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Parallelism and transversals in geometry: Experiences of fresh senior high school graduates into teacher education

Stephen Rowland Baidoo [1*], Josephine Celestine Baidoo [2]

[1] School of Education, University of Tasmania, Hobart, AUSTRALIA
[2] Department of Mathematics and Science, Holy Child High School, Cape Coast, GHANA
*Corresponding Author: stephenrowland.baidoo@utas.edu.au

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ABSTRACT

This study was set up to investigate the newly admitted senior high school graduates' geometric representation of corresponding and alternate angles in contexts where parallel and non-parallel lines are cut by a transversal. The study also examined their reasoning about parallelism. 25 volunteers, through a pilot study, responded to a series of geometric tasks meant to assess geometry reasoning and understanding. This study reports on the data dealing with the afore-mentioned concepts.

The findings indicate that: the participants were more able to identify geometric representation of alternate angles (64%) than they were with corresponding angles (44%); participants' written narratives demonstrated evidence of imprecision in their reasoning about parallelism; and most participants showed limited knowledge and use of necessary keywords to justify parallelism. The findings suggest participants showed diverse conceptual understanding of alternate and corresponding angles and demonstrate insufficient and 'suspended' knowledge of parallelism.

Keywords: parallel lines, corresponding and alternate angles, pre-service teachers, geometric representations, senior high school graduates

INTRODUCTION

Parallel lines cut by a transversal introduce and help explore several important angles (e.g., corresponding, alternate angle, etc.), properties, and relationships that come with parallelism. Parallelism as a geometrical concept may seem an abstract entity among members of a classroom community, yet its practical application in many geometric components of our physical environment is awe-inspiring. Its uses in man's existence are helpful in solving some of the numerous issues in life. It includes construction of buildings, pillars, windows, doors, bridges, roads, railway lines and airports runway designs. What about drying and electricity lines?

Despite its practical applications, parallelism is identified as one of the geometric concepts which tends to be problematic and a challenge to students (Kusuma & Retnotwati, 2021, p. 391). This is probably because it demands simultaneous use of several principles and theorems to effectively engage in it. For instance, in dealing with parallelism, varied geometric concepts such as angles, properties and relationships related to parallelism, and relationship of parallel lines to transversals, are paramount. Thus, a study by Zuya and Kwalat (2015) revealed that most teachers were unable to indicate the missing knowledge of their students in terms of angles and parallelism. The teachers were found very limited in helping students because they were unable to offer any alternative ways to eliminate the difficulties that their students faced.

It is suggested that school students are uninterested in geometry, misunderstand geometric concepts (Mamiala et al., 2021; Ngirishi, 2015); and teachers still teach though students are reported to have been complaining of misunderstanding geometric concepts under study (Mamiala et al., 2021). These revelations do not come as surprises but as supports to literature that evinces that geometry is a hard mathematical concept (Ubi et al., 2018), difficult to teach (Mamiala et al., 2021), and to learn (Bikić et al., 2016; Fabiyi, 2017). Yet when they are learning mathematics, learners' conceptual understanding needs to be demonstrated when they are explaining the concepts learned. They need to re-express the already communicated concepts precisely; make use of concepts in various ways and contexts; and develop significance of such concepts when used in their correct sense to problem-solve (Duffin & Simpson, 2000).

This goal can be enhanced when schoolteachers are empowered to do so, more especially at a time when both students and teachers face difficulties in spatial thinking (Seah, 2015b), which is necessary for studying parallel lines and transversals. One way to empower teachers, is to investigate PSTs' geometrical concepts knowledge base to ensure their adequate content knowledge preparation so they can eventually become teachers with developed skills necessary to anticipate and understand how students' reason about concepts; and teachers who have facility to respond appropriately to students' queries.

The above narratives offer reasonable grounds for mathematics educators to begin paying attention to PST students' geometry preparation in Ghanaian context where only 27% of basic school teachers are said to inspire mathematical reasoning (Keaveney et al., 2018) and where research studies and national examinations results have indicated the struggle in geometry studies by both senior high school (SHS) students (West Africa Examination Council [WAEC], 2015, 2016, 2017a, 2018) and PST students (Armah et al., 2017; Armah & Kissi, 2019; Salifu et al., 2018; University of Cape Coast-Institute of Education [UCC-IOE], 2011, 2012, 2013, 2014). The narrative is not very different with the basic education certificate examinations (BECE) candidates. These candidates are final year junior high school (JHS) students. Their challenges with geometric facts that related parallel lines cut with transversals to find given angles were reported (WAEC, 2014). They evinced limited understanding of geometry involving conception of parallel and transversal during their final examinations. In 2017, BECE candidates had difficulty identifying opposite and alternate angles from two parallel lines with a transversal. The chief examiner reported that only few candidates identified a relationship that was fostered between given angles and established the correct facts about the unknown angles (WAEC, 2017b). The Ghanaian JHS graduates, about 90%, are observed to enter SHS with low level of geometry understanding (Baffoe & Mereku, 2010). However, no empirical study has sought to uncover what the issue is with the SHS graduates who enter teacher education, generally and, with reference to their knowledge of parallelism with transversals.

This study is significant on two counts. Firstly, numeracy is re-gaining traction in the Ghanaian education context, because of the newly introduced 2019 education reform. The reform focuses on the 4Rs (**R**eading, w**R**iting, a**R**ithmatic, and c**R**eactivity). This suggests that the basic (primary and JHS) schoolteachers' role to inspire and challenge their students to achieve higher in mathematics is important that their (schoolteachers') mathematics preparation in teacher education requires effective and intended investment. When they are adequately prepared, teachers can effectively spearhead this education paradigm (by offering quality instructions) to achieve its overall intention and success. Quality teaching and learning is identified as the greatest defining factor for students' mathematics attainment when other sources of variations are accounted for (Alton-Lee, 2003; Darling-Hammond, 2000).

Secondly, the struggle on the part of the teacher can hinder effective transfer of geometric reasoning and understanding to their students. A visual, logical, and deductive thinking is inhibited because of deficiently developed spatial and geometrical thinking capacity (Seah et al., 2016). This suggests newly admitted SHS graduates into teacher education (fresh PSTs) are investigated early enough to know their initial knowledge about parallelism and transversals. This study, therefore, was set up to explore the newly admitted SHS graduates' (fresh PST students') geometric representation of corresponding and alternate angles in contexts where parallel and non-parallel lines are cut by a transversal and how they reasoned about parallelism.

THEORETICAL FRAMING FOR THE STUDY

Parallel Lines with a Transversal

Parallel lines cut by a transversal introduces diverse geometrical concepts. Reasoning about parallelism with a transversal line is therefore not a straightforward thing, as is not about only identifying whether the lines involved are parallel or not. It brings with it some important angles such as corresponding and alternate angles. Parallelism must also be argued for. Students, apart from knowing what constitutes parallel lines, would need to determine if two lines are parallel by investigating the relationships between the angle formed and the lines involved.

Investigating reasoning and understanding of parallelism, corresponding and alternate angles among PST students provides an enabling avenue for them, not only to use their visualizing skills in geometry but an opportunity to assess how they can analyze their own understanding and offer reasoning of geometric properties of figures/shapes. This, in turn, helps identify how well they have formed mental pictures of concepts and their representations. The fact is, nearly every reasoning done in geometry, and ability to make sense to solve problems are linked with spatial reasoning (Battista & Frazee, 2018). This supports the suggestion that "underpinning success in mathematics and science is the capacity to think spatially" (National Research Council [NRC], 2006, p. 6). To reason geometrically, it is recognized that using property-based conceptual system is very essential (Battista, 2007); and is testified by Battista and Frazee (2018) that such system include concepts such as "angle measure, length measure, congruence, and parallelism" (p. 231).

Thus, to succeed, students would need to visualize such parallel lines with a transversal and angles formed and their respective properties. Students would employ such terms as *corresponding*, *alternate interior*, *vertical*, *alternate external* angles, and their relative positions as circumstantial evidence suggests. All these abilities would require that students deeply understand and appreciate the concept, in addition, to how the concept relates other interconnected web of ideas, simultaneously.

This "simultaneous analysis of several elements (angles, lines, intersections, etc.) and the need to take reference (line, angles, sides) into account constitute a vast and stimulating field of geometrical exploration" (Bairral et al., 2022, p. 130), which enables visualization and representation of geometric objects. However, this is not the case often with many students. Wang (2016) argued that learners most times only commit to memory the patterns to get tasks done and never really develop conceptual understanding. She cited the issue of learners' recognition of "corresponding angles by finding the 'F' that is formed by parallel lines and the transversal" (p. 8). She argued that using such technique oversimplifies any relationship among angles and lines which, according to Pierre van Hiele in 1986, could be detrimental to learners' search for answers.

Representations in Geometry

In learning geometry, one must deal with various abstract entities (geometrical concepts). These includes mental entities that must be built by means of geometric representations (Seah, 2015a). According to Seah (2015a), these entities do not merely represent the real object we interact with in our physical environment. Geometric representation–symbols used to depict abstract concepts, making the unseen seen and comprehensible–comes in varied forms. These include lines, diagrams, symbols, points, among a host of them. Fischbein (1993) thought of these representations as being both conceptual and figural characters. Conceptual characters also refer to concept image (Seah, 2015a) and figural is pictorial.

Concept image is used to depict "all the cognitive structure in the individual's mind that is associated with a given concept" (Tall & Vinner, 1981, p. 1), and is simply, the pictures formed in one's mind about concepts, and their associated properties, and the processes that relate them (Vinner, 1991). To begin with, learners make meaning of representations in geometry through their visual appreciation (Seah et al., 2016). Seah et al. (2016) acknowledged that, as one's geometrical knowledge base expands and deepens, later; the capacity to visualize becomes more robust and that enables individuals to make inference and deductions about geometric relationships presented by images.

Developing Mathematical Discourse

By commognitive foundations, thinking has been defined handily to comprise any action "of communicating with oneself" (Sfard, 2012, p. 2). In doing mathematics, learners engage in communicating ideas. Sfard (2012) referred to that as a discourse–a type of communication. According to Sfard (2012), certain features characterize mathematical discourse, and these are listed as special keywords; visual mediators; distinctive routines; and endorsed narratives.

Special keywords are unique words that are used in mathematics. For example, parallel lines, transversal line, angle, and rectangle. *Visual mediators* are symbols, diagrams, numerals that are used to represent concepts. *Distinctive routines* are procedures or ways through which learners perform mathematical tasks. Finally, the set of theorems, rules, and definitions that are commonly accepted by the community of mathematics practitioners are the *endorsed narratives.* They are said to be a group "of spoken or written utterances" that learners use in mathematics discourse and can be accepted or rejected when they are subjected to a specified procedure for confirmation (Seah et al., 2016, p. 586).

The Issue of Concept Image and Concept Definition

The functionality of the human brain is recognized to operate in a complex manner, which frequently defies the tenet of mathematics (Tall & Vinner, 1981). The authors reasoned that it was not entirely the logical reasoning that made one insightful, neither were chances which led people to commit mistakes. They admitted that one's understanding of successful or erroneous processes depends on the thin line that can be drawn between mathematical concept–defined formally; and the mental (cognitive) processes by which these concepts are comprehended. Tall and Vinner (1981) explained that a lot of concepts are used that are not formally defined. They specified that to be able to use these concepts in their appropriate contexts depended on one's experiences and frequent use.

These authors maintained that as the meaning and interpretation attached to these concepts get refined later (with or without recourse to exact definition), symbols or names are employed to enable their effective communication. This, in turn, helps one's mental operation. Thus, a single symbol used to elicit meaning from a concept, does not fully explore the meaning as would the entire cognitive structure (of a person) which makes the meaning explicit. When learners recall and manipulate a concept mentally–in this single process–there are many other related processes that become involved (Tall & Vinner, 1981). These processes affect the meaning and how the concept is used. Take, for instance, the concept of corresponding angles which are normally introduced to learners as being congruent (with only parallel lines cut by a transversal). At the time, learners may realise that this would work with parallel lines cut with a transversal. For these learners, this observation has become part of their concept image. This knowledge is likely to be challenged later should non-parallel lines cut with transversal be encountered. Again, the extent to which this might pose a future problem might depend on experiences individuals may go through later in life. This illustration, thus, reinforces the fact that personal concept image and concept definitions are acquired individually by each learner through their various experiences (Seah et al., 2016).

Examining how fresh PSTs represent the concepts of *corresponding* and *alternate* angles geometrically in different contexts, identify these angles in the contexts of parallel and non-parallel cut by a transversal, and their reasoning about parallelism with a transversal can help educators in their instructions. Literature suggests that PSTs usually lack the necessary knowledge and reasoning and understanding in geometry content (Adolphus, 2011; Aslan-Tutak, 2009; Aslan-Tutak & Adams, 2015) and that their restricted mathematical knowledge hinders didactical and pedagogical knowledge training (Brown & Borko, 1992).

In this manuscript, attempt is made to present an initial research study focused on providing information about fresh PSTs' representation of ***corresponding*** and ***alternate angles*** at instances where there were two parallel lines cut with a transversal and two non-parallel lines cut with a transversal. An investigation was made to explore their conception of parallelism and transversals in geometry as they communicated their mathematical ideas (mathematics as communicating) and reinforced their logical reasoning (mathematics as reasoning) abilities as they compared various underpinning relationships.

The study's aim, therefore, was to assess their geometric representation of corresponding angles and alternate angles (their facility to identify corresponding and alternate angles) under a context where parallel lines and non-parallel lines are cut by a transversal. We also examined how they applied the concept of parallelism in a novel situation and offered reasoning.

METHODOLOGY

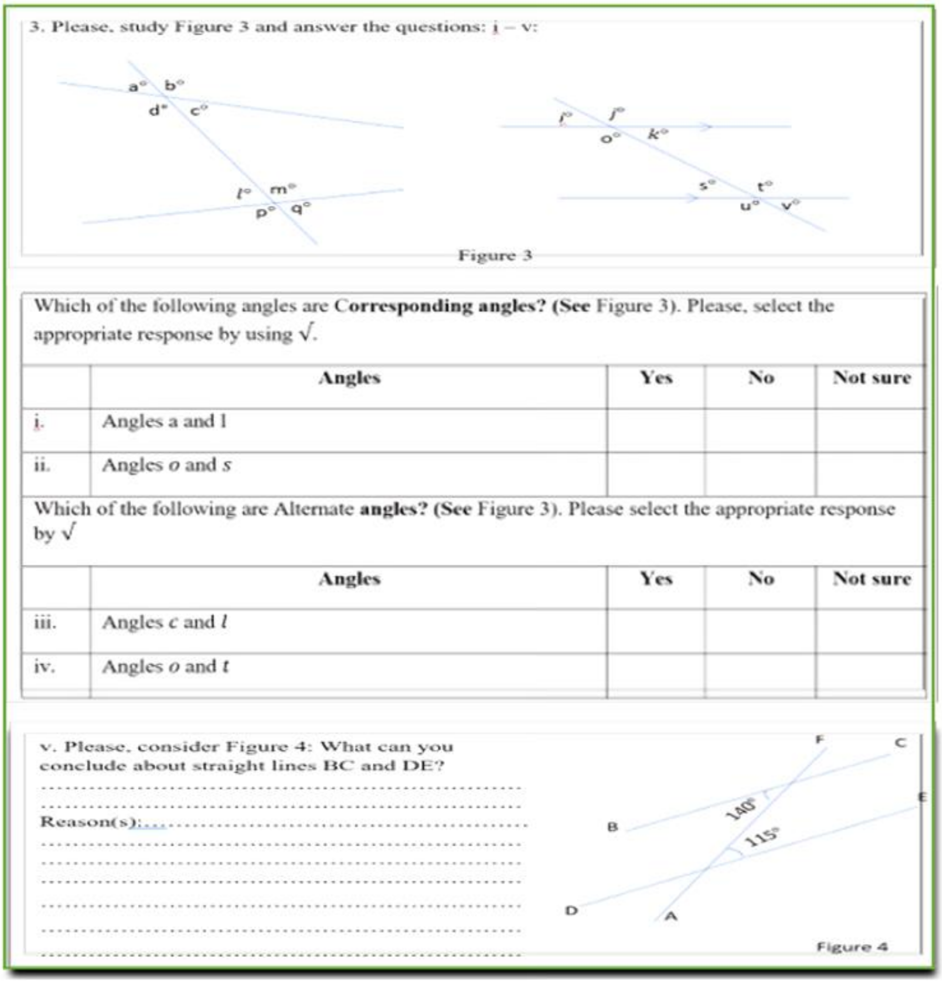
Research Design

Participants

The participants (n=25) were a section of SHS graduates who have been admitted in one particular college of education (CoE) in the Central-Western Zone of Ghana, as first-years. These participants have enrolled to commence a four-year bachelor's degree in education (BEd) in JHS program and taken mathematics as their major teaching subject. Since geometry content course is offered in semester 2 in that college, they had not been delivered geometry content course at the time the study was conducted. They were part of volunteers who were recruited for a trial on an instrument for a project meant to investigate reasoning and understanding in geometry.

Instrument

This manuscript reports a data set taken from the responses to a task (**Figure 1**) in the pilot study.



3. Please, study Figure 3 and answer the questions: i – v:

Which of the following angles are **Corresponding angles? (See** Figure 3). Please, select the appropriate response by using √.

	Angles	Yes	No	Not sure
i.	Angles a and l			
ii.	Angles *o* and *s*			

Which of the following are Alternate **angles? (See** Figure 3). Please select the appropriate response by √

	Angles	Yes	No	Not sure
iii.	Angles *c* and *l*			
iv.	Angles *o* and *t*			

v. Please, consider Figure 4: What can you conclude about straight lines BC and DE?

....

Reason(s):..

Figure 1. Tasks used for data collection: ***I-ii*** for ***task* a**; ***iii-iv*** for ***task b***; and **v** for ***task* c** (Source: Authors' own elaboration)

The task (**Figure 1**) and scoring rubrics (**Table 1**) were designed, trailed, and re-designed for validation purposes. The task was coded GPARTRANS, meaning geometry parallelism and transversals. GPARTRANS contains two aspects: GPARTRANS a (i & ii) is framed about the definition of corresponding angles when there are two parallel lines with a transversal and two non-parallel lines with a transversal. GPARTRANS b (iii & iv) is framed about the definition of alternate angles when there are two parallel lines with a transversal and when there are two non-parallel lines with a transversal.

The chief aim of tasks a and b was to test participants' geometric representation of corresponding angles and, thus, reflects the representation aspect of reasoning and understanding geometry (Seah & Horne, 2019). GPARTRANS c (v) examined on applying known results into different situation and was meant to assess participants' facility in geometric discourse. As seen in **Figure 1**, sub-items i-ii are represented in this study as task a; iii-iv, represented as task b in this study and v is represented and used as task c. This change is done to ease reporting. Similarly, as presented in **Table 1**, the scoring rubrics for all the tasks used to collect data are shown. The instrument was given to other two experts in the field of mathematics education and one measurement assessment expert for review for relevance, language tone, and coverage.

Table 1. Scoring rubrics

Score	Description	Remarks
0	No response or incorrect response	Scoring rubric for ***task a***
1	Partial response, i.e., identifies one corresponding angle correct	
2	Identifies All the corresponding angles: ***a*** & ***l*** are corresponding angles; ***o*** & ***s*** are not	
0	No response or incorrect response	Scoring rubric for ***task b***
1	Partial response, i.e., identifies one alternate angle correct	
2	Identifies All the alternate angles; ***c*** & ***l*** are corresponding angles; ***o*** & ***t*** are not	
0	No response or incorrect/irrelevance response	Scoring rubric for ***task c***
1	Partial response, i.e., they are not parallel	
2	Partial response, i.e., they are not parallel; with some reasoning [i.e., the 2 labelled angles are not same]	
3	Correct response [i.e., lines BC and DE are not parallel] without reasoning or incorrect reasoning	
4	Correct response [lines BC and DE are not parallel] with some reasoning [i.e., the 2 labelled angles (140° and 115°) are not the same]	
5	Correct response [i.e., lines BE and DE are not parallel] with correct reasoning [the 2 labelled angles, 140° & 115°, are not the same, though they are alternate angles. They would be if BC//DE or BC would be //to DE if the named angles are equal	

In this study, we employed percentages to explore participants' responses to **GPARTRANS tasks a** and **b** and the Sfard's (2012) interpretive framework of mathematical discourse to analyze participants' written narratives to **task c**. This was used to assess depth of participants' mathematical discourse about parallelism and transversal.

RESULTS

As noted in the previous section, participants' responses to **GPARTRANS *tasks a*** and ***b*** are presented in percentages. Analysis on data from ***task c*** is reported in terms of *endorsed narratives* and *keywords*. It was not possible to report on *visual mediators* and *distinctive routines* in this study. None of the correct response categories of tasks ***a*** and ***b*** received 100% performance.

Corresponding Angles

GPARTRANS 'a' (i & ii) was framed around the definition of corresponding angles. The participants were to observe two diagrams (**Figure 1**): one is a depiction of parallel lines cut by a transversal and the other, at an instance when non-parallel lines are cut by a transversal. They were then to indicate their agreement by ticking 'yes', 'no', or 'unsure' to two pairs of given angles (one of which is corresponding angles, the other is not). As noted earlier, it was principally to test their facility in geometric representation of corresponding angles.

In **Table 2**, the scores of responses are categorized into three as 'incorrect', 'partial', and 'correct' with their respective associated percentages. Whilst 28% of the participants gave nil or incorrect responses (i.e., either did not attempt to or wrongly represent *corresponding* angles geometrically), 44% responded correctly (i.e., represented *corresponding* angles geometrically). At the same time, 28% of them gave responses that were partially correct (i.e., partially represented *corresponding* angles geometrically). Overall, 56% of the participating SHS graduates either did not respond or incorrectly responded, or partially responded to the task. The response categories are presented in **Figure 2**. It appears that this percentage of participants have, from their previous geometry experiences and exposure, formed incomplete understanding of corresponding angles. Hence, the difficulties with representation, which also speaks to the issue of their limitedness in identifying corresponding angles. This might stem from their lack of visualization facility or their weak conception of what corresponding angles might be altogether from their previous schooling since they had not yet taken any geometry course in their teacher education since admission.

Table 2. Scoring of responses for corresponding angles

Score	Description	Percentage correct
0	No response or incorrect response [no/incorrect response]	28
1	Partial response, i.e., identifies one corresponding angle correct	28
2	Identifies all the corresponding angles: ***a*** & ***l*** are corresponding angles; ***o*** & ***s*** are not [correct response]	44

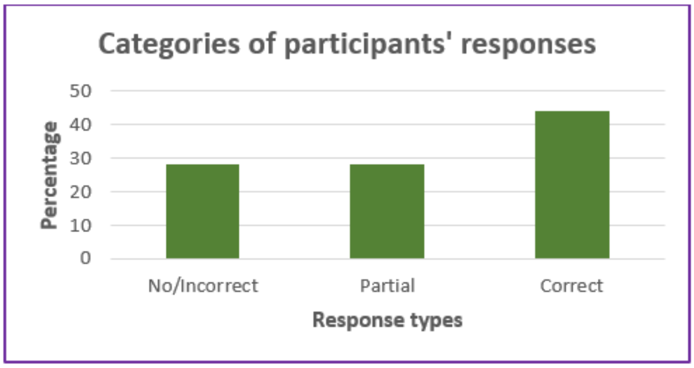


Figure 2. Response categories for corresponding angles (Source: Authors' own elaboration)

As seen in **Table 2**, participants who gave nil response or responded incorrectly received score zero. A score of one was awarded to those who partially responded–thus only identified the angle correctly, whereas score two was awarded for correct response.

Alternate Angles

Similarly, assessing participants' representation of alternate angles, **GPARTRANS b (iii & iv)** requested them to observe the same diagrams (**Figure 1**) and indicate their agreement by ticking 'yes', 'no', or 'unsure' to two pairs of given angles (one of which is alternate angles, the other is not). **Table 2** shows the percentage scores of responses in three categories: 'incorrect', 'partial', and 'correct'. It emerged that 12% of the participants gave nil or incorrect responses whilst 24% gave partial responses (i.e., partially represented *alternate* angles geometrically). Those who gave correct responses (64%) to this sub-task were the majority. Unlike the result in the previous task, the freshmen and women appeared comfortable in their knowledge of the alternate angles than the corresponding angles as they had higher correct percentage responses in looking to represent the former geometric concept. **Table 3** illustrates the score categories whereas **Figure 3** depicts the various response types.

Table 3. Scoring of responses for alternate angles

Score	Description	Percentage correct
0	No response or incorrect response [no/incorrect response]	12
1	Partial response, i.e., identifies one corresponding angle correct	24
2	Identifies all the corresponding angles: ***a*** & ***l*** are corresponding angles; ***o*** & ***s*** are not [correct response]	64

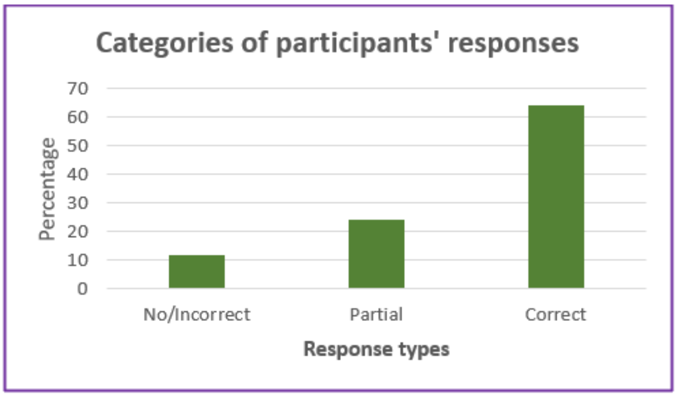


Figure 3. Response categories for alternate angles (Source: Authors' own elaboration)

Of a particular interest is, 45.45 % more of those who gave correct response to the task on corresponding angles were able to respond correctly to the task on the alternate angle.

Reasoning About Parallelism

GPARTRANS c (v) examines on applying known results into different situation and requires communicating reasoning (discourse). We applied Sfard's (2012) framework of mathematical discourse in this aspect of our analysis of data, using *endorsed narratives* and *keywords*. The first of this current section discusses participants reasoning about parallelism (*endorsed narratives)* and the next addresses their use of *keywords.*

Narratives

The description: "the given lines BC and DE, in the diagram/figure, are not parallel. This is because the labelled angles 140° and 115° are not equal or the same, though they are alternate angles. Lines BC and DE will be parallel, or line BC will be parallel to line DE if angles 140° and 115° are equal/same", is the endorsed narrative to explain the context of '**GPARTRAN c**' as whether BC//DE or not. **Table 4** presents some selected responses of the participants as they reasoned about parallelism.

Table 4. Some selected participants' reasoning about parallelism

Score	Participants	Conclusion and reasoning offered
	k-026	BE and DE are parallel to each other. BC and DE both follows the same pattern.
	k-023	Line BC and DE are parallel.
	k-021	The line BC and DE are parallel lines because they do not meet. With the line BC and DE, we used line AF to cut across to enable us to get the types of angles as angles are formed when two or more lines meet.
	k-019	Line BC and DE are parallel, moving line (opposite lines). This is because on projecting the line BC and DE, they will be moving in a straight line, which cause them not to meet.
	k-018	Line BC and line DE are parallel line. Line BC and line DE do not meet.
	k-017	the straight lines BC and DE are not parallel. Parallel lines do not meet each other.
	k-015	The straight lines BC and DE are parallel. Because the two straight lines can travel a long journey without meeting anywhere.
	k-012	The straight lines BC and DE are parallel lines.
	k-007	BC and the line DE are called parallels lines. This is because any time you see parallel line in the diagram it means it help us to find an unknown variable easily. This means that both of the lines are facing each other.

From the explanations offered by the participants (samples in **Table 4**), such as K-026, most participants' written narratives demonstrate issues of imprecision in their reasoning about parallelism. K-026 incorrectly reasoned that "BC and DE both follows the same pattern". This response indicates a misconception held by the participant about parallelism. Several misconceptions were detected from the responses of the participants and are presented in **Table 5**. This finding suggests participants were struggling with parallelism with a transversal. Only one of them mentioned that the involved angles 140° and 115° were not equal or the same; and that they should have been equal for the lines to be parallel. Overwhelming majority (96%) did not realise that line BC would be parallel to line DE if angles 140° and 115° were equal/same. This was not seen by the participants as a necessary and sufficient condition underlying the context in the task. Though some participants mentioned ***alternate*** and ***corresponding*** angles, they were not mentioned in reference to the involved angles.

Table 5. Participants' misconceptions about parallelism cut by a transversal

Misconceptions	**Number**
They move in the same direction.	1
Both follow the same pattern.	1
Any time you see parallel lines in the diagram it means it helps us to find an unknown variable easily.	1
On projecting the line BC and DE, they will be moving in a straight line, which cause them not to meet.	1
Because the two straight lines can travel a long journey without meeting anywhere.	1
It is only parallel lines which we can get corresponding and alternate angles in them.	1
Parallel lines with transversal.	2
Moving in one direction, without a curve.	1
The straight lines BC and DE are not equal.	1

Table 5 illustrates the misconceptions held by some SHS graduates who took part in this study.

These participants have undergone schooling where geometry is a major component of school mathematics curriculum in Ghana. One therefore would expect that they would respond beyond their shallow statements such as shown in **Table 4**. They tended to interpret what they saw in the diagram with what they knew: that parallel lines do not meet. Hence, most popular conclusion was '*BE and DE are parallel',* which seriously lacks any good attempt of justification.

Their conclusion was prompted by the aspect of ***task c*** which required them to mention how they were going to conclude about the straight lines BC and DE based on the prevailing circumstantial evidence (see **Figure 1**). **Figure 4** is a diagrammatic view (from NVIVO output) depicting how they concluded about the parallelism of lines BC and DE, cut by a transversal AF.

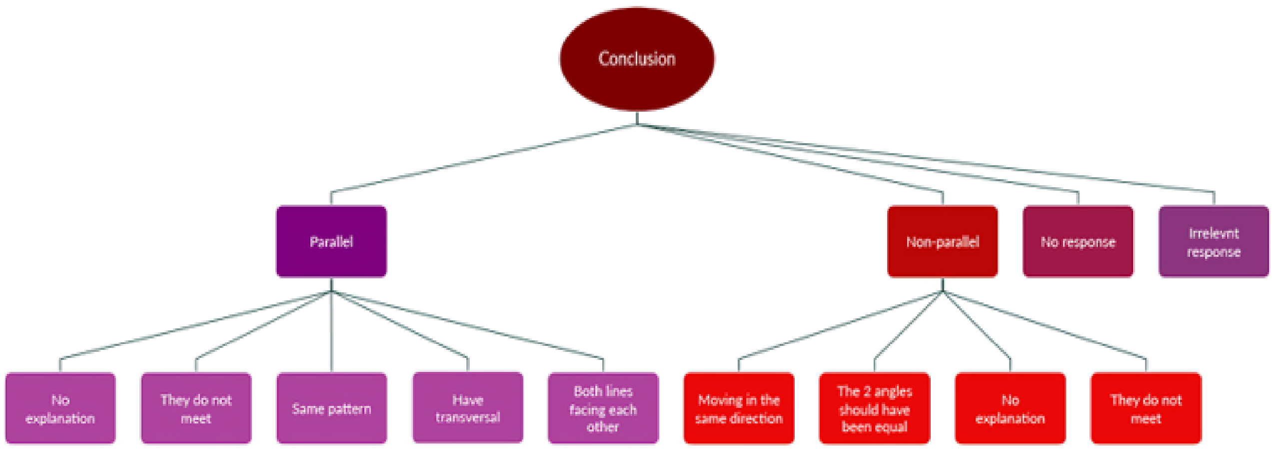


Figure 4. Diagrammatic view of participants' conclusion (Source: Authors' own elaboration)

The misconceptions of the participants were deduced from their conclusions and their justifications. Of those (n=14) who concluded that lines BC and DE are parallel, some had various underlying reasons: they do not meet, they have same pattern, the lines have a transversal, and both lines are facing each other. Others offered no explanations. Of the four participants who concluded that lines BC and DE are not parallel, one offered no explanation; one reasoned that they were moving in the same direction; another said, they do not meet; and the other mentioned that "Since <BFA≠<EAF [it shows] that BC and DE are not parallel lines. Thus, they are moving in different directions and are not parallel because the angle <BFA and <EAF should have been the same". The latter was the only one among the participants who reasoned correctly to justify the conclusion made. Whilst some participants gave nil responses (n=6) for their conclusion, one person specified an irrelevant response: "BC is alternate to DE respectively on a straight line whereby angles on a straight line sum up to 180. BC+140=180=BC=40".

Use of keywords

Table 6 depicts the summary of participants' use of keywords in their attempt to justify their decision of whether the context in question is a parallel or not. As indicated in **Table 6**, more mathematics students' frequent use (19.6%) outshone their counterparts as they used keyword "parallel" in their justification of parallelism of lines BC and DE, crossed by a transversal AF. Though the idea of 'equal' angles is consequential in ascertaining the parallelism of lines BC and DE in the given context, the results showed that only the students of Mathematics and Mathematics and ICT used that word, accounting for 2.2% (with each group attaining 1.1%) of the word use. Other groups seemed not to have any idea of the type of necessary keyword needed to explain parallel lines in the task as they never used any.

Table 6. Use of keywords in attempt to explain parallel lines

Keywords	Mat	MatSci	MatICT	Sci	Not declared	Total
Lines/lines	19.6	-	6.5	4.3	1.1	32.5
Line BC	7.6	1.1	-	-	-	8.7
Line DE	2.2	-	1.1	-	-	3.3
Lines BC and DE	4.3	-	1.1	1.1	-	6.5
Angle/angles	9,8	-	2.2	-	-	12.0
Alternate	3.3	-	3.3	-	-	6.5
Parallel	18.6	4.3	2.2		1.1	26.1
Equal/same	1.1	-	1.1	-	-	2.2
Diagram/figure	2.2	-	1.1	-	-	3.3

Note. Calculated based on words used

Participating mathematics students were the group which used the keyword '*parallel'* (18.5%) and *'angle/angles'* (9.8 %) most in their narratives, followed by the Mathematics and Science students (for *paralle*l) and Mathematics and ICT (for *angle/angles*). This connotes that some of these students had retained key information from their SHS geometry education, but such knowledge was misapplied, or misconstrued, or was not sufficient to be useful in their justification in **task c**. It might seem that others too had forgotten or were not equipped with the information at all, as some students never responded to task c.

DISCUSSION

The findings from this study revealed how the sample of newly admitted SHS graduates (fresh PSTs) exhibited their understanding of *representations* of geometrical concepts (specifically *corresponding* and *alternate* angles), and their level of communicating reasoning/understanding (geometric discourse) of parallelism and transversal as shown in their written narratives.

The data revealed participants' inadequate understanding in both concepts of corresponding and alternate angles, though they demonstrated better understanding in the latter concept. The results indicated that many newly admitted SHS graduates were unable to connect the diagram to their previous knowledge on corresponding and alternate angles to solve the problem. This is somewhat contrary to the asserted view of Sunzuma et al. (2020) that learners can connect diagrams to their previous knowledge to solve problems in geometry. Probably, this failure might stem from their incomplete and suspended understanding of these geometrical concepts which in turn created the misconceptions they hold about parallelism. Their responses seem to strengthen the argument of Clements and Battista (1992) and Wang (2016) that learners often only learn by rote to get tasks accomplished, and not acquiring the underlying conceptual understanding. Wang (2016) stressed that learners learn to identify corresponding angles by using the 'F' technique associated with parallel lines and transversal lines. The use of such shortcuts has not been helpful to learners as any relationship among angles and lines are overlooked (Wang, 2016). van Hiele (1986) noted that such techniques are detrimental to their learning. Accordingly, Berenger (2018) worried that this would not help in delivering learning experiences that could extend geometric thinking of their students.

Our findings show that most participants' written narratives demonstrated evidence of imprecision in their reasoning about parallelism with a transversal. Although, the participants could somehow visualize – in this case, lines BC and DE cut by a traversal line AF (as these were mentioned in their written narratives)–they were yet to develop any visual appreciation of what is meant by parallel in a context such as where two lines are cut by a traversal. It emerged that only one (4%), out of 25 participants, was able to justify the parallelism of *lines BC and DE, cut by a transversal AF.* To engage meaningfully in geometric thinking, it is required that there is a connected synergy between representations, visualization, and discourse (Seah & Horne, 2021). All the participants, except one, failed to consider the relationships between the angles formed by the transversal and the lines involved. Much to our chagrin, we noted that their sterling performance in the previous sub-task on alternate angle did not appear to support their justification for parallelism, since they failed to recognize that 140° and 115° were alternate angles and yet were not congruent. This clearly shows the underlying inadequacy and unsupported nature of PSTs' figural (pictorial) and conceptual understanding of parallelism with a transversal. This lends support to the finding of Đokić et al's. (2020) investigation into Serbia pre-service primary school teachers' reasoning in geometry. Their study focused on how figural and conceptual properties of geometric objects are connected. The authors found participants' geometric reasoning was dominated by figural structure of an angle over its conceptual issues. They adduced that the participants, though could define angle and demonstrate understanding of several types of lines, were unable to demonstrate any conceptual understanding of an angle when they must observe an image.

Quite clearly, it could be inferred from the explanations of our sample that they have not personally reconstructed the image of parallel in their mind, let alone on a paper because of their inability to interpret and interact with the concept of parallelism cut by a transversal. Seah and Horne (2021) argued that to make sense of a concept is to "visualize and communicate the multiple representations used to express the idea" (p. 5). Thus, their inability suggests their weak visualizing facility and lack of conception of parallelism with transversals. It is suggested that majority of students experience difficulties doing tasks that demand them to visualize and do logical and deductive reasoning as a result of want of spatial and geometric thinking facility (Marchis, 2012). Literature indicates that geometric concepts that are misconceived and misunderstood by teachers stem from the time they were in school (Cunningham & Roberts, 2010; Fujita & Jones, 2006; Marchis, 2012). This is proven to be true with our sample since they have not been taught geometry: whether content or its methods in their teacher preparation period. Thus, their limitedness in geometry content knowledge was because of their experiences in their previous schooling. Strong visualization capacity would have enabled informed interpretation of what they saw in the diagram (see **Figure 1**, 3v) from within their network of own experiences and understanding.

Further, the findings indicate limited knowledge and use of necessary keywords needed to offer justification for parallelism. For instance, the word '*equal' or 'same'* accounted for only 2.2 % of frequency of keywords used in their written discourse. Seah et al. (2016) articulated that meaning of concept is derived out of geometric representations through our visual appreciation and that, we increase the store of our geometrical knowledge later as our visualizing capacity becomes more stronger to make inferences and deductions about geometric relationships. However, this might not be seen about these sample, as they failed to infer and deduce the inherent geometric relation in the task. Their visualizing capacity may be weak and not very robust to enable them to infer or deduce the equality of or compare the angles involved. Some participants tended not to have any knowledge of appropriate keywords to use to explain parallelism. Such participants gave nil or irrelevant responses. This finding is consistent with that of Youkap (2021) who found that the students in his study demonstrated difficulties as they defined straight lines and parallel lines. Youkap (2021) explained that the students struggled to find suitable terms to express their understanding of straight and parallel lines. However, it should be noted that these students (n=28) were 8th graders, aged between 13 and 14, and were examined in French language as it is their first language.

This inadequacy in knowledge of this study's sample about parallelism resulted in all of them, except one participant, failing to justify the parallelism of lines BC and DE. Thinking parallel lines do not meet, might have informed their responses. They might have 'borrowed' this idea to extend to two lines cut by a transversal, as found in **task c**. As a result, they tended to hold a stereotypical view of what the concept of parallel lines are. This supports the argument of Arcavi (2003) that the context within which individuals perceive concepts influence their concept image.

CONCLUSION

Understanding geometric concepts, their properties and relationships is catalytic to reasoning and performance in geometry. This study explored the newly admitted SHS graduates' (fresh PSTs') understanding in geometric representation of corresponding and alternate angles and assessed their geometric discourse facility in parallelism and transversal. The findings reveal that, among other things, most students who apply to enter teacher education come with diverse conceptual understanding of geometric concepts – specifically, varied conception of, and limited ideas about corresponding and alternate angles as well as insufficient and 'suspended' knowledge of parallelism. Specifically, the study found:

- participants' inadequate understanding in both concepts of *corresponding* and *alternate* angle,
- that most participants' written narratives demonstrated evidence of imprecision in their reasoning about parallelism with a transversal,
- limited knowledge and use of necessary keywords needed to offer justification for parallelism, and
- more participants hold misconception about parallelism.

Based on the findings, we conclude that vast majority struggle with formal figural concept knowledge that focuses on parallelism, corresponding, and alternate angles. They hold misconception about parallelism. This notwithstanding, they appeared to have strong facility in *alternate* than in *corresponding* angles. The results obtained in this study serve as an insight into the newly admitted SHS graduates' world of *corresponding* and *alternate* angles within the context of parallel and non-parallel lines and t*ransversal*. This can enable and equip geometry teacher educators with the necessary foresight to interact with their first-year PST students in their engagements. We also suggest that mathematics teachers at the SHSs be interested in and ensure their students' conceptual understanding of key concepts in geometry, especially *parallelism* and *transversal*. This study can be extended to investigate other geometrical concepts not covered in this study. It would be a worthwhile study to track how these newly SHS graduates overcome their misconceptions and inadequacy during their teacher preparation in geometry.

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REFERENCES

Adolphus, T. (2011). Problems of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. *International Journal of Emerging Sciences, 1*(2), 143-152.

Alton-Lee, A. (2003). *Quality teaching for diverse students in schooling: Best evidence synthesis iteration (BES)*. https://www.educationcounts.govt.nz/publications/series/2515/5959

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*, 215-241. https://doi.org/10.1023/A:1024312321077

Armah, R. B., & Kissi, P. S. (2019). Use of the van Hiele theory in investigating teaching strategies used by college of education geometry tutors. *EURASIA Journal of Mathematics, Science and Technology Education, 15*(4), 1-13. https://doi.org/10.29333/ejmste/103562

Armah, R. B., Cofie, P. O., & Okpoti, C. A. (2017). The geometric thinking levels of pre-service teachers in Ghana. *Higher Education Research, 2*(3), 98-106.

Aslan-Tutak, F. (2009). *A study of geometry content knowledge of elementary preservice teachers: The case of quadrilaterals* [Doctoral dissertation, University of Florid].

Aslan-Tutak, F., & Adams, T. L. (2015). A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal of Elementary Education, 7*(3), 301-318.

Baffoe, E., & Mereku, D. K. (2010). The van Hiele levels of understanding of students entering senior high school in Ghana. *African Journal of Educational Studies in Mathematics and Sciences, 8*, 51-61. https://doi.org/10.4314/ajesms.v8i1.69103

Bairral, M., Henrique, P., & Assis, A. (2022). Moving parallel and transversal lines with touches on smartphones: A look through screen recording. *The Mathematics Enthusiast, 19*(1), 114-135. http://doi.org/10.54870/1551-3440.1546

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Coucil of Teachers of Mathematics* (pp. 843-908). Information Age Publishing.

Battista, M. T., & Frazee, L. M. (2018). Middle school students' use of property knowledge and spatial visualization in reasoning about 2D rotations. In P. Herbst, U. H. Cheah, P. R. Richard, & K. Jones (Eds.), *International perspective on the teaching ang of geometry in secondary schools*. https://doi.org/10.1007/978-3-319-77476-3_14

Berenger, A. (2018). Changes in students' mathematical discourse when describing a square. In J. Hunter, P. Perger, & I. Darragh (Eds.), *Proceedings of the 41st Annual Conference of Mathematics Education Research Group of Australasia* (pp. 170-177). Mathematics Education Research Group of Australasia.

Bikić, N., Maričić, S. M., & Pikula, M. (2016). The effects of differentiation of content in problem-solving in learning geometry in secondary schools. *EURASIA Journal of Mathematics, Science and Technology, 12*(11), 2783-2795. https://doi.org/10.12973/eurasia.2016.02304a

Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). Macmillan.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook on mathematics teaching and learning* (pp. 420-464). Macmillan.

Cunningham, R. F., & Roberts, A. (2010). Reducing the mismatch of geometry concept definitions and concept images held by pre-service teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*, 1-17.

Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives, 8*(1), 1-44. https://doi.org/10.14507/epaa.v8n1.2000

Đokić, O. J., Jelić, M. S., & Ilić, S. M. (2020). The correlation between figural and conceptual properties of angle and cube in pre-service teachers geometric reasoning. *Teaching Innovations, 33*(1), 1-20. https://doi.org/110.5937/inovacije2001001D

Duffin, J. M., & Simpson, A. P. (2000). Search for understanding. *Journal of Mathematical Behavior, 18*(4), 415-427. https://doi.org/10.1016/S0732-3123(00)00028-6

Fabiyi, T. R. (2017). Geometry concepts in mathematics perceived difficult to learn by senior secondary school students in Ekiti State, Nigeria. *Journal of Research and Method in Education, 7*(1), 83-90. https://doi.org/10.9790/7388-07010018390

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*, 139-162. https://doi.org/10.1007/BFO1273689

Fujita, T., & Jones, K. (2006). Primary trainee teachers' understanding of basic geometrical figures in Scotland. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings 30th Conference of the International Croup for the Psychology of Mathematics Education* (pp. 129-136). Psychology of Mathematics Education.

Keaveney, E., Epstein, A., Duthie, M., Siddiqui, H., Dawuda, M., Wang, M., & Hoo, B. (2018). Ghana numeracy pilot impact evaluation: 2017 baseline report. *USAID*. https://pdf.usaid.gov/pdf_docs/PA00SWTG.pdf

Kusuma, I. A., & Retnotwati, E. (2021). Worked example as a scaffolding on problem posing instruction of parallel lines and transversal. *Advances in Social Science, Education and Humanities Research, 528*, 391-395. https://doi.org/10.2991/assehr.k.210305.056

Mamiala, D., Mji, A., & Simelane-Mnisi, S. (2021). Students' interest in understanding geometry in South African high schools. *Universal Journal of Education Research, 9*(3), 487-496. https://doi.org/10.13189/ujer.2021.090308

Marchis, I. (2012). Preservice primary school teachers' elementary geometry knowledge. *Acta Didactica Napocensia, 5*(2), 33-40.

Ngirishi, H. (2015). *Exploration of FET mathematics learners' understanding of geoemtry* [Master's thesis, University of Kwa-Zulu Natal].

NRC. (2006). *Learning to think spatially*. National Academy Press.

Salifu, A. S., Yakubu, A.-R., & Ibrahim, F. I. (2018). van Hiele geometric thinking levels of pre-service teachers' of E.P. college of education, Bimbilla-Ghana. *Journal of Education and Practice, 9*(23), 108-119.

Seah, R. (2015a). Reasoning with geometric shapes. *The Australian Mathematics Teacher, 71*(2), 4-11.

Seah, R. (2015b). Understanding geometric ideas: Pre-service primary teachers' knowledge as a basis for teaching. In *Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 571-578). MERGA.

Seah, R., & Horne, M. (2019). A learning progression for geometric reasoning. In D. Siemon, T. Barkatsas, & R. Seah (Eds.), *Reasoning and using progressions (trajectories) in mathematics education* (pp. 157-180). Brill Sense Publishers. https://doi.org/10.1163/9789004396449_007

Seah, R., & Horne, M. (2021). Developing reasoning within geometric learning progression: Implications for curriculum development and classroom practices. *Australian Journal of Education, 0*(0), 1-17. https://doi.org/10.1177/00049441211036532

Seah, R., Horne, M., & Berenger, A. (2016). High school students' knowledge of a square as a basis for developing a geometric learning progression. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Proceedings of the 39th Annual Conference of the Mathematics Education Group of Australasia* (pp. 584-591). Mathematics Education Research Group of Australasia.

Sfard, A. (2012). Introduction: Developing mathematical discourse–some insights from communicational research. *International Journal of Educational Research, 51-52*, 1-9. https://doi.org/10.1016/j.ijer.2011.12.013

Sunzuma, G., Chando, C., Gwizangwe, I., Zezehwa, N., & Zinyeka, G. (2020). In-service Zimbadwe teachers' views on the utility value of diagrams in the teaching and learning of geometry. *LUMAT: International Journal on Mathematics, Science and Technology Education, 8*(1), 1-18. https://doi.org/10.31129/LUMAT.8.1.1316

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*, 151-169. https://doi.org/10.1007/BF00305619

Ubi, E. E., Odiong, A. U., & Igiri, O. I. (2018). Geometry viewed as a difficult mathematics. *Internal Journal of Innovative Science and Research Technology, 3*(11), 251-255.

UCC-IOE. (2011). *Chief examiner's report on the 2011 three-year post-secondary teacher training college diploma in basic education mathematics II (geometry & trigonometry) examination*. University of Cape Coast-Institute of Education.

UCC-IOE. (2012). *Chief examiner's report on the 2011 three-year post-secondary teacher training college diploma in basic education mathematics II (geometry & trigonometry) examination*. University of Cape Coast-Institute of Education.

UCC-IOE. (2013). *Chief examiner's report on the 2013 three-year post-secondary teacher training college diploma in basic education geometry examination*. University of Cape Coast-Institute of Education.

UCC-IOE. (2014). *Chief examiner's report on the 2014 three-year post-secondary teacher training college diploma in basic education geometry examination*. University of Cape Coast-Institute of Education.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.

Vinner, T. (1991). The role of definitions in teaching and learning of mathematics. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 65-81). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47203-1_5

WAEC. (2014). Chief examiners' reports on the basic education certificate examination for school candidates, 2014. *West Africa Examination Council*. https://www.waecgh.org/uploads/examinersReport/2014/Mathematics.pdf

WAEC. (2015). Chief examiners' reports on the basic education certificate examination for school candidates, 2015. *West Africa Examination Council*. http://www.waecgh.org/examiners-report

WAEC. (2016). Chief examiner's report on the 2016 senior high school certificate examination. *West Africa Examination Council*. https://www.waecgh.org/uploads/examinersReport/2016/General%20Resume%20W16.pdf

WAEC. (2017a). *Chief examiner's report on the 2016 senior high school certificate examination*. West Africa Examination Council.

WAEC. (2017b). Chief examiners' reports on the basic education certificate examination for school candidates. *West Africa Examination Council*. https://www.waecgh.org/uploads/examinersReport/2017/Mathematics.pdf

WAEC. (2018). Chief examiners' repot on West African senior secondary certificate examination 2018. *West Africa Examination Council*. https://www.waecgh.org/uploads/examinersReport/2018/MathematicsW18.pdf

Wang, S. (2016). *Discourse perspective of geometric thoughts*. Springer. https://doi.org/10.1007/978-3-658-12805-0

Youkap, P. T. (2021). Student comprehension of the concept of a geomtrical figure: The case of straight lines and parallel line. *Journal of Mathematics Education, 6*(2), 149-158. http://doi.org/10.31327/jme.v6i2.1408

Zuya, H. E., & Kwalat, S. K. (2015). Teacher's knowledge of students about geometry. *International Journal of Learning, Teaching and Educational Research, 13*(3), 100-114.