathematical reasoning during the teaching process. The verbalizations between the facilitator and the students were examined to explain students' mathematical thinking and reasoning. How students can think and reason in identifying mathematical relationships and justified inferences were noted and examined. For example, if a student formed 5 groups of 2 and noting that both 2 and 5 are factors of 10. Therefore, there is no need to form groups of 5 to determine whether it's a factor of 10, then that constitutes mathematical thinking and reasoning. Second, when possible elements of mathematical thinking and reasoning skills were identified as predict, deduce, justify, explain, interpret, and critique have been written in brackets. How factor game influences participants' mathematical thinking and reasoning were identified and its implications for mathematics teaching were discussed.

RESULTS

The result section presents the participants' response to tasks and questions and questions asked by the facilitator that direct students' learning. The analysis examined students' mathematical thinking and reasoning.

Participants on Factors of 15

Nancy and Awoe observed that since some number of unit blocks are equal in length with the long stick of length 15 units, then 1 is a factor of 15. Similarly, since 15 unit blocks equal in length with one long stick of length 15, then 15 is one of the factors [**deduce**]. The participants noted that 2 is not a factor of 15. The participants formed 5 groups of 3 and concluded that 3 and 5 are factors of 15 [**mathematical thinking and reasoning skills**]. The participants were able to form 3 groups of 4 with three-unit blocks left and therefore conclude that 4 is not a factor of 15.

In this case, the *facilitator* asked "Is it necessary to form groups of fives?" The participants responded "yes" [**incorrect answer**]. They then formed the groups and were asked to compare their results with the results they had when they formed groups of three and they realized there were no differences in the two results [**valid conclusion**].

Learners were not able to form exact groups of six and concluded that "6 is not a factor of 15". **Figure 3** shows a student explaining why 6 is not a factor of 15 [**explain**].

Figure 3. Student offering explanation to her results

When the facilitator sought an explanation for what they mean by not being able to form exact groups of six, Nancy has this to say "I formed two groups of six with three-unit blocks left. So 6 is not one of the factors of 15" [**explain**]. Facilitator: "does 6 divide 15?" Awoe: no. Facilitator: why? Awoe: "because I can't form a group of sixes exactly" [**justify**].

Learners noticed it's impossible to form groups of seven from 15 without remainder. Therefore 7 is not a factor of 15 [**conclude**].

Awoe concluded that 8 is not a factor of 15 since she "Has only one group of 8 and 7 unit blocks" [**justify**]. Facilitator: "if only one group of 8 are possible to be formed from 15 unit blocks, then is it possible to formed exact groups of 9 from 15 unit blocks?" The participants: "no, it is not possible" [**answer**]. Facilitator: why? Nancy: if 15 unit blocks gave me only 1 group of 8 with 7 unit blocks, then the same 15 unit blocks cannot give me two or more groups of 9, since 9 is greater than 8" [**explain, justify**]. Awoe responded, "When I move 9 unit blocks to one place, then the rests will not be up to 9" [**justify**]. The participants noticed there are no new groups to be formed to give factors of 15 that they had not formed [**deduce**]. Therefore, factors of 15 were listed as {1, 3, 5, 15}. The participants' observations and justifications here constitutes mathematical thinking and reasoning.

Participants Finding the Factors of 30

Nancy: "2 is a factor of 30". Facilitator: "Awoe do you agree or not and why?" Awoe: I agree because I formed 15 groups of two with no remainder" [**explain**].

Learners form 10 groups of 3 from 30 units with no object left. Hence concluded that 3 and 10 are factors of 30 [**conclude**].

Learners form 7 groups of four from 30 unit's blocks with 2 blocks left and noted that 4 is not a factor since there's a remainder [**justify**].

The Learners form 6 groups of fives from the 30 unit's blocks with no remainder and concluded that 5 is a factor of 30. The participants: since 6 groups of 5 gives 30 [**reason**]. So, 6 is a factor of 30 [**conclude**].

Nancy noted that, "I need not to form groups of 6", when asked why, she said 6 is a factor of 30 from the formation of five groups [**correct answer, reason**]. So, doing so will not produce different results [**deduce**]. When Awoe was asked what she has to say about Nancy's response, she said, "I can't tell" [**no answer, no reason**]. Awoe was therefore asked to form the groups of six. She then agrees with Nancy that 6 is a factor of 30. The reasoning of Nancy is inductive. **Figure 4** is a photo of five groups of six blocks.

Figure 4. A sample of student's forming groups and counts groups

Learners form 4 groups of seven with 2 units' blocks left and concluded that 7 is not a factor of 30.

From the grouping, learners realized that they have 3 groups of 8 with 6 unit blocks. Therefore, 8 is not a factor of 30.

Learners form 3 groups of nines with 3 units' blocks left. They, therefore, conclude that 9 is not a factor of 30.

Awoe said, "10 is a factor of 30 from the earlier activity on grouping in threes. So, we do not need to form the group again" [**justify**]. This position has been supported by Nancy who added that "grouping in tens will not produce any different number" [**observe**]. This shows that students are able to identify patterns and justify the same when teaching activities are well planned. The factors of 30 were listed as {1, 2, 3, 5, 6, 10, 15, 30} [**answer**]. **Figure 5** presents visual tools like groups of objects of equal members led to students' counting the group one by one and listing the number of groups and the number of objects in one group as the factors of the given number.

Figure 5. Student forming groups and counts groups

Facilitator: "how would you tell whether or not 3 divides 30 using thirty unit blocks?" Awoe: "I will form groups of 3 and if no block is left, then I know 3 divides 30" [**explain**]. Awoe concluded that she has 10 groups of 3 without any unit block remaining, so, 3 divides 30 [**justify, reason**]. The explanation and the justification demonstrate mathematical thinking and reasoning. **Figure 6** is a photo of the participant proving that 3 divides 30 [**demonstrate**].

Figure 6. Student forms groups to test divisibility

Participants on Brain Game

Awoe mentioned the first number as 1 and the second number was mentioned by Nancy as 2 [**predict**]. This continued until all the factors of 42 were listed by the participants as {1, 2, 3, 6, 7, 14, 21, and 42} [**predict, conclude**]. The game was won by Nancy since Awoe could not mentioned all the factors within the stipulated time. At the end of the game, Awoe had 3 scores while Nancy obtained 5 scores.

DISCUSSION

Block Factor Game: Factors of a Number

Long stick and unit blocks were used in finding the factors of a number. Alternatively, factors of a number can also be found using block units only as was done in determining the divisibility of a number by another number. In this case, the number of block units is equivalent to the number we want to determine its factors. Learners were able to identify all the factors of a given number and concluded that factors of a number are all numbers that divide a particular number exactly. However, Awoe had difficulty in finding factors of a number correctly without the unit blocks. Nevertheless, this was not surprising since early graders tend to rely more on concrete materials in learning mathematics (Carbonneau et al., 2012). Similarly, the participants were able to form groups of equal numbers and drew conclusions on the divisibility of a number by another number. The lesson started with manipulation of concrete materials, later the participants were made to predict before manipulation. This helped to develop predictive skills among the participants. The 'brain factor game' which is metal operations prepares students to engage in the division, addition and subtraction daily which is needed in real-life situations which in most cases does not require pen and paper or calculator and improve their problem-solving skills. The strategy is developmentally suitable for many primary school students and can be adapted for teaching prime numbers and composite numbers. It was observed that students enjoyed this game a lot even though they were not able to predict all the factors correctly. The number of times the game can be played depends on the number of people in the group. This is important so that each member has an equal opportunity to predict. Additionally, in planning for the number of times the game can be played, one should take into account the number of factors as well. The participants were able to identify patterns and relationships, and justify the same and this constitute mathematical thinking and reasoning. The participants' in most cases apply inductive principles of reasoning in mathematics in responding to tasks or questions. In inductive reasoning, inferences are made based on some examples which may not be sufficient for generalization as compared to deductive reasoning. Therefore, inductive is subject to deductive for conclusion on its generalizability. But at the early stage of mathematics learning, inductive reasoning is essential for learners' later mathematical concepts which require much deductive reasoning. Deductive reasoning is when a conclusion follows necessarily from the given premises. Its truthfulness depends on the premises.

Learners observed that when they started forming groups of objects starting from 1 up to half of the number they want to find the factors of, then all possible factors of the given number are determined. Therefore, there's no need to try to form a group of any integer greater than half of the number they want to find the factors of. Additionally, learners noted that 1 is a factor of every positive integer. It was also observed that every number is a factor of itself. These observations indicate conceptual understanding. *Nancy* explains "factors of a number as all numbers that can divide the given number exactly" while *Awoe* explains factors of a number as "all numbers that can be organized into equal groups of the given number". This suggests that when the "block factor game" is used to teach factors of numbers, students will be able to explain 'factors of a number'.

Brain Factor Game

The participants were able to do a lot of mental operations as such were able to predict most of the factors correctly. However, Awoe had some difficulty in predicting factors of numbers compared to Nancy. This was understandable since Nancy had more experience with numbers than Awoe. Interestingly, apart from 42 that the participants mentioned as one of the factors of 42, none of them predicted a factor greater than 21. Students realizing that there is no number between 21 and 42 which is a factor of a 42 constitutes mathematical thinking. The participants were able to identify patterns, explain relationships or provide justifications to their results. This means the block factor activities had a good impact on participants' thinking and reasoning.

The activity games especially the 'brain factor game' were effective in promoting students' creativity and critical thinking. Kula (2021) found that mind games contribute to students' communication, creativity, problem solving, mathematical and logical thinking, and academic skills for their functioning in the world. Every student irrespective of the geographical location should have the opportunity to learn to think mathematically and reason mathematically (both deductive and inductive) in classrooms (OECD, 2021b), and teachers can provide this opportunity through mind games. Therefore, mind games should be encouraging in mathematics classrooms since it plays essential roles in students' development of mathematical skills. The mathematics behind the games should guide and direct teachers' questioning to keep students focus to acquire the mathematical skills behind the games. Mohd and Shahrill (2021) examined the effectiveness of a non-digital game-based learning approach on students' mathematics achievement and found positive effect on students' achievement scores. Their study further found that a non-digital game-based learning enhance their mathematical skills. The non-digital game-based learning is less expensive to implement since they do not require the use of computers or digital devices. Therefore, can easily be used in an emerging economy.

Mathematical Thinking and Reasoning and Assessment

The fundamental laws that govern mathematical concepts, and how we acquire mathematical knowledge are dependent upon critical thinking and reasoning. Students' mastery of mathematical concepts should be to convince us that a certain statement is true or false and explain why the statement is true or false. The findings of this study portrayed reasoning skills that are predictive of mathematical abilities or achievements should be at the center of assessments of cognitive development in mathematics (Nunes & Csapo, 2011). Correct answer or solution without justification should not be the overall premise to determine the ability of the students. This implies that in teaching and assessment, students should have opportunity to justify their results or answers. Someone who understands the meaning of any concepts of mathematics must be able to think and reason in the meaning of that concept. Developing reasoning in mathematics learning is crucial because learners need it at time of decision-making when solving problems. Decision-making capacity is needed to respond to rapidly changing conditions of life (Oiknonomidis, 2018) and this decision-making require critical thinking skills.

> "People who think and reason analytically tend to observe structure, patterns or differences in both the real world and mathematical situations" (Alhunaini et al., 2021, p. 3).

Teaching and learning should seek to develop critical thinking skills in learners and provides opportunity to assess learners' use of critical thinking and reasoning skills in making choices in life. For what use will it be to accompany to produce goods for its customers and not able to transport these goods to the customers? The inclusion of mathematical thinking and reasoning skills is only complete when teachers can integrate it effectively and efficiently in lesson for learners' acquisition, and not just its inclusion in mathematics curriculum. The results of this study are in support of the findings of Mueller et al. (2014) that teachers questioning can promote effective learning. These questions should demand from students to offer explanation or justifications to their solutions or answers. All teachers need to be exposed to various forms or components of a constructivist theory in their training if indeed we expect them to use critical questions to guide the learning of their students. Teachers' philosophical ideologies influence their way of teaching. So, teacher educators must inculcate into student-teachers the relevant philosophy for teaching right from the beginning of their training by their way of instruction. In this regard, scholars have the responsibility to investigate the mathematical thinking of the expected learners of their preservice teachers. Understanding this reality and using them in an appropriate context would be a great example to preservice teachers. For example, if we (researchers or teacher educators) expect teachers to consider the history of mathematical concepts and the learners' background in their teaching, then we must equally provide the historical, cultural, philosophical, and post-epistemological perspectives on how it stimulates learning. This will give both preservice and in-service teachers cogent reasons for why they should teach their students in that particular way. Jonsson et al. (2014) posited that mathematics invokes three basic competencies which are problem-solving abilities, reasoning ability, and conceptual understanding. This suggests that mathematics instructions should develop such skills in students. Therefore, mathematical skills together with students' ability should determine the instructional strategies of the teacher.

Solutions to most problems in mathematics require precision and accuracy which are premised on critical thinking and reasoning. Based on this, many view Mathematics as an exact science. It is the most discipline that can claim certainty of results generally. In many Mathematics problems, the results are either right or wrong; true or false. The mathematical statement is either true or false but not both. Mathematicians do determine whether or not their conclusions are right. Although there is an emphasis on approximation or estimation in this century, mathematical results are still expected to obtain some level of accuracy. It is the responsibility of the teacher to lead the students in attaining this degree of accuracy which is essential for measurement or decision-making. One main reason for the study of Mathematics is for learners to apply the skill acquired to new situations meaningfully. Students with mathematical thinking and mathematical reasoning skills can always determine the validity of mathematical rules and relationships when using them in a new situation to solve problems. This is highly achievable through probing and pushing questions which are sound ways to develop mathematical thinking skills in students (Titin, 2014). Critical questions inspire, promote, and direct students' mathematical thinking and reasoning which is necessary for their intellectual growths. It must be noted that the development of mathematical thinking and reasoning takes time, but a conscious effort should be made to accelerate its growth in a student right from early grades. Teachers' questions should be timely and connected to the skills the teacher expects the learners to acquire. In this regard, learners become reflective learners who assess their thinking and that of their colleagues which is good for active intellectual discourse. This reflection enables students to reconstruct the meaning of concepts by reviewing their strategies for solving problems and examine their experiences. At this point, learners can challenge their misconceptions and that of others. This intends prepare them to become great problem solvers in the world.

CONCLUSION AND IMPLICATIONS

The study concludes that, the students were able to list factors of a number and test the divisibility by another number in block factor games successfully. While Nancy succeeded in translating that understanding from block factor games to list factors of a number without the unit blocks (concrete materials), Awoe had difficulty listing the factors without unit blocks. This implies teaching factors of a number at primary 2 should focused more on the use of concrete materials (Furner & Worrell, 2017). In this study, students' response to tasks and questions with explanation and justification constitute mathematical thinking and reasoning. This study demonstrates how mathematical skills like thinking and reasoning can be cultivated in students at primary school for their late mathematical concepts. When mathematical skills are the bases for the teaching strategies adopted by the teacher which inform questions and assessment, students tend to acquire critical thinking and reasoning skills. The participants were able to use direct and indirect reasoning to support their answers in most cases. This study does not only better teachers' understanding of how to incorporate mathematical skills into their lessons but also advocates for factor games for teaching factors of a number and other number concepts (e.g., prime numbers, composite numbers). Critical questions provoke students' mathematical thinking and also direct their reflections. So, teachers should construct their questions carefully to support students in teaching and learning mathematics. Additionally, teachers need to vary their questioning techniques to check consistencies in students' reasoning. Students justify their answers to convince each other. This was encouraged to allow students to have confidence in their thinking which is good for their intellectual growth and enhance their problem-solving skills. This tends to enhance their mathematical literacy.

> "But mathematical literacy goes beyond solving problems in the traditional sense of the word to include making informed judgements about those important family or societal issues to depend on reasoning" (OECD, 2021a, p. 8).

In this century and the century to come students must be guided to learn to reason and reason to learn to be economically powerful through thoughtful decision-making. In this regard, they will not just accept anything in life, but accept things based on principles or rules and always conscious of the implications of their actions and inactions.

Limitation

Firstly, this study is limited in method; that is only qualitative method was used. The quantitative research method should be added to enable an in-depth understanding of students' skills acquisitions. Also, this study was limited with a small sampling of students; therefore, additional studies involving large participants are warranted to confirm or contradict the current findings. The study activities were carried out in a day may be considered a limitation, but it was necessary considering the COVID-19 guideline requesting people to stay home more often and limit social gathering.

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