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Mathematics teachers' problem-solving content knowledge for teaching in disadvantaged contexts: Insights from a design-based intervention

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ABSTRACT

Mathematical problem-solving content knowledge (PSCK)–a core component of mathematical problem-solving knowledge for teaching–is essential for supporting learners' problem-solving (PS) proficiency. However, little is known about how PSCK develops among teachers working in disadvantaged contexts. This study examines the development of mathematics teachers' PSCK in disadvantaged South African schools through a design-based professional development intervention. Four grade 9 teachers participated in two six-month cycles of workshops and classroom-based activities designed to strengthen their PS pedagogy. Data were collected through classroom observations and semi-structured interviews and analyzed using reflexive thematic analysis. Findings show that teachers' understanding of what makes a mathematical problem meaningful is foundational to their professional growth in PSCK. Growth in teachers' ability to interpret students' unconventional solutions was closely linked to their capacity to infer reasoning from diverse solution strategies. Teachers also developed greater skill in problem posing when encouraged to reflect on their own processes of creating and reformulating tasks. While growth was evident among all participants, differences in teaching experience influenced the extent of development. The study broadens our understanding of mathematics teachers' PSCK in disadvantaged contexts and shows that these contexts do not merely constrain PS instruction; they actively shape the forms of PSCK teachers develop.

Keywords: design-based professional development, disadvantaged contexts, mathematical problem-solving proficiency, mathematical problem-solving knowledge for teaching, problem-solving content knowledge

INTRODUCTION

Over the past decades, problem-solving (PS) has primarily influenced research in mathematics education (Chirinda, 2021; Chirinda & Barmby, 2017, 2018; Lester, 2013; Polya, 1957; Santos-Trigo, 2024; Schoenfeld, 1985; Schreiber, 2025) and is regarded as one of the primary goals of mathematics instruction. It is viewed as an end result of mathematics learning and the means through which mathematics is learned (Lester, 2013). As Halmos (1980) famously noted, 'the mathematician's main reason for existence is to solve problems, and that, therefore, what mathematics consists of its problems and solutions' (p. 519).

Mathematical problems are tasks without immediate or apparent solutions (Polya, 1957). Accordingly, PS is simply what a problem-solver does to find a solution to the problem presented (Schoenfeld, 1985). This definition implies that for learners to be successful problem solvers, they must have applicable experience in PS, deep content knowledge, skillful use of various representations, and a solid understanding of recognizing and building inference patterns (Lester, 2013). If learners are required to possess the indicated attributes to become successful problem-solvers, one would wonder what knowledge teachers need to help learners become better problem-solvers. This study focuses on the knowledge teachers need to support learners in doing PS competently. Appropriate knowledge of instructional practice is the backbone of teachers' professional work.

Building on calls for a more practice-based conceptualization of mathematical problem-solving knowledge for teaching (MPSKT) that is contextually responsive (Chapman, 2015; Jacinto & Carreira, 2023; Lester, 2013; Piñeiro et al., 2021), this study focuses on one core component of this knowledge: problem-solving content knowledge (PSCK)–teachers' understanding of the nature and structure of mathematical problems, PS processes, and problem-posing.

This study, which is positioned within a larger project, stems from my ongoing endeavor to address the identified gap in the mathematics education body of knowledge. Little is known about how PSCK develops among teachers working in disadvantaged contexts, where multilingual classrooms, large class sizes, and scarce resources shape instructional practice (Tibane et al., 2024).

This study addresses this gap by examining how grade 9 mathematics teachers' PSCK evolves during a design-based professional development intervention in under-resourced South African schools. It is guided by the following research question:

How does grade 9 mathematics teachers' PSCK develop during participation in a design-based professional development intervention in disadvantaged South African schools?

This study makes three contributions to the field of mathematics PS pedagogy. First, it offers an empirically grounded account of how teachers' PSCK develops under the real constraints of under-resourced, multilingual classrooms. Second, it provides evidence that the contextual features of disadvantaged school environments not only shape the enactment of PS pedagogy but also actively shape the forms of PSCK that teachers develop. Third, the study identifies design principles for professional development that are responsive to such disadvantaged contexts and can guide future efforts to strengthen mathematics teachers' PS instruction.

THEORETICAL PERSPECTIVE

Effective mathematics teaching requires specialized knowledge that goes beyond the ability to solve problems. Teachers must unpack mathematical ideas for learners and orchestrate learning around them–what Ball et al. (2008) refer to as MPSKT. Being able to solve a problem does not, in itself, prepare a teacher to teach PS well (Ball et al., 2008). For that work, teachers need MPSKT–the knowledge required to design, facilitate, and assess rich PS experiences (Chapman, 2015).

In this study, MPSKT refers to the knowledge teachers need to hold to teach PS effectively and help learners become proficient problem-solvers. Following Chapman (2015), MPSKT comprises three interrelated components:

(1) PSCK,

(2) pedagogical PS knowledge: instructional strategies for fostering learners' PS skills, and

(3) teachers' knowledge and understanding of the positive and negative impact of the affective factors and beliefs on learning and teaching of PS.

Among these three components, this study focused on teachers' PSCK, which provides the conceptual foundation for these dimensions, shaping how teachers interpret learners' thinking and select or design meaningful mathematical tasks.

As conceptualized earlier, PSCK underpins teachers' instructional decision-making by shaping how they select tasks, interpret learners' solution strategies, and design opportunities for mathematical inquiry. Drawing on Chapman's (2015) framework, in this study, PSCK was operationalized across three interrelated strands:

(1) teachers' knowledge of mathematical problems, including what constitutes a meaningful problem and how problem characteristics influence learner engagement,

(2) teachers' knowledge of mathematical PS processes, and

(3) teachers' knowledge of mathematical problems posing.

In this study, these three strands form the analytic framework for examining teachers' learning.

Teachers' knowledge of mathematical PS: This includes teachers' understanding of what makes a problem "meaningful," how task characteristics influence learner engagement, and how non-routine tasks differ from routine exercises (Chapman, 2015). A core aspect of PSCK is the ability to select or design tasks that create productive cognitive challenge.

Knowledge of mathematical PS processes: Teachers require understanding of

(1) what successful PS entails–conceptual understanding, strategic competence, adaptive reasoning,

(2) the use and purpose of heuristics (e.g., Polya's, 1957 stages), and

(3) how to interpret learners' solution strategies.

Knowledge of mathematical problems posing: Teachers must be able to generate or adapt to problems before, during, and after PS episodes. Problem posing allows teachers to adapt tasks to learners' needs, explore alternative pathways, and extend or deepen mathematical ideas (Grigaliūnienė et al., 2025; Li et al., 2022). The framework provides the conceptual structure for analyzing how teachers understand problems, interpret learner reasoning, and engage in problem posing–key aspects of effective PS pedagogy in under-resourced contexts.

Studies on Mathematical Problem-Solving Knowledge for Teaching

While scholars have extensively studied students' mathematical PS proficiency (e.g., Lester, 2013) and curriculum design, relatively few have examined the MPSKT required to teach PS effectively (Jacinto & Carreira, 2023; Piñeiro et al., 2021).

Early studies emphasized teachers' PS process knowledge. For instance, Foster et al. (2014) found that secondary teachers often struggled to facilitate reasoning-focused PS discussions, highlighting gaps in teachers' process knowledge and their ability to work with learner-generated strategies–areas central to PSCK and directly relevant to this study's focus on developing such knowledge. Similarly, Piñeiro et al. (2021) showed that prospective elementary teachers displayed inconsistent and sometimes incorrect conceptions of mathematical PS instruction, suggesting that PSCK is frequently underdeveloped even before teachers enter the profession. This underscores the need to understand and support the development of PSCK among in-service teachers in challenging contexts.

Other studies have explored technology integration and its implications for MPSKT. Jacinto and Carreira (2023) demonstrated that teaching non-routine mathematical PS with digital tools requires specialized knowledge that integrates mathematical and representational fluency. Their work shows how PS knowledge becomes context-dependent, aligning with this study's interest in how disadvantaged school conditions shape the form and development of PSCK.

Taken together, these studies illustrate three themes:

1. **Fragmentation of focus:** Many studies focus on isolated components of MPSKT (e.g., PPSK and technology integration) rather than a holistic understanding of PSCK and its classroom application.
2. **Limited attention to context:** Most research is situated in well-resourced settings; studies addressing teachers' knowledge in disadvantaged contexts remain scarce.
3. **Insufficient intervention research:** Few studies trace teachers' development of PSCK over time through structured professional learning, making it difficult to identify effective supports.

This study addresses these gaps by investigating how teachers' PSCK evolves during a design-based professional development intervention in disadvantaged South African schools. Unlike prior work, it examines PSCK systematically across three core dimensions–teachers' knowledge of mathematical problems, PS processes, and problem posing (Chapman, 2015)–and documents how these dimensions change over iterative cycles of professional learning.

Design-Based Research Implemented for the Large Project

The study reported in this paper was positioned within a larger design-based research (DBR) project focused on developing a professional development intervention to strengthen mathematics teachers' PS instruction, particularly in disadvantaged contexts (Chirinda, 2021; Chirinda & Barmby, 2017, 2018). DBR was chosen because it emphasizes iterative design, implementation, and refinement of educational interventions while generating both practical solutions and theoretical insights (Komatsu et al., 2025; McKenney & Reeves, 2018). The project comprised two six-month cycles in which workshop activities and classroom tasks were implemented, analyzed, and redesigned. Cycle 1 informed the initial design of the intervention, while insights from teachers' participation guided refinements in cycle 2. Across cycles, teachers worked collaboratively in a series of structured workshops that combined engagement with non-routine mathematical problems, analysis of learner solution strategies, and opportunities to design and adapt tasks for their own classrooms. Workshop tasks were aligned with Polya's (1957) four-phase PS model, and the workshop content was grounded in both research evidence and teachers' local instructional realities, ensuring relevance to the South African CAPS curriculum (Department of Basic Education, 2011). The project's iterative design enabled tracking changes in teachers' PSCK over time while adapting the intervention to the realities of disadvantaged school contexts.

METHODOLOGY

Study Design

This qualitative study was embedded within a larger two-cycle DBR project aimed at designing, implementing, and refining a professional development intervention for mathematics teachers' PS pedagogy; details of the DBR process are described earlier in the paper. A qualitative design was chosen to provide an in-depth understanding of teachers' knowledge in a specific context (Creswell & Creswell, 2023).

Participants

Four grade 9 mathematics teachers from public secondary schools in Gauteng, South Africa, were purposefully selected. A small, information-rich sample was intentionally chosen to allow for in-depth tracing of PSCK development over time, consistent with qualitative and DBR methodological principles. Purposeful sampling was guided by three criteria:

(1) teachers taught grade 9 mathematics,

(2) they worked in schools characterized by multilingualism, large classes, and resource constraints–the conditions central to the study's focus, and

(3) they were willing to participate intensively across both DBR cycles.

These criteria enabled the selection of cases where the phenomenon of interest–PSCK development–was most likely to be visible.

The sample size of four teachers was appropriate for the study's analytic goals. DBR emphasized depth of engagement with a small number of cases rather than breadth, and sustained observation over an extended period generated a large volume of data per participant (McKenney & Reeves, 2018). This allowed the study to construct detailed case trajectories and examine within-teacher and cross-teacher variation in PSCK development without aiming for statistical generalization. For confidentiality, the four teachers–three females and one male–were assigned pseudonyms (Olivia, Sophia, Robert, and Emma). **Table 1** gives the demographic data of the teachers.

Academic qualifications refer to the highest mathematics teaching qualification held at the time of the study. The bachelor's degree represents a four-year initial teacher education qualification specializing in secondary mathematics. The Diploma refers to a three-year teaching qualification with mathematics as a major subject. The master's degree refers to a postgraduate qualification in mathematics education. While teachers differed in academic qualifications and years of teaching experience, these

Table 1. Demographic data of the participant teachers

Participant	Age in years	Highest mathematics teaching qualifications	Experience in teaching secondary school mathematics
Olivia	31	Bachelor's degree	13
Sophia	39	Bachelor's degree	19
Emma	25	Master's degree	1
Robert	30	Diploma	6

Table 2. Data collection frequency for each participant teacher

Data collection activity	Cycle 1	Cycle 2
Pre-intervention classroom observations	4	4
Pre-intervention interview	1	1
Classroom observations during the intervention	5	3
Reflective interviews	5	3
Post-intervention interview	1	1

characteristics were not treated as explanatory variables. Instead, they provide context on the range of professional backgrounds represented in the study.

School Contexts

The participating schools were all public secondary schools serving multilingual, low-income communities in Gauteng. Class sizes ranged from 38 to 52 learners, with instruction occurring in English as the language of learning and teaching. However, learners frequently drew on their native languages, such as isiZulu, Sesotho, Setswana, and isiXhosa, during peer discussions. Schools had limited access to mathematical manipulatives, minimal technology infrastructure, and experienced heavy curriculum pacing pressures due to large enrolments and frequent absenteeism. These shared contextual features enabled examination of PSCK development under conditions typical of disadvantaged South African schools.

Data Collection Tools and Procedures

The qualitative data were collected through classroom observations and semi-structured reflective interviews, which were triangulated to strengthen the credibility of the findings. The semi-structured reflective interviews were audio-recorded and transcribed verbatim with the teachers' consent. During the semi-structured interviews, a schedule was used to determine what to probe or follow up on (see **Appendix A**).

The reflective interviews, each lasting 20-30 minutes, encouraged teachers to reflect on their instructional decisions and evolving understanding of PS and provided valuable insights into their PSCK. Selected classroom audio clips were used during interviews to stimulate discussion. Observations, which were recorded using a structured comment card (**Appendix B**), captured teachers' enacted practices during PS lessons, while reflective interviews provided insight into their interpretations and decision-making. The two sources were compared iteratively, with interview statements used to confirm, clarify, nuance, or challenge patterns identified in classroom episodes. Additionally, a post-intervention interview was conducted to assess the impact of the professional development intervention on teachers' PSCK. Teachers were required to reflect on their experiences participating in the professional development intervention and evaluate if they felt their PSCK had evolved. **Table 2** presents the data collection frequency for each participant teacher.

Data Analysis

Data were analyzed using reflexive thematic analysis, following Braun and Clarke's (2022) six-phase process: familiarization with the data, initial coding, generating themes, reviewing themes, defining and naming themes, and producing the report. This approach allows for flexible, interpretive analysis while acknowledging the researcher's active role in meaning-making.

The primary data sources included classroom observation notes and verbatim transcripts of semi-structured reflective interviews. The unit of analysis was a meaningful segment of data–such as a teacher utterance, reflective explanation, or classroom episode–that conveyed a coherent idea related to mathematical PS instruction.

Initial coding was conducted inductively across the whole dataset to capture patterns in teachers' talk and practice related to problem selection, learner engagement, solution strategies, and task design. In subsequent analytic phases, these inductive codes were clustered and interpreted using Chapman's (2015) PSCK framework as a set of sensitizing concepts rather than predetermined categories.

Theme development was iterative and recursive, with continual movement between data extracts, analytic memos, and emerging interpretations to ensure internal coherence within themes and clear distinctions between them. To trace change over time, data from pre-intervention, cycle 1, and cycle 2 were compared within and across cases, allowing for analysis of both individual teacher trajectories and shared patterns of development. Although not intended as an evaluative rubric, PSCK growth was traced using indicators aligned with Chapman's (2015) framework, including:

(1) ability to distinguish routine exercises from meaningful problems,

(2) ability to interpret unconventional solutions, and

(3) ability to pose and reformulate problems.

Table 3. An example of analytic movement from raw data to themes

Data excerpt	Initial code	Pattern/subtheme	PSCK component
"If they can answer it immediately, then it's not really a problem"–Olivia	Distinguishes exercise vs. problem	Recognizing cognitive demand	Knowledge of meaningful problems
Learner decomposes shape differently; teacher asks, "Can you explain how you saw it?"	Probes reasoning	Interpreting unconventional strategies	Knowledge of PS processes
Robert alters goal of textbook task to generate new version	Changes problem goal	Problem reformulation	Knowledge of problem posing

These indicators guided the initial–final comparisons. Frequency counts of coded instances were used descriptively to indicate analytic emphasis across PSCK components and data sources. These frequencies show the distribution of coded segments in the dataset, not statistical generalizability or comparison.

Analytic Procedures: Movement From Data to Themes

Initial open coding generated 312 inductive codes across interview transcripts and classroom observation notes. These codes captured discrete instructional actions (e.g., "redirects to formula" and "asks for justification"), teacher statements (e.g., "I want them to think first"), and contextual influences (e.g., "language mixing" and "resource workaround"). Codes were then grouped into provisional clusters, including task interpretation, strategy use, teacher prompts, and problem adaptations.

As analysis progressed, these clusters were re-examined using Chapman's (2015) PSCK framework as sensitizing concepts rather than preset categories. This process led to three analytic theme families that corresponded to the strands of PSCK:

(1) knowledge of meaningful problems,

(2) knowledge of PS processes, and

(3) knowledge of problem posing.

Within each, subthemes (e.g., "selecting tasks with cognitive demand," "interpreting unconventional strategies," and "reformulating problems during lessons") were refined through iterative comparison (**Table 3**). These examples demonstrate how teacher talk and classroom episodes were linked analytically to PSCK dimensions. Thick description and verbatim excerpts were used to ensure that final themes remained grounded in actual practice.

Ensuring Rigor and Trustworthiness

Credibility was established by triangulating data collection tools and spending a prolonged time in the field doing professional development, observing, and interviewing participants. Rather than generalizability, transferability addresses how qualitative findings can be applied to other contexts. This was upheld by providing detailed, thick descriptions of the South African context, participant teachers, schools, the data collected, and the themes generated to communicate the findings. Nonetheless, transferability is only possible if a study's contextual design is modified to fit a comparable setting. In this regard, I provided a comprehensive evaluation of prior research on mathematical PS, both within and outside South Africa, for additional reference. Confirmability was established in three ways. Firstly, the data was checked and rechecked throughout the data collection and analysis processes. Secondly, member checking was performed with the participants to validate and, if necessary, confirm, nuance, or challenge the accuracy of the data interpretations. I served as both facilitator of the professional development intervention and primary analyst of the data. This dual role enabled close insight into teachers' learning processes but also required ongoing reflexivity. To address this, I kept a reflexive journal documenting assumptions and emerging interpretations throughout the research process.

Ethical Considerations

Ethical approval was obtained from the university ethics committee, and permission was granted by the South African Department of Basic Education and participating schools. Written informed consent was obtained from all participants, who were assured of confidentiality and the voluntary nature of their participation. Ethical considerations were strictly observed to safeguard participants' well-being, confidentiality, and privacy.

Design Principles That Guided the Professional Development Intervention

The professional development intervention was refined across two DBR cycles. Analysis of teachers' participation, classroom enactments, and reflective interviews during cycle 1 identified four design principles that informed the redesign implemented in cycle 2. These principles are presented in **Table 4**, along with their empirical origins and implications for intervention refinement.

The refined PD program implemented in cycle 2 thus represented a theoretically informed and context-responsive iteration of the cycle 1 prototype. By grounding redesign decisions in empirical evidence and in teachers' lived instructional challenges, the DBR process enabled a more coherent and sustainable integration of PSCK principles into everyday classroom practice.

FINDINGS

This study explored the development of the PSCK of four grade 9 mathematics teachers. Because this study was conducted in under-resourced, multilingual schools with large class sizes, teachers' learning and enactment of PSCK were shaped by contextual constraints and affordances. Rather than functioning as background variables, features of disadvantage–multilingualism, limited

Table 4. Design principles emerging from cycle 1 and refining cycle 2

Design principle	Emergence from cycle 1	Implication for cycle 2 redesign
1. Anchoring problem-solving in teachers' own mathematical work	Teachers showed limited confidence in facilitating PS discussions and often reverted to procedural explanations. Their difficulty with non-routine problems suggested that personal struggle illuminated cognitive demands experienced by learners.	Workshops centered teachers' engagement with rich mathematical problems before pedagogical analysis. Teachers articulated their strategies, challenges, and heuristic choices, strengthening PSCK and empathy for learners.
2. Making student thinking central through analysis of unconventional solutions	Teachers frequently overlooked unconventional learner strategies and expressed uncertainty about interpreting such approaches.	Workshops incorporated analysis of learner-generated solutions, using classroom audio and written work to practice probing reasoning, interpreting strategies, and identifying instructional implications.
3. Structured reflection on task design and problem reformulation	Teachers' problem-posing efforts focused on producing tasks without reflecting on how they were generated or adapted.	Cycle 2 introduced structured reflection tools in which teachers documented and justified design decisions. Teachers reformulated problems before, during, and after PS episodes to deepen understanding of how task features support learner engagement.
4. Iterative alignment with CAPS and contextual constraints	Teachers experienced tension between PS pedagogy, curriculum pacing, assessment pressures, and contextual constraints such as large classes, multilingual learners, and limited resources.	Tasks were redesigned to align explicitly with CAPS and assessment requirements while remaining feasible under resource constraints and adaptable to local classroom realities.

Table 5. Categories, frequency, and codes

Category (components of PSCK)	Frequency (f = 208)	Codes (understanding of)	n = 4
Knowledge of mathematical problems	46 (22.1%)	The nature of meaningful problems	3
		The structure and purpose of different types of problems	2
		The impact of problem characteristics on learners	1
Knowledge of mathematical PS	84 (40.4%)	What is needed for successful mathematical PS?	4
		PS models, the meaning, and use of heuristics	4
		How to interpret learners' unusual solutions	3
		Implications of learners' different solution strategies	3
Knowledge of problem posing	78 (37.5%)	Problem posing before, during and after a PS episode	4

material resources, and large classes–interacted with teachers' developing PSCK and influenced how particular aspects of PS pedagogy were taken up. These contextual influences are made explicit in the findings that follow. The findings are presented in terms of teachers' initial MPSKT, the growth in knowledge resulting from the professional development intervention, and the nature of their PSCK.

Table 5 presents the distribution of coded units across the three components of PSCK. In this study, the unit of analysis for coding was a bounded segment of meaning, evident in classroom interaction or interview talk. A coding unit consisted of

(1) a coherent classroom episode (e.g., a teacher's introduction of a problem, facilitation of a PS discussion, or response to a learner's solution) or

(2) a teacher's utterance during reflective interviews that revealed an aspect of PSCK.

Frequency counts in **Table 5** are reported descriptively to indicate patterns of emphasis across PSCK components. They do not imply statistical generalization, comparison between participants, or estimates of effect size; instead, they are included to enhance analytic transparency within the qualitative analysis. The interpretation of teachers' learning is therefore grounded primarily in qualitative evidence, including classroom vignettes and interview excerpts, which are presented below to substantiate the analytic claims. These excerpts demonstrate how teachers' thinking and instructional decision-making evolved and exemplify the types of meaning units that contributed to the frequency counts reported in **Table 5**.

The findings are organized around the three components of PSCK, beginning with teachers' knowledge of mathematical problems.

The Teachers' Knowledge of Mathematical Problems

The teacher's knowledge of mathematical problems was examined under three codes: understanding the nature of significant problems, the structure and purpose of different types of problems, and the impact of problem characteristics on learners (Chapman, 2015). As indicated in vignette 1 in **Appendix C**, baseline observation data revealed that teachers predominantly viewed mathematical problems as routine textbook word problems. For example, Sophia explained:

> "For me, a problem is basically a word sum from the book. Learners must just identify the formula and apply it correctly."

The professional development workshop discussions challenged this view by emphasizing that meaningful mathematical problems are tasks with a goal but no immediate solution path. With the given definition in mind, teachers were presented with several tasks and required to select those that fit as worthwhile and appropriate problems and justify their reasoning. This activity helped improve the teacher's understanding of mathematical problems. As the professional development intervention progressed, teachers' descriptions of what constituted a mathematical problem shifted. In a cycle 2 reflective interview, Robert articulated a revised understanding, noting that

> "a problem is not about giving them a method first. It must be something where they do not immediately know what to do, so they have to think and decide on their own approach."

Olivia similarly reflected that she had begun to evaluate tasks in terms of the opportunities they created for learner engagement:

> "If they can answer it immediately, then it is not really a problem."

These interview excerpts illustrate growth in teachers' conceptualization of meaningful problems and provide qualitative grounding for the increased frequency of codes related to the nature and purpose of mathematical problems shown in **Table 5**. However, despite this growth, only one teacher, Robert, consistently demonstrated awareness of how specific problem characteristics influenced learners' engagement, suggesting uneven development within this PSCK component. In one lesson, Robert designed a group task involving textbook ratios that prompted extended learner engagement and productive struggle:

> At the beginning of August, the PNA bookstore had grade 9 mathematics and science textbooks on its shelves at a ratio of 2:5. Mr. Jones, a grade 9 teacher, went to the bookstore on the 15th of August and realized that a fifth of each type of textbook had been bought and sold. A total of 560 books were unsold. How many of each were there at the beginning of August?

The above problem was clearly stated to help learners in their PS and used a familiar context that learners could easily relate to. During the reflective interview, teacher Robert reported intentionally choosing tasks to '*encourage collaborative problem-solving*,' reflecting an evolving understanding of the purpose of problems and the cognitive demand they entail. This finding suggests that teacher Robert had developed knowledge to support learners' PS processes, including the nature of significant problems, the structure and purpose of different types of problems, and the impact of problem characteristics on learners. During the activity, I noticed that Robert's learners persisted for an extended period of time in solving the problem. This suggests that the problem may have been at an appropriate difficulty level for the learners, as they remained engaged in productive struggle. As the intervention progressed, I noticed Robert increasingly selecting worthwhile problems that created meaningful contexts for learners' PS development.

Classroom observations revealed that large class sizes further influenced how teachers enacted their PS pedagogy. Managing whole-class PS discussions with over forty learners led teachers to increasingly rely on collaborative group work as a practical and pedagogical strategy. Group PS enabled teachers to circulate, listen to multiple solution strategies, and select examples for whole-class discussion.

Olivia reflected on this during a post-intervention interview:

> "With so many learners, group work helps me hear different ideas without everyone talking at once."

Over time, collaborative PS became more than a classroom management tool; it supported teachers' PSCK by creating opportunities to observe diverse strategies and to pose follow-up problems based on learners' collective thinking. In this sense, large class sizes shaped teachers' learning by pushing them toward instructional practices that foreground reasoning, discussion, and shared PS.

The classroom observations and reflective interviews revealed that three participant teachers (Olivia, Sophia, and Emma) did not understand the impact of problem characteristics on learners. In addition, I observed that Emma had difficulty grasping the concept of meaningful problems. This affected her comprehension of the structure and purpose of different problem types, as well as the impact of problem characteristics on learners.

Teachers' evolving understanding of meaningful problems was also shaped by material constraints within their schools. Limited access to manipulatives, technology, and supplementary resources meant that teachers relied heavily on contextualized word problems as entry points for PS. Rather than viewing this reliance as a deficit, several teachers began to use familiar everyday contexts strategically to support learner engagement.

Robert explained this shift during a cycle 2 interview:

> "We do not have many resources, so I try to use situations learners know, like shops or taxis. It helps them imagine the problem better."

This reliance on contextualized problems reflects how resource limitations shaped teachers' PSCK by foregrounding the role of problem context and representation. Teachers learned to select and design problems that compensated for material scarcity while still offering cognitive challenge and opportunities for mathematical reasoning.

Teachers' Knowledge of Mathematical Problem-Solving

Analysis of teachers' knowledge of mathematical PS focused on four codes: understanding what is required for successful PS, familiarity with PS models and heuristics, ability to interpret learners' unconventional solutions, and awareness of the instructional implications of learners' different solution strategies (see **Table 1**). Early interview data indicated that teachers prioritized answer correctness over reasoning (vignette 2 in **Appendix C**). Emma reflected on her initial practice by stating,

> "When learners give an answer, I usually just check if it is right or wrong. If it is wrong, I show them the correct way."

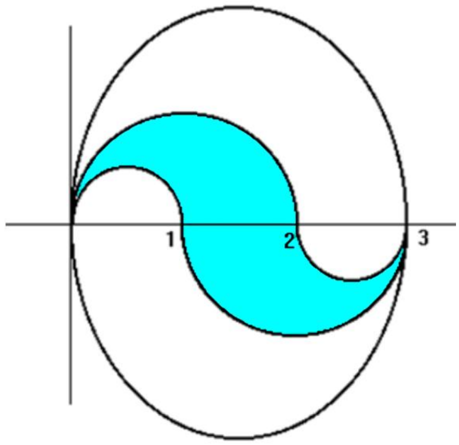

Figure 1. Decoration for the problem in the **Appendix C** vignette (Source: Author's own elaboration)

I observed all teachers struggling to sustain classroom discussions during mathematical PS sessions because they lacked adequate MPSKT. They partially understood or held misconceptions about what mathematical PS proficiency entails, emphasized procedural explanation over exploration, and needed help formulating conjectures about the possible solution strategies their learners would use to solve a given problem.

Participation in the professional development intervention was associated with changes in teachers' knowledge of mathematical PS in several ways. I supported teachers in teaching mathematical PS as a process and in integrating heuristics in their teaching. In addition, teachers watched short videos from Japan, Singapore, or the USA that modelled the mathematical PS pedagogy and participated in workshop sessions on interpreting learners' unusual solutions and the implications of learners' different solution strategies.

By cycle 2, all four teachers displayed an understanding of the requirements for successful mathematical PS, PS models, and the meaning and use of heuristics. Teachers could easily sustain mathematics discussions with learners during PS sessions. The teachers' reflections suggested a marked shift in how they interpreted learners' responses. Sophia explained:

> "Even if the answer is wrong, I now ask how they were thinking. Sometimes the method is actually clever, and then I can build on that."

Robert similarly described how his questioning practices evolved:

> "Now I ask them to explain why they chose that strategy, and it helps me see what they understand."

These excerpts from Sophia and Robert demonstrate teachers' growing capacity to attend to learners' reasoning rather than solely to correctness, aligning with the increased frequency of codes related to interpreting learners' unusual solutions and drawing instructional inferences (see **Table 5**). The data suggest that this aspect of PSCK development was closely associated with teachers' increasing confidence in facilitating sustained PS discussions.

The multilingual nature of their classrooms strongly shaped teachers' growing attention to learners' unconventional solution strategies. Learners frequently expressed mathematical reasoning orally, in their home languages or in informal English, before translating their thinking into symbolic representations. As a result, teachers were required to interpret incomplete or non-standard explanations.

Sophia explicitly linked her learning to this context, noting in a reflective interview:

> "Because learners explain in different ways and sometimes mix languages, I had to focus more on what they are thinking and not just on how they write it."

This shift supported her ability to attend to reasoning rather than surface correctness. Vignette 2 and vignette 4 (**Appendix C**) demonstrate how teacher Sophia's class systematically engaged in understanding problems, devising plans, and reviewing solutions while solving the following problem: Mr. Jones is decorating house windows for Thanksgiving and needs to know the amount of material required for the shaded part in **Figure 1**, in square meters.

Three teachers–Sophia, Robert, and Olivia–demonstrated improved skill in interpreting learners' unconventional solution strategies. As teachers interpreted learners' unusual solutions, I observed that they applied what they had gained from the professional development through reflective questioning–such as "What helped you understand the problem? What did you do that helped you understand the problem? Did you find any information that you did not need? Did you think about your solution after you got it? How did you decide that your solution was correct?"–to guide learner reasoning and validate diverse approaches. These questions helped learners reflect on their thinking and enabled teachers to identify the inferences from learners' different solution strategies. This suggests that teachers' ability to analyze learners' strategies and draw instructional inferences was closely intertwined with their growing confidence in facilitating classroom discussions.

Teacher's Knowledge of Mathematical Problem Posing

Teachers' knowledge of problem posing was examined in terms of their ability to pose problems before, during, and after PS episodes. Baseline interviews indicated that problem posing was largely absent from teachers' instructional repertoires (see vignette 3 in **Appendix C**). Before the intervention, teachers relied solely on textbook problems and rarely engaged students in problem-posing activities. Olivia acknowledged:

> "I don't usually ask learners to create their own problems. I just take them from the textbook because it's faster and safer."

The professional development intervention workshops emphasized designing tasks both before and during lessons, as well as reformulating problems after they were solved. Teachers practiced problem posing and simulated posing these problems to their learners. Teachers were required to consider, create, and pose mathematics problems they anticipated their learners would pose in a given scenario. The tasks posed by teachers provided rich insights into their mathematical thinking about problem-posing. **Appendix D** shows an example of a scenario in which teachers were required to pose mathematical problems. Following sustained engagement in the professional development workshops, teachers began to describe problems posing as a deliberate instructional practice. Robert explained in a cycle 2 interview:

> "Now I think about how to change a problem after we solve it–like asking what happens if we change the numbers or the goal."

Emma reflected on the impact of structured reflection during workshops, noting:

> "When we had to explain how we created a problem, I realized I never thought about the process before."

These interview excerpts illustrate a shift from viewing problem posing as an optional or risky practice to recognizing it as integral to PS instruction. This qualitative evidence supports the relatively high frequency of problem-posing codes reported in **Table 5** and highlights the role of reflective task design in strengthening teachers' PSCK.

As the intervention progressed, I asked the teachers to write, explain, and reflect on their approach to developing and reformulating problems during the problem-posing process. Overall, the classroom observations and reflective interviews revealed that the professional development supported all four teachers in understanding problem-posing before, during, and after PS sessions. I observed that the reflection activities encouraged teachers to articulate the reasoning behind their problem design choices, leading to more creative and purposeful tasks.

DISCUSSION

The findings illustrate how teachers' PSCK developed across the three strands and how this development was intertwined with the contextual realities of under-resourced, multilingual classrooms. These contextual features shaped their opportunities to notice learner thinking, adapt tasks, and orchestrate PS discussions. Rather than functioning solely as constraints, these conditions helped shape the specific forms of PSCK that emerged–a pattern consistent with current practice-based accounts of mathematics teacher knowledge (Grigaliūnienė et al., 2025; Tibane et al., 2024).

Multilingual classrooms appeared to heighten teachers' attention to reasoning as learners explained their thinking using mixed-language resources, supporting growth in interpreting unconventional solution strategies–an essential component of PSCK. In this way, multilingualism functioned as a catalyst for developing PSCK related to interpreting learners' unusual solutions. Teachers' need to make sense of diverse forms of expression appeared to deepen their sensitivity to underlying mathematical reasoning, reinforcing the importance of listening and probing during PS discussions. This echoes research demonstrating that learner explanations are central to teachers' development of PS pedagogy (Schreiber, 2025).

Limited access to manipulatives or technological tools shaped how teachers selected and designed meaningful problems. Teachers increasingly relied on contextualized, familiar word problems as a substitute for physical representation, thereby making the structure and context of problems more central to their instructional decisions. In this way, material scarcity shaped the development of teachers' knowledge of mathematical problems by foregrounding the roles of context, representation, and problem structure. Similar patterns have been documented in studies showing how material constraints influence teachers' task selection and representation in mathematics classrooms (Remillard, 2005). Large class sizes prompted the use of collaborative PS, which expanded teachers' opportunities to observe and respond to diverse solution strategies, interpret unconventional approaches, compare solution paths, and pose follow-up problems–core aspects of PSCK development. This aligns with research showing that collaborative PS structures in mathematics classrooms can both manage instructional complexity in large classes and expand teachers' access to learners' reasoning (Liljedahl, 2016). Accordingly, professional development interventions in similar contexts should be leveraged rather than bypass contextual realities when supporting teachers' PS pedagogy.

Teachers' evolving understanding of meaningful problems appeared foundational. While cycle 1 primarily supported teachers' engagement with PS processes and learner strategies, cycle 2 revealed that deeper conceptual work on the nature of meaningful problems was necessary for sustained PSCK development. As they gained awareness of how problem characteristics influenced engagement in cycle 2, teachers selected and designed tasks that placed greater cognitive demand.

While growth in PSCK was evident across all four teachers, the extent and pace of development varied, highlighting the differentiated and non-linear nature of teacher learning. Emma, the least experienced teacher, demonstrated more limited progress in certain aspects of PSCK, particularly in distinguishing between routine exercises and meaningful problems and in interpreting learners' unconventional solution strategies. Although classroom observations indicated that Emma continued to rely on procedural explanations and closed questioning during PS lessons, even in cycle 2, this pattern should be interpreted cautiously, given the study's small sample size and qualitative design. Rather than implying a causal relationship, Emma's case illustrates how experience and prior pedagogical orientation may shape teachers' readiness to engage with PSCK (Solomon et al., 2023).

In addition, novice teachers may require additional or differentiated support. This interpretation aligns with prior research indicating that novice teachers often prioritize procedural clarity and classroom control when implementing unfamiliar instructional approaches such as PS (Chapman, 2015).

Emma's difficulty distinguishing between exercises and authentic problems underscores the importance of a strong conceptual foundation for defining meaningful problems as a prerequisite for broader PSCK development. When considered alongside the design principles in **Table 4**, this pattern suggests that the professional development intervention placed greater emphasis on analyzing learner strategies and reformulating tasks (design principle 2 and design principle 3) than on explicitly foregrounding the conceptual characteristics of meaningful problems. Consequently, the intervention was less effective in strengthening teachers' knowledge of mathematical problems, indicating a need to redesign this component of the professional development with more sustained and explicit attention to what constitutes a worthwhile mathematical problem.

Growth in teachers' ability to interpret learners' strategies reflected the combined influence of the professional development structure and the instructional context. This aligns with international work highlighting the importance of attending to learner strategies and forms of reasoning in developing PS pedagogy (Santos-Trigo, 2024)

Structured reflection supported teachers in articulating the purpose of task adaptations. The study concluded that participants attempted to devise unique and mathematically intriguing problems as a result of their reflection on the problem-posing process. The nature and quality of the problems posed by teachers indicated an emerging internalization of PSCK, particularly in their ability to adjust tasks responsively during instruction. This aligns with research highlighting how problems posing deepens teachers' understanding of mathematical structures (Cai et al., 2015; Koichu & Kontorovich, 2013).

The Study's Implications for Professional Development

The findings suggest several implications for professional development aimed at strengthening teachers' PSCK in disadvantaged contexts. First, professional learning should prioritize developing teachers' understanding of what constitutes a meaningful mathematical problem, as this knowledge appears foundational to other aspects of PSCK. Second, professional development should explicitly support teachers in interpreting learners' unconventional solution strategies and linking these interpretations to instructional decision-making. Third, structured reflection on problem design and reformulation should be embedded within professional learning to cultivate teachers' problem-posing expertise. Importantly, professional development interventions should leverage contextual realities–such as multilingual classrooms, large class sizes, and limited resources–as sites for learning rather than treating them solely as constraints.

CONCLUSION

This study examined the development of grade 9 mathematics teachers' PSCK through participation in a design-based professional development intervention implemented in disadvantaged South African school contexts. The findings demonstrate that targeted, context-sensitive professional learning can support teachers in strengthening their understanding of meaningful mathematical problems, interpreting learners' diverse solution strategies, and engaging in purposeful problems posing as part of PS instruction.

A key contribution of this study lies in showing that teachers' development of PSCK is inseparable from the contextual conditions in which they work. Multilingual classrooms, large class sizes, and limited material resources did not merely constrain teachers' PS pedagogy; rather, these features actively shaped the forms of PSCK that developed over time. Teachers' growing attention to learners' reasoning, increased reliance on contextualized problems, and use of collaborative PS emerged as adaptive responses to these conditions. In this sense, the study extends existing conceptualizations of PSCK by illustrating how knowledge for teaching PS is co-constructed with, rather than applied despite, contexts of disadvantage.

The study makes clear the specific forms of knowledge teachers need to teach mathematical PS in disadvantaged contexts. Teachers require an understanding of what constitutes a meaningful mathematical problem, the ability to interpret and respond to learners' varied solution strategies, and the capacity to pose and adapt problems through instruction. Importantly, these forms of knowledge develop in close interaction with the contextual conditions of under-resourced classrooms, highlighting that PSCK is not only conceptual but also situated, and that it emerges through teachers' ongoing efforts to make PS accessible and meaningful for their learners.

The study highlights that growth in PSCK was uneven among participants and strands. Teachers' understanding of what constitutes a meaningful mathematical problem appeared foundational to subsequent development in interpreting learner strategies and posing problems. While most participants demonstrated substantial growth across the intervention cycles, one teacher exhibited more limited change, suggesting that teaching experience and prior pedagogical orientations may influence how teachers engage with PS professional development. These differences underscore the non-linear and differentiated nature of teacher learning in complex classroom environments. The small number of participants enabled detailed tracing of individual teacher trajectories across two DBR cycles, offering insight into the uneven and non-linear nature of PSCK development.

Although the study involved a small number of participants, its strength lies in the detailed tracing of individual teacher learning trajectories across two DBR cycles. This fine-grained analysis offers insight into how PSCK develops over time and under real-world constraints, contributing practice-based knowledge to the literature on mathematical PS knowledge for teaching. Future research could build on this work by examining how similar interventions function across a broader range of contexts and by exploring how differentiated supports may further enhance teachers' engagement with PS pedagogy.

Limitations

The sample size is often a limitation in mathematical PS studies, and this study involved only four teachers. This study's findings should be interpreted within the constraints of its small, qualitative sample, which prioritized depth of analysis over generalizability. Theoretical generalizability lies not in the specific practices observed, but in the mechanisms demonstrated by which context interacts with teachers' PSCK development. The primary purpose of this study was not to investigate the PSCK with all grade 9 South African mathematics teachers, but to comprehend the particulars of the cases studied in their complexity. Therefore, the results can inform future efforts to develop MPSKT and PSCK in similar educational settings.

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APPENDIX A: SEMI-STRUCTURED INTERVIEW GUIDE

(constructs + sample questions for replication)

Purpose

To explore teachers' developing **PSCK** across three strands:

(a) knowledge of mathematical problems,

(b) mathematical problem-solving processes,

(c) problem posing,

while also examining contextual influences.

Section 1. Knowledge of Mathematical Problems

Construct: Understanding what constitutes a meaningful mathematical problem.
Sample questions:

1. "How do you decide whether a task is a real problem rather than an exercise?"
2. "Can you describe a problem you used recently that required learners to think before choosing a method?"
3. "How do problem characteristics (context, numbers, wording) influence learner engagement?"

Section 2. Knowledge of Mathematical PS Processes

Construct: Understanding PS phases, heuristics, and strategy use.

4. "How do you support learners in making sense of a problem before solving?"
5. "What strategies or heuristics do you encourage learners to use?"
6. "Describe a situation where a learner used an unexpected strategy. How did you interpret it?"
7. "How do you decide when to intervene and when to let learners persist?"

Section 3. Interpreting Learners' Unconventional Solutions

Construct: Ability to infer reasoning from non-standard approaches.

8. "What do you look for when a learner gives a method different from what you expected?"
9. "How do you respond when a learner uses an unconventional strategy that is partially correct?"

Section 4. Problem Posing (Before, During, After PS Episodes)

Construct: Designing, modifying, and reformulating problems.

10. "Before a lesson, how do you decide whether to adapt a textbook problem?"
11. "Do you pose new problems during a lesson–for example, changing the goal or numbers? Why?"
12. "After the class solves a problem, do you ever ask: 'What if we changed _________________'? How do learners respond?"

Section 5. Contextual Influences

Construct: How multilingualism, large classes, and limited resources shape PS instruction.

13. "How do multilingual explanations influence how you interpret learners' thinking?"
14. "How does limited access to resources affect the problems you choose?"
15. "What strategies help you manage PS in large classes?"

Section 6. Post-Intervention Reflection (Final Interview)

16. "How has your understanding of PS changed through the workshops?"
17. "Which activities or tasks helped you the most?"
18. "Which aspects of PS instruction do you still feel uncertain about?"

APPENDIX B: STRUCTURED COMMENT CARD (CLASSROOM OBSERVATION INSTRUMENT)

(aligned with PSCK components and contextual influences)

Lesson Information

Teacher: ___________________________School: _____________________________Grade/topic: _________________________
Date & time: ________________________Class size: ___________________________Observer: ___________________________

Section A. Problem Characteristics (Knowledge of Mathematical Problems)

Indicators (tick all observed):

- ☐ Task represents a genuine mathematical problem (goal present and no immediate method).
- ☐ Problem requires cognitive demand beyond procedural application.
- ☐ Context is familiar/meaningful to learners.
- ☐ Problem structure supports mathematical reasoning (e.g., multiple representations possible).

Evidence (teacher actions, learner responses, quotes):

Section B. Problem-Solving Processes (Knowledge of Mathematical PS)

1. **Understanding the problem**
    - ☐ Teacher prompts learners to restate the problem in their own words.
    - ☐ Learners identify knowns/unknowns.
    - ☐ Teacher clarifies unnecessary or distracting information.
2. **Devising a plan**
    - ☐ Teacher encourages strategic choices (diagram, table, pattern, equation).
    - ☐ Multiple strategies are welcomed.
3. **Carrying out the plan**
    - ☐ Teacher supports without demonstrating prematurely.
    - ☐ Learners justify reasoning during execution.
4. **Looking back**
    - ☐ Teacher asks learners to check answers for sense-making.
    - ☐ Alternative strategies or generalisations discussed.

Evidence:

Section C. Interpretation of Unconventional Solutions

- ☐ Teacher probes reasoning behind unexpected strategies.
- ☐ Teacher uses learner-generated approaches as instructional resources.
- ☐ Teacher avoids redirecting prematurely to the standard method.

Evidence/quotes:

Section D. Problem Posing (Before, During, After PS Episodes)

- ☐ Teacher reformulates tasks before instruction.
- ☐ Teacher poses follow-up questions that alter constraints or goals.
- ☐ Teacher encourages learners to pose their own problems.

Examples observed:

Section E. Contextual Adaptations (Multilingualism, Class Size, Resources)

- Multilingual strategies used? (Y/N) ______________________.
- If yes, describe: _______________________________________.
- Are resource limitations evident? (Y/N) ___________________.
- Adaptations used: _____________________________________.
- Large class strategies (grouping, rotation)? ____.

Section F. Overall Observational Notes

Strengths:

Challenges:

Key quotes/critical incidents:

APPENDIX C: VIGNETTES

Vignette 1. Task Selection and Problem Characteristics

At the start of the intervention, teachers typically equated mathematical problems with textbook exercises or routine word problems aligned directly with CAPS examples. For instance, during a baseline observation, Olivia introduced a ratio task by first demonstrating the solution procedure on the board and then assigning similar items for individual practice, leaving learners little opportunity to engage in PS.

By cycle 2, Olivia's task selection had shifted. In one observed lesson, she presented learners with a contextual proportion word problem involving two family members' trips. Rather than demonstrating a method, she asked learners to work in groups and decide how to represent the situation mathematically.

The problem had a clear goal, but no immediately apparent solution path, and learners debated whether to use tables, equations, or proportional reasoning. During the reflective interview, Olivia explained that she now looked for tasks that would "make learners think first before choosing a method." This episode illustrates growth in Olivia's understanding of the nature of meaningful problems and how problem characteristics–such as context familiarity and cognitive demand–can support productive struggle and sustained engagement.

Vignette 2: Interpreting Learners' Unconventional Solutions

Early classroom observations showed that teachers often evaluated learners' solutions primarily on correctness. When learners produced unconventional strategies, teachers tended to redirect them toward standard methods. For example, during a cycle 1 lesson, Sophia dismissed a learner's visual strategy for calculating area and instead demonstrated a formula-based approach.

In contrast, during cycle 2, Sophia facilitated a whole-class discussion in which learners solved a geometry problem involving composite shapes. One learner decomposed the figure differently from the expected approach. Rather than correcting the learner, Sophia asked, "Can you explain how you saw the shapes?" and followed up with questions such as "How does this connect to what we already know about area?" This questioning enabled other learners to compare strategies and evaluate their efficiency.

In her reflective interview, Sophia noted that she had learned to "listen for the thinking behind the answer." This vignette demonstrates growth in teachers' ability to interpret learners' unconventional solutions and to use these interpretations to inform instructional decisions–an essential element of PSCK.

Vignette 3. Shifts in Problem-Posing Practices

Before the intervention, all four teachers relied almost exclusively on textbook problems and did not engage learners in problem-posing activities. When asked to create their own problems during early workshops, teachers focused on producing a final task rather than reflecting on how or why the problem was constructed.

As the intervention progressed, teachers were required to design problems and explicitly articulate the reasoning behind their design choices. In one cycle 2 workshop, Robert reformulated a routine algebra problem by changing its goal and embedding it in a familiar shopping context. He explained that he wanted learners to "see the same mathematics differently" and to consider alternative solution paths.

Classroom observations later showed Robert posing follow-up problems during lessons, such as asking learners how the solution would change if certain quantities were altered. This before-and-after contrast illustrates growth in teachers' knowledge of problem posing before, during, and after PS episodes, supported by structured reflection on the problem-posing process.

Vignette 4. Implementation of the Mathematical PS Pedagogy by Sophia

Understanding the problem

The first step was for learners to understand the problem. Sophia established first if learners understood what was being asked for in the problem by asking them to do the following:

- State the problem in their own words
- Identify the unknowns
- Decide what information was important and irrelevant to the problem

In the given problem, I observed that all the learners took time to realise the important information. The teacher worked with the learners to understand that the question involved identifying circles and finding their areas.

Devising a plan

After understanding the problem, Sophia required learners to devise a plan to solve the problem by considering various strategies:

- Making a table, diagram, or chart
- Trying a more straightforward form of the problem
- Writing an equation
- Guessing and checking

- Looking for a pattern

In the given problem, Sophia's learners had to figure out how many circles or half-circles they could see in the decoration to devise the plan.

Carrying out the plan

During the plan's execution, Sophia required learners to implement the strategy chosen in the second step. I observed that all learners could easily label the circles. Nonetheless, half of the class struggled to find the shaded area using the labelled circles.

Looking back

To ensure their understanding, learners should review both their solution strategy and the solution itself. I noticed that learners in Sophia's class tended to skip this step and considered it unnecessary. During our professional development workshop discussions, we emphasized the importance of reflecting on a PS episode after completing it. Sophia highlighted to learners that miscalculations, such as multiplying instead of dividing, can occur during PS, leading to incorrect answers. She stressed the importance of checking their answers to pinpoint any errors. Sophia also prompted learners to consider whether the answer to the given problem made sense.

APPENDIX D: EXAMPLE OF PROBLEMS USED IN PROBLEM POSING SESSIONS


--- e Card #: ********4263
1 GM COOKIE CRSP CRL 10.6 1.77F
ORIGINAL PRICE
1.99 EACH
COUPON SAVINGS
8.29
6.30 -
.22 -
1 GM COOKIE CRSP CRL 10.6 1.77F
ORIGINAL PRICE
1.99 EACH
COUPON SAVINGS
8.29
6.30 -
.22 -
1 GM COOKIE CRSP CRL 10.6 1.77F
ORIGINAL PRICE
1.99 EACH
COUPON SAVINGS
8.29
6.30 -
.22 -
1 FBRZ AE LINEN 8.8Z 3.99T
ORIGINAL PRICE
COUPON SAVINGS
4.99
1.00 -
1 CVS CCBTR PETROLM 7.5Z 3.82T
ORIGINAL PRICE
COUPON SAVINGS
5.99
2.17 -

***********COUPONS APPLIED***********
1 2% BACK IN EXTRABUCKS R 1.89 - CVS
1 20% OFF ONE FULL-PRICE 1.07 - CVS
1 10% OFF YOUR PURCHASE! .87 - CVS

5 ITEMS
SUBTOTAL 13.12
CA 10.25% TAX .80
TOTAL 13.92
CHARGE 13.92
************6023 RF
VISA CREDIT ************6023
APPROVED# 07052D REF# 042279
TRAN TYPE: SALE AID: A0000000031010
TC: 989D50B98070E98C TERMINAL# 06848588
NO SIGNATURE REQUIRED CVM: 280000
TVR(95): 0000000000 TSI(9B): 0000

CHANGE .00

1. Create and solve a story problem using information from the exit till slip from CVs. Your problem must be about grade 9 algebra.
2. Write the approach you used to develop the problem in the first question.