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Investigating the effectiveness of flipped classroom model in a mathematics education course in Greece

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ABSTRACT

An alternative instructional model, which due to the social reality created by the ongoing COVID-19 pandemic is gaining more and more popularity in the global educational community, is the so called "flipped classroom". In this paper we give the details of a one-week qualitative research we conducted in order to explore the effectiveness of the flipped classroom model in an undergraduate "mathematics education" course in Greece. The study involved thirty third-year math students, whose achievements and perceptions were captured with the help of questionnaire, task, personal interviews, and open discussion in the class. The analysis of the research data indicated increased engagement of the students, development of positive attitudes about the learning, control of the learning pace, autonomy in managing the time required for study and relative improvement of students' performance in problem solving and teamwork. At the same time, several challenges emerged. The main ones concerned the demand for frequent active students' participation in the educational process, the management of queries during the homework, the doubts about the effectiveness of the method on difficult teaching subjects, as well as the time and skill requirements for preparing such a lesson.

Keywords: flipped classroom, mathematics education, tertiary education, problem solving, qualitative research

INTRODUCTION

Within the COVID-19 pandemic, many university institutions have encountered obstacles in providing quality education or even suspended their operation due to a lack of technological infrastructure, while others saw the crisis as an opportunity to adapt their pedagogical methods appropriately and to create or extend innovative and flexible learning approaches (Crawford et al., 2020; Divjak et al., 2022; Lassoued et al., 2020; Rizos & Gkrekas, 2022). One such innovative teaching approach, which has recently caught much attention, is the so called "flipped classroom".

Flipped or inverted or upside down classroom is a pedagogical approach that is gaining ground both at school and at university around the world. It is based on the simple but groundbreaking idea of reversing the educational process and can potentially transform teaching. Specifically, the lesson is not conducted face-to-face in a group learning space, but in an individual one with the help of appropriate digital material, while the tasks are done, usually collaboratively, in the classroom. Students are prepared using digital resources, designed to cover the traditional lecture part, while taking initiatives on issues of space, time, and the pace of learning.

Especially in Greece, with the exception of a few isolated cases (e.g., Aidinopoulou & Sampson, 2017; Gariou-Papalexiou et al., 2017), flipped classroom first appeared in the public debate and was then used, rather hesitantly, by instructors, during the COVID-19 pandemic and the ensuing incarceration–mainly the academic year 2020-2021. Today it officially comes to the fore in the context of the new curricula announced by the Greek Ministry of Education. However, many educators have serious reservations, considering that this new method weakens their pedagogical role and replaces knowledge with visualized information.

In this paper, we describe a qualitative research we carried out aiming to identify the teaching-learning benefits and the limitations that arise from the application of the flipped classroom instructional model in tertiary education and specifically in a Mathematics Department in Greece. Our sample was thirty undergraduate students, and the subject of our teaching intervention was the divisibility. The participants had minimal to zero experience in the flipped classroom and were encouraged to play a dual role during the teaching intervention: we asked them on the one hand to face the educational process as students, while on the other hand to judge the flipped classroom model as potentially math teachers. In addition, we look into the way flipped classroom can operate as a team learning method while working on problem-solving mathematical tasks in tertiary education. To the best of our knowledge all of the above constitutes a new approach, at least by Greek standards, for which not much previous work has been done.

In order to familiarize the reader with the topic, we compose a definition of the flipped classroom grounded on the available literature and we analyze the rationale of this approach based on the use of information and communication technology (ICT) in distance learning. Then, we focus on empirically investigate students' attitudes and their degree of participation in the educational process, we intend to examine the contribution of the model to the effectiveness of teaching methods and techniques, while we are also interested in the role of the educator especially in relation to the design and construction of the educational material. In the following paragraphs the necessary theoretical background is set and information about the course "mathematics education", in which our research was conducted, are given. Then the methodology is developed and the data of the qualitative research we conducted are analyzed. The article ends with a discussion and drawing some conclusions.

As revealed by our research, the flipped classroom model seemed to be a *structural change* in education (at least in Greek tertiary one) and at the same time a *good practice* in which activity holds a dominant position in contrast to the traditional teacher-centered lecture. However, the effect of flipped classroom in enhancing students' academic performance remains a challenge (Cabi, 2018; Fung et al., 2021).

THEORETICAL BACKGROUND

An Updated Social Context

The COVID-19 pandemic brought rapid and radical changes in the education systems of all the countries of the world. For instance, more than 1.5 billion children and youth from 188 countries around the world have been badly influenced by the sudden "lock" in schools and institutions (UNSDG, 2020). The urgent need for social distancing has led to the revision of classical face-to-face teaching, establishing practices in which digital resources play a crucial role. The changes in the educational reality generated from the coronavirus pandemic demand new working methods, different from the traditional exhibition classroom (de Sousa & Alves, 2022). Thus, ICT was de facto entrenched as the basic and in many cases unique tool of synchronous and asynchronous education (Karalis & Raikou, 2020) and were more prominently used by teachers to interact with students in comparison to verbal feedback in class or educator notes in workbooks (Kalogeropoulos et al., 2021). In fact, given the extent of the challenge, the claims of researchers who state that digital tools can be considered as resources with better learning outcomes than traditional teaching are grounded (Reimers & Schleicher, 2020). At the same time, was caused a global economic and political crisis, and several problems were revealed, such as the lack of digital education strategy, which indicate that almost all education systems in the world were not prepared to deal with such a situation. In this updated social context, the pedagogical model of flipped classroom should be seen.

Literature Review About Flipped Classroom

According to the Flipped Learning Network (2014) the word "FLIP" is an acronym for the word-pillars *flexible environment* (spaces and time frames where educator-student communication develops), *learning culture* (both cultivated by the educator to facilitate and encourage his/her students, as well as students by actively and independently participating in the learning process), *intentional content* (depending on grade level and subject matter), and *professional educator* (equipped with more qualifications and skills on the subject and the use of technology). A distinction is also made between the terms "flipped classroom" and "flipped learning". For the purposes of this paper, we will not insist on this distinction, but we will use the rather more common term "flipped classroom", giving it the following content:

In general, flipped classroom is an innovative pedagogical approach that reverses the educational process (Bergmann & Sams, 2012) and consists of three stages:

a) the pre-class,

b) the in-class, and

c) the post-class (Estes et al., 2014).

In pre-class stage students are introduced to a new subject with the help of appropriately designed and easily accessible educational material (videos, podcasts, PPT presentations, quizzes, etc.) posted on a digital platform. They view the content at home on their computer, tablet, or mobile phone, interact with the educational material and take notes. In in-class stage, students make use of what they have learned in the previous stage, formulate their observations or queries, clarify the new concepts, and do the tasks with the guidance and support of the educator (Bishop & Verleger, 2013), having more available time and working, mostly in groups, on various activities (e.g., problem solving). Finally, in post-class stage, the students, with the teacher's encouragement, evaluate the achievement of the learning objectives and assess the whole process. If they want to check their knowledge, they have the option to go back to the digital learning material, which they now study from a different viewpoint (Estes et al., 2014).

Direct instruction is combined with constructivist pedagogical practices (Vygotsky, 1978), inquiry-based learning (Love et al., 2015), and the use of digital tools, in order to facilitate differentiated learning (Lawrence-Brown, 2004; Tomlinson, 2017) and in-depth understanding (Warter-Perez & Dong, 2012). The learning process is not confined to the physical classroom space, but students are given the opportunity to take initiative and receive immediate feedback (de Araujo et al., 2017), move at their own pace, overcome any misconceptions, and direct their efforts based on their needs, thus personalizing the teaching. Flipped classrooms can be used to support student-centred learning through the pedagogical dimensions of personalization, higher-order thinking, collaboration, and self-direction (Koh, 2019).

The above approach is inextricably linked to the use of new technologies and teachers, as is natural, give their own personal touch to its implementation each time, however the concept of the flipped classroom remains essentially the same. The role of teacher shifts from following in-class schedules to planning activities and providing learning resources that can be used as and when required (Davies et al., 2013). In general, educator's role is broadened and enriched with pedagogical, technological, and administrative parameters. Educators also gain the advantage of managing a learning management system (LMS) that allows them to post digital material and create quizzes, personalized and mass communication and feedback, as well as informing their students' progress.

The flipped classroom is part of the blended/ hybrid learning model, which combines face-to-face teaching with distance learning (synchronous or asynchronous) using ICT (e-learning), so that one can substantially supports the other (European Union, 2020). Blended learning is not learning that takes place in the classroom with online reinforcement, nor online learning that is enhanced by teaching in the classroom. Synchronous and asynchronous activities are both important as they serve different purposes and can complement each other. Synchronous e-learning increases arousal and motivation, while asynchronous e-learning increases the ability to process information (Hrastinski, 2008). This model tends to become common practice at all levels of education (Alonso et al., 2005; Owen & Dunham, 2015), especially since e-learning emerged in the period of the pandemic and the subsequent lock-down (March 2020-June 2021) as a stand-alone form of education and became experientially known to the entire educational community. The main reason for this seems to be, beyond practical issues of time management, learning pace and digital tools, the fact that students understand the concepts taught more easily and in greater depth when they are actively engaged in the educational process rather than passively watching (Datig & Ruswick, 2013). Even theoretically weaker students having dynamic participation, continuous support from the teacher and more instructional time to experience group collaborative activities, achieve better learning outcomes (Tucker, 2012).

Focusing on Higher Education

Although the idea of the flipped classroom originated in schools and most of the international literature concerns its application in secondary education, in recent years more and more researchers have turned their attention to the use of the flipped classroom in higher education, with a focus on problem solving (Alias et al., 2020; Arnold-Garza, 2014; Hwang & Chen, 2019). In fact, there are university teachers who, while they had not planned from the beginning to implement the flipped classroom model in their courses, discovered it along the way of the academic year, either seeking to create a more engaging learning experience that leverages available technology (Dotson & Diaz, 2008), or because they discerned that doing so could improve certain skills of their students (Findlay-Thompson & Mombourquette, 2014). Research, however, converge that the utilization of flipped classroom in higher education is mainly advocated to promote students' engagement, metacognition, attitude, performance, understanding, and achievement, as well as other learning outcomes (Al-Samarraie et al., 2019).

According to Hertz (2012), the core idea of the flipped classroom is not to demote the educator or change the educational content, nor of course to substitute in-depth understanding of concepts and processes–especially in mathematics–with blindly executing algorithms. It is therefore interesting to see how an innovative teaching approach, which advocates the active involvement of learners over procedural teaching and learning, i.e., the direct provision of formulas and solution steps (Legesse et al., 2020), can be implemented in university practice. However, to the best of our knowledge, there is no domestic research that focuses on the teaching of mathematics modules in Greek higher education, particularly in mathematics departments, using the flipped classroom model. This paper aspires to contribute towards filling this research gap.

To design and implement our research in a university department, we took into consideration the factors which, according to the literature, play an inhibiting role in the application of the flipped classroom model. Thus, at a first-order barriers (Wang, 2017), it should not be taken for granted, especially with regard to the Greek educational reality, that all students and teachers have the appropriate technical infrastructure (modern equipment, high speed internet connection, etc.) and are familiar with ICT. This is also confirmed by a recent survey conducted at the same university department (see Rizos & Gkrekas, 2022), according to which the vast majority of students during e-learning were systematically experiencing technical problems with low internet speed and other network errors. Moreover, for teachers, the selection and development of appropriate digital material is a generally demanding process that requires knowledge and long preparation (for example, we spent more than a week on the design and construction of the teaching material), while for learners, depending on how they manage personalized learning, there is a small but real risk of replacing knowledge with skills and visualized information.

At a second-order barriers (Fullan, 2015), some teachers, depending on their beliefs/ attitudes about technology and their degree of adaptability to change, may not be willing to adopt the flipped classroom model immediately. According to Ertmer (1999, 2005), priority should be given to addressing such barriers because teachers may not be willing to adopt ICT in their teaching even if all the first level barriers have been removed.

The Course "Mathematics Education" in a Greek University Department

In the university department, where we conducted our research, "mathematics education" is an elective course for third-year students. The main aim of the course is the discussion of basic theoretical issues (e.g., learning theories, instructional models, lesson plans, integration of ICT in teaching practice, classroom management, psychological notions about mathematics education, etc.) alongside the involvement of students in the design of mathematics teaching in education. In practice, in addition to the development of theory, independent activities and tasks are implemented, which are characterized by experimentation, problem solving strategies, construction of mathematical models and teaching scenarios in the context of interdisciplinarity (Rizos, 2018; Rizos et al., 2021). Thus, undergraduate students, i.e., prospective math teachers, have the opportunity to cultivate their own way of thinking and acting in the modern society of knowledge. It should however be noted that the COVID-19 pandemic has changed the agenda of mathematics education (Borba, 2021), especially in terms of the role of digital technology.

Table 1. Statistics for the research participants

Participants		Sex		
		Male	**Female**	**Total**
Number theory	Yes	2	16	18
	No	4	8	12
Total		6	24	**30**

A Few Words About the Traditional Method of Teaching

In the traditional teaching method, the teacher is charged with the responsibility of imparting knowledge to the students, while the students are obliged to listen carefully, take notes, and learn what their teacher says. The focus is on the educator, who dictates to the pupils what is "correct" or "important" and they in turn have a duty to commit the information to memory (Hu et al., 2018). Particularly in mathematics, the traditional teaching model focuses on procedural knowledge, i.e., providing instructions, formulas, and solution steps (Legesse et al., 2020) without emphasizing the underlying mathematical structures. The theoretical background of this approach is essentially the Behaviorism, according to which learning is a change in behavior, while behavior is shaped through interaction with the environment (Krapfl, 2016). As can easily be seen, the models of traditional teaching and the flipped classroom are diametrically opposed.

METHODOLOGY

In the spring semester of the academic year 2021-2022 we designed and implemented, within the course "mathematics education" of a mathematics department of a Greek university, a research project for the theme of "divisibility". We chose the specific course because

i) it provides the possibility of implementing interactive classroom activities and experiences,

ii) it includes in its syllabus the involvement of students in problem solving processes, and

iii) it highlights the significant role of technology in education.

Furthermore, we considered that the mathematical content of the project could arouse participants' interest.

The research, which lasted a total of one week, was attended by 30 students (24 girls and six boys) of the above department. The participants belonged to the so-called "alpha generation", i.e., they were born after 2000 (dos Reis, 2018), had at least one electronic device (smartphone, tablet, and laptop) with an internet connection and used it with ease. Moreover, they had gained significant experience in e-learning, and they had already been examined remotely five times (June and September 2020, January, June, and September 2021) using MS Forms, eClass, etc. at their institution (see Rizos & Gkrekas, 2022).

The research was carried out at the beginning of April 2022, when all the courses of the university were taught face-to-face. We verbally informed the third-year students (average 21 years old) in the class, students who were attending the course "mathematics education", for the conduct of a three-phase project entitled "divisibility" and we invited them to participate voluntarily in it. At the same time, we posted an announcement in LMS eClass, an announcement, which was sent by email to all 48 students who had registered for the course, of which 30 responded. Of the 30 students, 18 when they were sophomores had attended the elective course "number theory" (prime numbers, the fundamental theorem of arithmetic, divisibility, greatest common divisor, least common multiple, the Euclidean algorithm, linear Diophantine equations, arithmetic functions, the Chinese remainder theorem, Fermat's little theorem, etc.). No student had been taught number theory in high school (grades 10-12), as this course is not part of the Greek school curriculum (see **Table 1**).

The students had minimal to zero knowledge/ experience in the flipped classroom and were encouraged to play a *dual role* during the teaching intervention. More specifically the participants were asked to face the new for them, educational process as students, while at the same time to judge the flipped classroom model as prospective math teachers.

Research data were collected by three methods: questionnaires, personal informal interviews (Mishler, 1991) and discussion in the class; three different time periods, from three different perspectives: the researcher, the two observers and the students. More specifically, the first author of the paper was "the observer as participant", a person who actively participated in the project by making crucial interpositions, and the two other authors were "the participants as observers", persons who participated in all phases only by observing the research activities and taking notes (Burgess, 1984). Essentially, we followed the multiple triangulation strategy (see Denzin, 2009), which encourages several data collection methods at various time periods, and the involvement of investigators with diverse expertise and roles. Thus, the validity and reliability of the research data was achieved.

Our main aim was to investigate the effect that the flipped classroom has on a mathematics education course at a Greek university department, identifying and highlighting the benefits as well as the points of concern that the application of the model can bring to the teaching practice. Thus, our research can be considered as *applied* (Hedrick et al., 1993), in the sense that any information that emerges from it, could be immediate used in the context of university teaching and solve practical problems.

To approach the above objectives, we pose the following interrelated research questions:

1. Is the flipped classroom methodology effective in higher education (in terms of understanding the educational material and cultivating positive attitudes towards learning)?
2. To what extent can the flipped classroom contribute to the active involvement of undergraduate students in the educational process and the control of the learning pace?

Similar questions have been raised by other researchers (see Galindo-Dominguez, 2021) who have applied the flipped classroom model in university departments.

First Phase of the Project

On the first day of the research, we held an hour-long classroom meeting with the 30 participants in order to give details about the content and process of the "divisibility" project, and to explain them in detail what the flipped classroom is. None of the participants had any experience with this teaching model when they were high school or undergraduate students; only five participants had recently read some things about it on social media and educational websites.

In the "documents" tab of the eClass of the course "mathematics education" we had already created a folder entitled "divisibility", in which we added the following educational material designed by us. We spent almost a week preparing the educational material:

1. Notes in .pdf format, which included the basic theory of divisibility, i.e., introductory activity, definitions (when we say that an integer divides another integer, what is it called divisor, etc.), as well as a theorem (about the properties of divisibility) with its proof,
2. Notes in .pdf format, where we listed six exercises solved in detail (e.g., prove that $4 \nmid (\alpha^2 + 2)$, for each $\alpha \in \mathbb{Z}$),
3. Two concept maps on the distinction between exact divisions and divisions with remainders, and on the distinction between even and odd integers. In general, the use of concept maps helps to avoid rote learning and has been successfully applied at both school and university level (Ullah, 2020),
4. Four self-assessment exercises with optional hints (e.g., find integers that divide the integers $2a + 3$ and $3a - 4$, $a \in \mathbb{Z}$),
5. Material for further study (links leading to websites with solved and unsolved exercises on divisibility, as well as *Python* codes that computed the greatest common divisor and the least common multiple of two numbers, and
6. Detailed instructions to students on how they should work on the project.

In "multimedia" tab in eClass we had attached two videos of a total duration of eight minutes, which consisted of a PPT presentation based on the notes 1 and 2 described above, while one of us was reading the texts of the slides with constant rate and giving clarifications and additional information where necessary. To create the video, we used the free streaming and recording program OBS Studio, as well as the free editing application Shotcut.

In the online survey creator "MS Forms" we had created a self-assessment quiz consisting of ten multiple-choice and true-false questions, which students could answer at the end of the second phase in order to check for themselves if and to what extent they understood the educational material provided. Some typical questions were, as follows:

1. If 7/(α+5) and 7(40-b), then
   a. 7/(a+b),
   b. 7/(a+b+1),
   c. 7/(a+b+2), and
   d. 7/(a+b-3).
2. The product of three consecutive integers is divisible by three. True or false?
3. If a/b, then there is an integer k such that a=kb. True or false?

Second Phase of the Project

During the second phase of the project the students had three whole days at their disposal in order to study the material described above, to take notes and to answer the self-assessment questions. The success rate in each of these self-assessment questions was over 75%. During these three days students, as already had been informed, could address their questions or comments via email to the first author of the paper. However, the students had no questions, because, as they later mentioned, the educational material was understandable, and the study went smoothly.

Third Phase of the Project

On the fifth and final day of the project, students participated in a two-hour classroom session. The meeting was facilitated by the first author of the paper while during the meeting the other two authors took notes, independently of each other, on the participants' comments and the whole process. We divided the students into two main groups based on whether they had attended the elective course "number theory" or not, and then we randomly split each group into teams of four or five students. More specifically, we created three teams of four for students who had not attended "number theory", and two teams of four and two teams of five for those who had. The students, based on what they had studied in the second phase of the project, were asked to solve a problem on divisibility that we handed out to them (see below). During the task each team could ask the first author for clarifications.

Task

> Find all rectangles with integer side lengths, whose area is numerically equal to their perimeter.

The teams were given enough time (one and a half hour and an additional ten minutes for anyone who wished) to work on the task, which in general was dealt with satisfactory. Many ideas were formulated, and different solution strategies were followed,

Table 2. Evaluation by students (results)

Question	Choice a	Choice b	Choice c	Choice d	Choice e
Q1	0 (0.00%)	3 (10.00%)	20 (66.67%)	6 (20.00%)	1 (03.33%)
Q2	1 (3.33%)	7 (23.33%)	11 (36.67%)	8 (26.67%)	3 (10.00%)
Q3	0 (0.00%)	2 (06.67%)	13 (43.33%)	8 (26.67%)	7 (23.33%)
Q4	0 (0.00%)	1 (03.33%)	11 (36.67%)	15 (50.00%)	3 (10.00%)
Q5	2 (6.67%)	25 (83.33%)	3 (10.00%)		

but the most important thing was the terrific cooperation between the students. This is of particular importance, especially considering that in the Greek educational system students are not used to working in groups.

At the end of the meeting the researcher encouraged the students to evaluate the project anonymously by filling in a form and take part in a final discussion. The form was consisted of the following six questions.

Form

1. **Q1.** *If we define the full guidance from the teacher to the student as one and the full autonomy of the student as five, to what degree of this scale do you personally feel comfortable during the educational process?*
   a. 1
   b. 2
   c. 3
   d. 4
   e. 5
2. **Q2.** *Do you face difficulties with time management in study, preparation of homework, assignments, etc. at the university?*
   a. Never
   b. Rarely
   c. Sometimes
   d. Often
   e. Always
3. **Q3.** *Do you think that the project "divisibility" takes into consideration the diversity of each student (in the way of learning, in the pace of learning, in the management of time, etc.)?*
   a. Not at all
   b. A little bit
   c. Enough
   d. A lot
   e. Very much
4. **Q4.** *To what extent did the "divisibility" project achieved to motivate you to actively participate in class discussions and tasks/problem solving?*
   a. Not at all
   b. A little bit
   c. Enough
   d. A lot
   e. Very much
5. **Q5.** *Would you like the flipped classroom model to be applied in the future, even on a trial basis, in other courses of your Department?*
   a. No, not to any
   b. Yes, in some
   c. Yes, in all
6. **Q6.** *Write a comprehensive review on the project "divisibility" (100-150 words)*.

Questions Q1 and Q2 were intended to help us to understand the profile of the participants, while Q3 and Q4 were more focused at the second research question. The remaining two questions were asked to obtain the necessary feedback, as intended for the post-class stage (Estes et al., 2014). The students' choices in the first five questions are summarized in **Table 2**.

From the answers we received to the sixth and last question of the evaluation form, as well as from the final class discussion, we quote indicatively, using pseudonyms, and comment briefly on the following:

> Achilles: "The process, i.e., to watch the video, read the texts, find extra information, and answer the self-assessment questions at home, seemed interesting and fun. It was something different."

Table 3. Table of values

a	b	a+b	ab
1	1	4	1
2	1	6	2
2	2	8	4
3	1	8	3
3	2	10	6
3	3	12	9

> Hera: "It was a method that I think most of us had never met before. I enjoyed the freedom to study the material alone when I had time. It would be nice if this method could be applied to other courses as well."
>
> Arethusa: "[…] there was a document with instructions and additional information, which was very useful, because someone who either could not attend the actual lesson for their own reasons or wanted to see more things or to delve deeper into the matter, had the opportunity not to be left behind."
>
> Helen: "I liked the interactive nature of the lesson. In addition to the home study, we were given the opportunity to collaborate and exchange ideas in order to solve a problem in the classroom. All of this motivated me to be more active in class […]."

As it turns out for most students the flipped classroom was a pleasant experience, completely different from the traditional teaching method, which considered the diversity of each learner and also provided the chance of cooperation in the classroom. In general, positive comments were made about the design of the teaching material, the clarity of the instructions, the high degree of autonomy in learning, the possibility of self-evaluation and further research, elements which the majority of the students seemed to seek.

It is worth noting, however, the point of view of a student, let us call her Clio, who stated that

> "the flipped classroom can be very easily applied to young children with the proper guidance of the teacher, but I wonder if it could be used entirely in a more difficult subject, a university course for example."

This question is pertinent and lays the groundwork for further study and reflection.

After the meeting, we asked a restricted number of students to discuss, individually and in greater depth, their views on the project. So, we had personal interviews of about five-ten minutes each with Clio, Electra, and Hector.

ANALYSIS

With the help of the "MS Forms" platform we collected the data from the self-assessment quiz of the second phase of the project. As we saw earlier, the success rate in this quiz was over 75%, which is a first, positive indication.

Regarding the task of the third phase of the project, many ideas were formulated and explored, most of which were common to all student groups. In the following we list the problem-solving techniques developed by a group of four students who had attended the "number theory" course. Similar techniques, and in roughly the same order, were followed by all the other groups:

- Students drew a rectangular parallelogram with sides a and b.
- From the exercise data and in combination with the previous figure, group was led to the relationship $2(a+b)=ab$ (1).
- The quotient $ab/(a+b)$, i.e., area/perimeter, must be an even number.
- Students created the values shown in **Table 3** to find out a pattern.
- The ratio $ab/(a+b)$ was calculated, for various values of a and b.
- Students realized that it is difficult to derive a pattern only by the table of values, because there are infinite combinations for a and b.
- The relation (1) was restated (e.g., $\frac{ab}{a+b}=2$).
- An obvious solution was found. One student observed that 3 and 6 verify relation (1).
- A second obvious solution was found. The relation (1) is also valid when a=4 and b=4.
- The relation (1) was restated, as follows: $ab=2a+2b \Leftrightarrow ab-2a=2b \Leftrightarrow a(b-2)=2b$.
- It should be $(b-2)/2b$, i.e., the number $\frac{2b}{b-2}$ should be an integer.
- The operations carried out did not lead to any result.
- The students then asked the facilitator's help on how to show that the number $\frac{2b}{b-2}$ to be an integer. It seems that the students at this point reached the limits of their zone of proximal development (Vygotsky, 1978). Facilitator encouraged them to factorize the numerator, possibly adding and subtracting a number, to enable simplification of the fraction.

- After much discussion among the group members and trying out different versions, they came up with the following equality: $\frac{2b}{b-2}=\frac{2b-4+4}{b-2}=\frac{2(b-2)+4}{b-2}=2+\frac{4}{b-2}$.
- So, students observed that, $\frac{4}{b-2}$ must be integer or equivalently when $b-2=1$ or $b-2=2$ or $b-2=4 \Leftrightarrow b=3$ or $b=4$ or $b=6$.
  - Therefore, due to relation (1), the requested rectangles have dimensions: $(6,3)$, $(4,4)$ and $(3,6)$.

The plethora of problem solving techniques and the smooth cooperation between the members of the teams–elements that are generally rare in the Greek educational system–allow us to conclude that the students performed quite well in the task. From our experience regarding the attitudes and academic performance of students at the specific university department and other similar departments in Greece, we can conclude that the learning objectives were largely achieved. Therefore, one approach to our first research question is that the teaching model of the flipped classroom seems to be quite effective in terms of understanding the educational material and fostering positive attitudes towards learning. However, because we did not use a control group in our research, we have some reservations, and we avoid generalizations. We believe that a new study that would utilize two groups of students (an experimental group taught through a flipped classroom and a control group following traditional instruction) and examine whether there are statistically significant differences between the groups would be very useful.

Analyzing now the results of the Form, we observe that about 90% of students feel comfortable when they have a moderate to very high degree of autonomy during the educational process (see Q1 in **Table 2**), while 73% of students often face difficulties when asked to manage the time for studying and preparing homework (see Q2). Therefore, if we combine the students' answers to the first two questions of the form with their answers to Q6 (see e.g., Hera and Helen) and their responses to the dialogue that took place at the end of the project, we can conclude that the flipped classroom enhances students' autonomy, regardless of their performance, supports their active participation in the educational process and enables them to manage their time more effectively. In this context we can also include Electra's view, who in her interview stated:

> "Many students, like me, are forced to work in order to study; so, they are not able to attend all the classes. The flipped classroom could be a solution; In essence, it is a way to discover the knowledge on your own."

At the same time, 93% of the students believe that the project "divisibility" considers the diversity of each student in the pace and way of learning, time management, etc. (see Q3 in **Table 2**). Concerning the content of the project, 96% of the participants answered that they were activated because of it (see Q4 in **Table 2**). In other words, it seems that the teaching model, i.e., the application of the flipped classroom, together with the content we chose (the divisibility), were able to engage students in the educational process and keep their interest throughout our teaching intervention. Consequently, in relation to our second research question, we can say that the flipped classroom has a positive effect on students' active involvement in the educational process.

In question Q5, 93% of respondents said that they would like the flipped classroom model to be applied to some other subjects. We therefore tried to explore what criteria the participants would like this option to be based on. In his personal interview Hector said that "*with divisibility, I can say that I did pretty well, but what if the subject was e.g., topology or measure theory?*" Consequently, according to the students who participated in the research, the flipped classroom model can be more easily applied in primary school (see Clio's view at the end of the previous paragraph) or even in upper classes, as long as the subject matter is "easy". In other words, there is a caution which is directly related to the degree of difficulty of the subject to be taught. However, in order to study this correlation in depth, if it does exist, a new, perhaps correlational, study is needed.

DISCUSSION AND SUGGESTIONS

The ongoing COVID-19 pandemic has in fact created a new social, economic, and educational context, integrating into the institutional framework of the Greek educational system methods, such as distance learning, which until recently were not part of it. The implementation of synchronous and asynchronous e-learning as the only counterbalance to the suspension of face-to-face school and university departments, highlighted both good practices and challenges regarding the effectiveness of teaching methods and materials. The flipped classroom, embedded in the blended learning model, has recently attracted the attention of researchers and educators at all levels in Greece, as in many other countries of the world. The fruit of this interest has been both the communication of individual papers and the publication of special issues of journals. At the same time, the Greek Ministry of Education, with the recent law 4823/2021, officially brought the flipped classroom into the teaching scene–not without reactions from a part of the educational community.

In our research we aimed to examine the effect of the instructional model of the flipped classroom on the learning process, focused on a mathematics education course in a university department in Greece. The benefits that undoubtedly emerged were:

i) increased students' engagement,

ii) the development of positive attitudes towards both our teaching intervention and the teaching and learning process in general,

iii) the control by the students of the pace of learning,

iv) autonomy in managing the cognitive load and teaching time,

v) the enhancement of communication and collaborative skills, and

vi) the relative improvement of students' problem solving abilities.

These findings are in full agreement with the results of recent related research (Al-Samarraie et al., 2019; Doman & Webb, 2017; Galindo-Dominguez, 2021; Hwang & Chen, 2019; Latorre-Cosculluela et al., 2021; Sen, 2022; Sen & Hava, 2020; Sharkia & Kohen, 2021; Ward et al., 2021). Moreover, as the participants themselves stated (e.g., Arethusa and Electra), the flipped classroom provides equal opportunities for all students, especially those who have to work alongside their studies and cannot attend all university lectures.

Regarding the achievement of the learning objectives, as shown by

a) the success rates of the students in the evaluation questions,

b) the plethora of solving strategies of the problem posed in the third phase of the project, and

c) the discussion we had–especially with those students who described themselves "low-achievers"–the flipped classroom was quite effective.

Besides, everyone had the opportunity, even after the end of the project, to reflect on the educational process and to refer to the teaching material, which remained accessible to everyone until the end of the spring semester. This finding, although converging with the literature (Bhagat et al., 2016; Cevikbas & Kaiser, 2020; Tang et al., 2017), may not be completely independent of the good quality of the educational material provided and the clarity with which the instructions are formulated, elements that were available to students in this case. However, the clarity of the instructions seems to be a necessary but not a sufficient condition for enhancing academic performance. Indeed, recent research indicates that although flipped classroom is an effective alternative teaching and learning strategy, the evidence is still not strong enough to conclude whether is better than the traditional approach in terms of students' academic performance (Fung et al., 2021). At the same time, we should not overlook the question raised by two students (Clio and Hector), namely what would have happened if the teaching subject had been more difficult (e.g., topology, functional analysis, or measure theory). In other words, there is a doubt about the effectiveness of the method on difficult subjects. Only a new research focused on the educational material and the teaching subject could shed light on these concerns.

Moreover, some limitations emerged from our research. The main ones concerned

i) the requirement for frequent active students' participation in the educational process,

ii) the management of queries during the homework, as well as

iii) the time and skill requirements for preparing such a lesson.

More specifically, given that Greek undergraduate students have a large amount of homework to do, and an already overloaded schedule of daily lecture attendance, it is probably expected that they have time management problems and/or experience stress. Therefore, the requirement for frequent active participation in the educational process, i.e., the study that needs to be done in the *pre-class stage* as well as the participation in *in-class activities* (Estes et al., 2014) can hardly be combined with the students' contractual obligations. In this context, Clio's view on the flipped classroom can be interpreted:

> "I think that for it to work, it is necessary to reduce the lecture hours in the class so that there is time for extracurricular activities like this. […] I would like it to be implemented as something different to attract more interest in the other subjects."

So, it seems that the flipped classroom model cannot coexist without serious problems with classical teaching methods in an institution.

However, we cannot exclude the fact that many students find it difficult to respond to the study of the pre-class material, which requires spending time in front of a computer screen, because they are exhausted of the extensive use of electronic devices during the pandemic and the subsequent lockdown. This phenomenon has been observed in a recent research in the same university department (see Rizos & Gkrekas, 2022) and relates to the skepticism with which some students view technology. More specifically, if the use of technology in recent years had not been so extensive due to distance learning (and working), students would appreciate its abilities, but after using it for more than eight hours a day for about two years it seems like torture to them.

A vulnerable spot, at least on a theoretical level, of the flipped classroom model that emerged from our research is the lack of direct communication between educator and learner during home study, with all the unpleasant consequences that this lack implies. Helen in response to question Q6 notes the following:

> "I liked the interactive nature of the lesson [...] but what can be considered negative is that in case of questions, the learner does not have the possibility to ask the teacher directly at the moment the query arises but has to save his/her questions for the next lesson or send an email. Thus, he/she will either stop studying until the question is resolved or proceed with a blank until he/she receives a response to the email he/she sent."

This is a reasonable assumption and should be taken seriously into account in the design of a future relative project.

The third limitation concerns educators. Summing up our experience from this project, we can say that we spent *a whole week* to plan and construct the educational material we considered relevant for the pre-class stage, even though all three of us had the necessary knowledge and skills to do so. We therefore find it de facto very difficult for an educator to develop educational material and plan activities alone on a daily basis in order to fully implement the flipped classroom model in his/her class, especially if he/she teaches two or three courses in the same semester. This is in line with recent related research (Aghaei et al., 2020; Al-Samarraie et al., 2019). It is therefore reasonable to suggest that the educators should receive practical support from their

institution (e.g., by creating an activity/ tasks repository or by organizing training seminars) in order to cope with their expanded role.

Another point to bear in mind is that our research, like most related research, focused on a specific course–that of mathematics education–in a limited sample. Therefore, it is risky to attempt to answer a generalized question of the type "to what extent can the flipped classroom model meet the requirements of teaching mathematics in higher education?". If all the courses in a university department's curriculum (e.g., in a mathematics' department) were redesigned to align with the flipped classroom model, would instructors and students be able to devote sufficient time to prepare themselves? (Akcayir & Akcayir, 2018). In addition, could the institution's e-learning platform support such a structural change? These questions are now gaining special importance and need further study, since the attention of instructors is required to well-designed tasks for students' collaborative learning (Fredriksen, 2021), while at the same time the importance of the use of ICT in e-learning is highlighting.

CONCLUSION

The flipped classroom is an original process of active learning, at least for the Greek educational reality, which places students at the center of interest, considering their diversity in the way and pace of learning. At the same time, it is a useful teaching model for all educational systems, with a number of positive outcomes, such as the development of positive attitudes for learning and improvement in certain processes like problem solving and teamwork. But in order to be effective in a university course and given that there are no technical problems, the flipped classroom presupposes responsibility and concentration on the part of the students, since learning becomes essentially their business. It also requires self-motivation and proper management of the queries or any challenges that may arise during the home study. After all, it upgrades the role of educators but also raises the bar of their preparation, mostly in terms of the development of educational material and the selection of activities, thus making it necessary to support them in order to cope with their expanded role.

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