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Realistic mathematics education and authentic learning: A combination of teaching mathematics in high schools

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ABSTRACT

Realistic mathematics education (RME) has been gradually asserting its influence on the development of many mathematics education in the world, including Vietnam. Through decades of application and development, RME is increasingly proving itself to be an effective approach in improving the quality of teaching and learning mathematics. The article focuses on clarifying some similarities between RME theory and authentic education method. The combination of the main ideas of these two approaches can help teachers in designing and selecting appropriate learning tasks to improve the effectiveness of teaching and learning mathematics in the new curriculum context.

Keywords: realistic mathematics education, authentic learning, authentic education, RME, problem-solving competency

INTRODUCTION

The twenty-first century sees an explosion of applications of mathematics. It's hard to find an area of study that doesn't use mathematical tools. Accordingly, mathematical theory and practice go hand in hand. Mathematics has more and more applications in life, basic mathematical knowledge and skills have helped people to solve real-life problems systematically and accurately, contributing to promoting development society. The mathematics curriculum 2018 (MET, 2018) emphasizes that mathematics in high schools contributes to the formation and development of key qualities, general competencies, and mathematical competencies for students; develop key knowledge and skills and create opportunities for students to experience and apply mathematics in practice; make connections between mathematical ideas, between mathematics and practice, between mathematics and other subjects and educational activities, especially with science, natural sciences, physics, chemistry, biology, technology, informatics to implement STEM education.

Inheriting and promoting the advantages of the current curriculum and previous curriculum, the mathematics curriculum 2018 thoroughly grasps the basic regulations stated in the master program; selectively acquire experience in curriculum development from advanced countries in the world, and approach modern and active educational methods. On that basis, the mathematics curriculum 2018 (MET, 2018) emphasizes a number of points:

(1) *ensuring simplicity*, *practicality, and modernity*,

(2) *ensuring consistency*, *consistency, and continuous development*,

(3) *ensuring integration and differentiation*, and

(4) *ensuring openness*.

In addition, the curriculum 2018 aims to form and develop for students a number of important competencies, including: mathematical reasoning and thinking competency competency; mathematical modeling competency; ability to solve mathematical problems; mathematical communication competency. These descriptions present both opportunities and challenges for teachers in designing learning tasks for students. These tasks must, on the one hand, ensure the curriculum's objectives, on the other hand, they must be able to arouse students' excitement and need to acquire new mathematical knowledge. In that spirit, the starting point of the learning process should be love or context taken from the real-world close to the students at least in their thoughts, or experiences. In our opinion, the combination realistic mathematics education (RME) and authentic learning (authentic learning), which can be a suggestion for teachers in designing learning situations towards establishing and develop the mathematical modeling competency and problem-solving competency for students in the new curriculum context.

RESEARCH RESULTS

Realistic Mathematics Education

RME is a teaching and learning theory in mathematics education first introduced and developed by the Freudenthal Institute in the Netherlands. Currently, RME theory is mainly defined by Freudenthal's (1991) view of mathematics (Freudenthal, 1991). In most of his research works, Freudenthal (1991) said that "teaching mathematics needs to be connected with situations related to everyday life, to society in general in order to be of value to learners". His two important views were that mathematics must be connected with reality and mathematics as a human activity. Firstly, mathematics must be close to children and relevant to all situations of everyday life. However, the word "reality" refers not only to the connection with the real-world, but also to real situations in the student's mind. For problems presented to students, this means that the context can be a real-world, but this is not always necessary. Second, he emphasized mathematics as a human activity. Mathematics education is organized as a guided re-invention (re-creation) process, where students can experience a similar process to the one in which mathematics was invented. Furthermore, the principle of reproducibility can also be inspired by informal processes or solutions. Informal student strategies can often be understood as intended for the formation of more formal processes.

Based on their research, Van den Heuvel-Panhuizen and Drijvers (2014) gave six core principles of RME theory:

(1) *Activity principle*: Students learn math by doing math, students are awarded the opportunity to perform horizontal mathematization and vertical mathematization.

(2) *Reality principle*: Real-life contexts and situations should be the starting point of the learning process.

(3) *Level principle*: This principle emphasizes that, in mathematics learning, students pass through various levels of understanding: from solutions involving informal contexts, through to making operations mathematics such as symbols, diagrams, and mathematical representations to gain insight into related concepts and strategies. Models are important for bridging the gap between "informal mathematics", in relation to context, and "formal mathematics".

(4) *Intertwinement principle*: According to this principle, areas with mathematical content such as arithmetic, geometry, measurement, and data processing are not considered separate curriculum chapters but are integrated with each other, so students need to mobilize their combined knowledge and diverse mathematical tools.

(5) *Interactivity principle*: learning mathematics is not only an activity of each individual learner but also a social activity. Therefore, RME encourages whole class discussions or group work, providing opportunities for students to share their strategies and inventions with others.

(6) *Guidance principle*: In RME, this principle refers to Freudenthal's (1991) idea of "guided reinvention" of mathematics. Specifically, teachers must take an active role in student learning, and educational programs must contain situations that are able to act as a lever to achieve change in student learning and students' understanding.

Authentic Learning

Some understandings of authentic learning

Authentic learning (practice-based learning) has been recognized as an important challenge for general education in the twenty-first century (Lombardi, 1997). Facilitating authentic learning as well as assessing learning through authentic activities has been identified as a core issue for technology-enabled school education, as technology. Technology is expected to facilitate real, authentic learning experiences that are integrated into the classroom environment (Santos et al., 2015; Williams & Penny, 2011). In particular, innovations in science education in schools have long been associated with classroom technologies that can facilitate complex problem-solving capabilities in real life (Edelson, 1997).

Authentic learning is learning in real life. It is a learning style that encourages students to create a tangible, useful, quality product/result to share with their world (Revington, 2016). Once a teacher presents a problem-solving challenge, or students choose their initiative, it is imperative to nurture and support the necessary criteria, planning, processes, resources and support to meet student success. Teachers become guides, supporting co-creation with their students. Processes become the dominant force while skills, knowledge, and behaviors are activated in relevant, real-life contexts.

Authentic learning has a multitude of different definitions and interpretations, but what it boils down to is making sense of what students are learning by engaging them in relevant learning and in the real world. real world. Piaget (1974) and other psychologists believe that learners must be actively engaged in real learning (Piaget, 1974). Learning becomes active when students are able to connect new knowledge with their previous understanding.

Constructivists take this concept a little further by saying that a meaningful context that brings the real world into the classroom learning environment is key to fostering learning (Brown et al., 1989). Learning is a process of interacting with the outside world, constantly analyzing and reinterpreting new information and its relationship to the real world (Brown et al., 1989; Lave & Wenger, 1991). Traditional learning situations in which students are passive recipients of knowledge are inconsistent with real-world learning situations.

The term "authentic" is defined as genuine, true, and genuine (Webster, 1998). If learning is authentic, then students should engage in authentic learning issues to provide opportunities for them to make direct connections between the new material being learned and their prior knowledge. These types of experiences will increase students' motivation to learn. A more formal definition emphasized (Donovan et al., 1999): Authentic learning is a pedagogical approach that enables students to meaningfully explore, discuss, and construct concepts and relationships in context to real-world problems and projects that are relevant to learners. To match student learning with real-life experiences, the learning environment must be authentic. "Authentic learning is defined as learning that is seamlessly integrated or integrated into meaningful 'real' situations" (Howland et al., 2012, p. 5). Lam (2013) argues that authentic learning is a pedagogical approach to engage students to solve real-world problems. Learning by doing, project-based learning and problem-based learning, etc., are some of the learning activities, which are included in authentic learning activities. Authentic learning gives students better motivation and learning opportunities making a complex concept easier to understand.

Authentic learning allows learners to manipulate factors and observe different outcomes. It is not necessarily important to get only positive results. Sometimes conflicting results can also help learners develop reasoning and logical thinking. Finding scientific phenomena in daily life activities can fulfill our teaching purposes. Students should not only learn in class, but students must be motivated to develop scientific arguments outside of the classroom. "The function of both an authentic learning environment and an authentic mission is to show students relevance and stimulate them to develop competencies relevant to their future careers or everyday lives" (Gulikers et al., 2005, p. 510).

As such, authentic learning, in terms of learning activities, is an instructional approach that enables learners to discuss, explore, and collaborate in order to construct new knowledge and create new real-world works. real-world contexts and tasks.

Some basic features of authentic learning (Lombardi, 2007)

Real-world relevance: Authentic education provides an authentic context that reflects how knowledge will be used in real life. Context needs to be inclusive, provide purpose and motivation for learning, and provide a sustainable and complex learning environment that can be explored over the long term.

The problem is not clearly defined: The challenge cannot be easily solved by applying an existing algorithm; instead, validation activities are relatively undefined and can lead to multiple interpretations, requiring students to define themselves the tasks and subtasks needed to complete the grand task.

Sustained investigations: Issues that cannot be resolved in minutes or even hours. Instead, authentic activities consist of complex tasks that are studied by students over a long period of time, requiring a significant investment of time and intellectual resources.

Multiple sources of information and perspectives: Authentic activities must facilitate and encourage students to consider tasks from a variety of theoretical and practical perspectives, using a variety of sources, and asking students to to distinguish relevant information from irrelevant information in a process.

Collaboration: A learning task that is difficult to achieve successfully by a learner working alone. Authentic activities make collaboration integral to the mission, both on the course and in the real world.

Reflection*:* Authentic activities allow learners to make choices and feedback on their learning, both as individuals and as a group or community.

Interdisciplinary perspective**:** Relevancy is not limited to one area or area of expertise within the discipline. Instead, authentic activities have outcomes that go beyond a specific area, encouraging students to adapt and have a certain understanding of interdisciplinary terms.

Integrated assessment**:** Evaluation is not simply summed up in validation activities but is seamlessly interwoven into the main task in a way that mirrors real-world assessment processes.

Valuable products**:** Conclusions are more than mere exercises, authentic activities that culminate in the creation of an entire product, valuable in its own right.

Multiple solutions and results: Instead of giving a single correct answer obtained by applying rules and procedures, validation operations allow for diverse interpretations and competing solutions.

Some similarities between theory of realistic mathematics education and authentic learning method

Authentic learning is often guided by constructivism. It is not a new approach guide (Lombardi, 2007). It was applied in several educational systems around the world many years ago. Constructivism is a theoretical approach that helps learners construct their own knowledge without the assistance of an instructor (Papert, 1990). The constructivist approach provides an opportunity for learners to collaborate with others and apply their existing knowledge to construct a new concept. Dewey (1987) argues that learners must engage in activities where they can apply the concepts they learn in the classroom. He strongly believes in the importance of experience and logical thinking for the development of problem-solving skills. Imagination and hands-on experience are important fundamentals of authentic learning. Although there has not been a study that does not suggest a link between RME and authentic learning, we believe that the goals, methods and characteristics that these two theories have indicated, they have many things in common, specifically:

First, in principle of operation: both support the view that mathematics is a creative human activity. Both RME-basef learning or "authentic learning" provide students with the opportunity to "learn by doing," and they gain the skills, background knowledge, and understanding that scientists have found really need and use in their profession. In RME, students are seen as active participants in the learning process. It also emphasizes that the best way to learn mathematics is by doing it, as is evident in the views of Freudenthal (1991), who viewed mathematics as a human activity, as well as in Freudenthal and Treffers' idea of mathematization.

Second, *reality*: In education, the term "authentic learning" refers to a range of educational and teaching techniques that focus on connecting what students are taught in school to real-life situations. real-world situations and applications. The basic idea is that students are more likely to be interested in what they are learning, more motivated to learn new concepts and skills; they are better prepared to succeed in college, career and adulthood if what they are learning reflects the real context, equipping them with practical and useful skills. Also address topics that are relevant and applicable to their lives outside of school. Practicality represents the importance attached to the goals of mathematics education including students' ability to apply mathematics in solving problems from "real life". In addition, practicality means that mathematics education should begin with situations (contexts) that make sense for the student. Instead of starting with teaching abstract concepts or definitions directly, in RME teachers should start with problems in different contexts.

Third, *interactivity*: This principle indicates that learning math is not only an activity of each individual learner but also a social activity. In both theories, students are given the opportunity to share their experiences with others; encourage whole class discussions or group work, providing opportunities for students to share their strategies and inventions with you in class. The interaction not only takes place between students and students, but also the presentation of opinions and personal views of students to students. Furthermore, the interaction helps students achieve a higher level of understanding as well as develop mathematical reasoning and critical thinking abilities, in addition, students have the opportunity to collaborate, exchange, and develop skills. Teamwork and presentation skills.

Fourth, both learning perspectives are aimed at forming and developing problem-solving competency for students. In the lesson study, the teachers observed that the skills of the process, e.g. decision making, comparison, reasoning, thinking, etc., the students' mathematical skills were reinforced and developed through practical problems-solving. Students have the opportunity to engage in more diverse and flexible thinking because real-world problems give them the opportunity to "choose and weigh" the appropriate solution. According to Kwon et al. (2006), the openness of tasks encouraged students to apply different thinking and reasoning and promoted active participation in the learning process. The use of real-life situations in teaching has made the math learning experience more meaningful and enhanced students' confidence and assessment of the nature of mathematics.

Understanding and knowledge are rarely divided into areas and topics, and as students mature, students will have to apply a variety of knowledge and skills flexibly to suit the particularities of their profession and life circumstances. In "less authentic" learning situations, students acquire knowledge largely for the purpose of getting high scores on tests. As a result, students may remember less of what they have learned because the concept is still abstract, theoretical, or unrelated to direct experience. And because students are rarely asked to use what they have already learned. learning in real-life situations, it will be difficult for teachers to determine if students can translate what they have learned into practical skills, applications and habits of mind that will be useful in life outside of school. Meanwhile, both RME-based learning or authentic learning aims to encourage students to think more deeply, look at problems from different perspectives, recognize nuances, spot contradictions, or navigate problems and difficult situations.

Fifth, teachers have the opportunity to engage in deeper discussion during the lesson planning process. The design of math problems using real-world context tasks requires knowledge in several respects, such as knowledge of the curriculum, knowledge of task design, and knowledge of about students' thinking and their beliefs (Cheng, 2013). When planning the tasks, the teachers created more open-ended questions in an ascending order of magnitude, pulling away from the standard answers in search of some reasonable response from the children. In order to enable students to complete the learning task, the facilitators had to anticipate and properly categorize the answers and the required mathematical skills with each answer. Teachers have found from their understanding of teacher thinking to create meaningful questionnaires and explanations to facilitate teaching.

Sixth, a greater opportunity for teachers to hear students' thoughts and insights: Teachers can hear what students are thinking, what they understand from the problem scenario, and obstacles. that students encounter when solving problems. For example, through the questions that students ask during group work, teachers become more aware of the diverse interpretations of key terms in the questions. According to Chan (2005), the "real world" description of these issues provides an opportunity for personal values and beliefs to be enhanced through discussion. Through solutions and unexpected student feedback, teachers can adapt and assimilate their schema based on a personal understanding of the type of solutions and interpretations students will create for those tasks. In addition, the teacher strengthened his understanding of the students' thinking.

Seventh, some challenges for teachers to complete assignments during the curriculum: Three major challenges teachers face in performing real-world tasks. The first challenge involves designing and implementing the learning tasks so that they are doable for the duration of the curriculum. Such real-world problems require a lot of planning and execution. This is consistent with the finding of Chan (2005). Real-world settings are naturally appealing to many students and are a great platform to motivate them to solve math problems. However, because of the richness of the context, students and teachers tend to spend more time understanding and expanding the context and problem. This results in part of the curriculum being devoted to discussing and clarifying the math in real-world situations. This can become a possible "noise" in the use of real-world problems in math class and inadvertently delay or deviate from the expected other math to be learned through the problem. Therefore, we need to clearly define when and "at what speed" we want students to approach mathematics. Another possible "noise" is the "thickness" of the context. One suggestion is to change the "thickness" of the context for different purposes of the math lessons and to "decorate" the context to suit the different needs and abilities of students.

Besides the commonalities between the two approaches to teaching mathematics, the two teaching methods still have certain differences: The main difference between realistic mathematics education (RME) and authentic learning (authentic) is that RME is only taught applied to mathematics education while authentic learning methods can be used in many subjects. In addition, authenticity in authentic learning is required but is not required in RME. It can be said that the authentic teaching method offers a significant extension or complement to RME theory and of course, RME theory provides the instructional theory to support teachers in designing instruction according to authentic learning methods.

Table 1. Worksheet no. 1

Question	Content of questions	Anwser
Q1	The total revenue of a milk tea shop is determined through which factors?	
Q2	What factors does the total cost of selling milk tea depend on?	
Q3	How is the profit of the milk tea business determined?	
Q4	Find selling price of a cup of milk tea to make biggest profit of Nam's family is related to which mathematical knowledge?	

Table 2. Expected answer of worksheet no. 1

Question	Anwser
Q1	Total revenue (income)=(Number of milk tea cups sold) "multiply by" (price per cup)+fixed revenue
Q2	Total cost (cost)=(Number of milk tea cups sold) "multiplied by" (cost per cup)+fixed cost (space rent)
Q3	Profit=Revenue-cost
Q4	Find the maximum and value of the function on the interval [a; b]

EXAMPLE OF A LEARNING SITUATION DESIGNED BASED ON A COMBINATION OF AUTHENTIC LEARNING AND RME-BASED LEARNING IN TEACHING AND LEARNING CALCULUS TOPICS FOR HIGH SCHOOL STUDENTS

Choose a Situation

The design of teaching situations is aimed at developing the mathematical competence of the Vietnam 2018 curriculum, with special attention being paid to two competencies: mathematical modeling and mathematical communication. RME principles and authentic learning characteristics will be referenced in the instructional design process. The following problem is designed for 12th graders when building knowledge about the maximum and minimum values of functions.

Situation

> Taiwanese milk tea is a drink made from green tea or black tea, developed by beverage shops in Taichung, Taiwan since the 1980s. Currently, milk tea is still suitable for consumption. Many customers, not only students, but also children and office workers love it. Grasping that trend, Mr. Nam's family decided to rent a space in the city to open a shop specializing in milk tea business. Mr. Nam said: every month, the store has to pay a fixed cost (space rent) of about 10 million VND; The cost of ingredients for a cup of milk tea is about 10 thousand VND. Moreover, each cup of milk tea will be sold for no more than 45 thousand VND. At the same time, Mr. Nam also confirmed that the number of milk teacups sold in a month will never exceed 4,500 cups. Please make a proposal for the selling price of a cup of milk tea so that Mr. Nam can maximize the profit in his business.

The situation is built on the real principle of RME. High school students have more or less experience and a certain understanding of basic economic relations. The profit (monthly) of a business depends on many factors, including the quantity (units) of goods sold (monthly) and the total cost per unit of those goods.

Activity 1. (13 minutes) (working independently and in groups). Explore and explain the problem in context: Teachers organize for students to learn and explain problems in real contexts:

Activity 1.1. (five minutes): After students have clearly understood the problem in the situation (**Table 1**), the teacher distributes worksheet No. 1 to all students in the class, the students answer (independently) on the sheet (**Table 2**).

Activity 1.2. (five minutes): After the members complete worksheet number 1, the teacher randomly divides all students in the class into 4 groups and distributes worksheet number 2 (with scratch paper) (**Table 3**). Based on the available results of the members, each group discussed and agreed to come up with the most reasonable answer for their group (**Table 4**).

Activity 1.3. (three minutes): Teacher gives expected answer.

Activity 2. (30 minutes). Contextual problem-solving:

Activity 2.1. (eight minutes): The teacher organizes (with instructions) for each group to propose solutions to the problem.

Proposed solution 1:

- Build some mathematical models (guessions) then conduct horizontal mathematization and vertical mathematization to solve the mathematical model.
- Solve mathematical models, thereby giving answers to real-life situations.

Proposed solution 2: Ask for some more sales information from Mr. Nam's family business in the last few months, thereby building a mathematical model based on the provided information.

Activity 2.2. (12 minutes): The teacher guides the students to solve the situation according to option 1 based on mathematical modeling through the suggested questionnaire in the worksheet No. 1.

Activity 2.3. (10 minutes):

- With the information available in the previous answers (Q1-Q7), the teacher guides the groups to set up a mathematical model showing the profit from the milk tea business of Mr. Nam's family.

Table 3. The content of the questions in the worksheet no. 2

No	Question	Object
Q1	What is the unknown factor (variable) in the real situation?	- Identify variables in mathematical models - Move from solving a real task to a math task (horizontal) - Developing mathematical modeling competence communication skills
Q2	Can we consider the number of cups of milk tea sold in a month as a function where the variable is the factor determined in Q1?	
Q3	How can total revenue in a month be expressed through number of cups of milk tea sold in a month (in Q2) & selling price of each cup of milk tea? (know that Mr. Nam has no other fixed income)	Set up mathematical expression for total revenue from Mr. Nam's milk tea business
Q4	How can cost in a month be determined by number of cups of milk tea sold & fixed cost for a month?	Set up a mathematical expression for business expenses for a month
Q5	How is profit from milk tea business determined through total revenue (in Q3) & expenses for a month (in Q4)?	Set up a mathematical expression for business expenses for a month
Q6	Information that each cup of milk tea will be sold for no more than 45 thousand VND & number of cups of milk tea sold in a month will never exceed 4500 cups	Set constraints, relationships between mathematical objects in model (vertical mathematization)
Q7	Propose some models (expressing dependence of number of cups of milk tea sold on selling price of a cup of milk tea)	Building a mathematical model for situation
Q8	Propose mathematical models-show profit from selling milk tea	Building a mathematical model for situation

Table 4. Expected answer worksheet no. 2

No	Anwser
Q1	Yes
Q2	
Q3	Revenue $T(x)=N(x).x+R_0$, where $N(x)$ is number of cups of milk tea sold in each month & R_0 is fixed income
Q4	Cost $R(x)=N(x).C+C_0$; $N(x)$: Number of cups of milk tea sold in each month, C: Cost per cup of milk tea (materials), & C_0: Fixed cost (space rent)
Q5	Profit $L(x)=T(x)-R(x)$
Q6	$N(45)=0$; $N(0)=4,500$
Q7	$N(x)=4,500(1-[x/45])$ (linear model)
	$N(x)=[-4,500/45^3](x-45)^3$ (3rd order model)
Q8	$L_1(x)=4.5(1-[x/45](x-10)-10$ (million VND)
	$L_2(x)=-[4.5/45^3](x-45)^3(x-10)-10$ (million VND)

- Students use mathematical knowledge and other supporting tools to solve math tasks.
- Using results in mathematical modeling, students pose questions in real situations.

Activity 3. (25-36 minutes). Compare and discuss the answers, then draw conclusions:

Activity 3.1. (16 minutes): The groups present summary of the main results (three-five minutes/one group/presentation):

- The teacher sends a representative of each group to the board to present their group's problem-solving options. The results of the group discussion were recorded.
- While representing one group, the other groups should take notes summarizing the main ideas and important information for later group discussion (*activity 3.2.*).

Activity 3.2: (10 minutes). Discussion between groups:

After finishing the presentation of one group, the remaining groups have the right to ask questions, exchange ideas and make comments, then give comments and supplements (if necessary).

Activity 3.3. (10 minutes): Conclusion, adjustment, comments, and some comments of the teacher:

- The teacher summarizes the answers of each group from which to make comparisons and comments (mathematical rationality and logic, teamwork skills, presentation skills, …).
- The teacher summarizes the process of solving practical problems by mathematical modeling, thereby emphasizing the mathematical knowledge that students need to acquire through real-life situations.
- For solving mathematical problem, teachers can guide students to process with GeoGebra software to support calculations and visual description, the results are shown in **Figure 1**, **Figure 2**, and **Figure 3**.

Comment

In **Figure 2**, we see that with a linear supply function, for each cup of milk tea sold for VND 27,500, each month Mr. Nam's family earns the highest profit of VND 20.63 million. Meanwhile, with a supply function of cubic form (**Figure 3**), we can observe that this is an inappropriate model because the monthly revenue will be lower than the expenses, so the business Mr. Nam will lose. This shows that the difference in returns, depending on the supply function that each student uses, is quite large because we have so little information. When starting a business, we should carefully keep a list to record our sales. These numbers can give us important information about what the supply function actually looks like. So, after a while, the seller should list the price of the store and the quantity of milk tea sold at that price. Let's say we have priced VND 30,000 per cup of bubble tea and sold 2,000 cups in a month. Now there is another point also on the graph of the supply function. Can the teacher suggest that students can find a reasonable function that goes through all three points? One simple method is to find a unique quadratic function that passes through all three points: A(45; 0), B(0; 4,500), and C(30; 2,000).

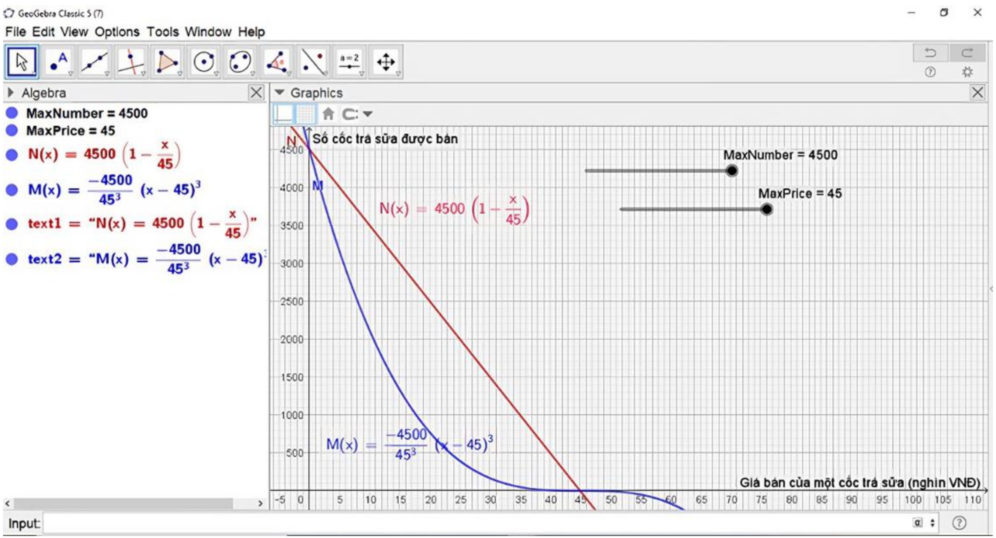

Figure 1. Two supply functions (Source: Author's own elaboration, using GeoGebra software)

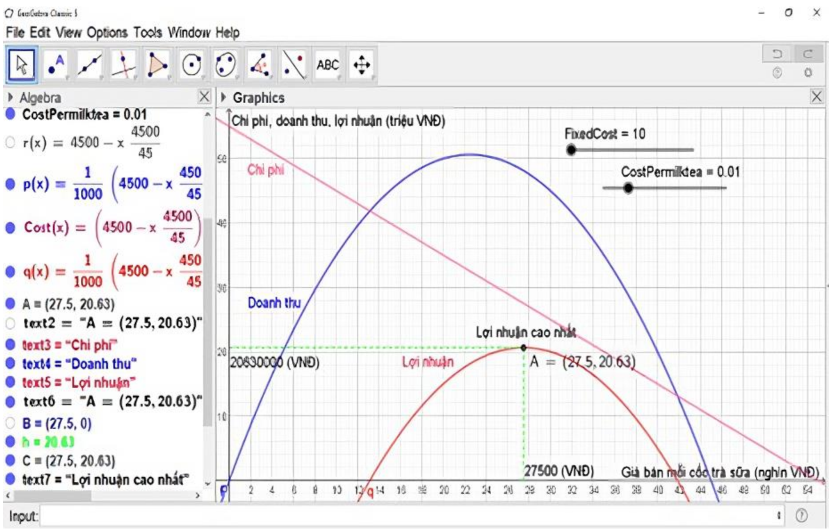

Figure 2. Finding the maximum profit from a linear supply function (Source: Author's own elaboration, using GeoGebra software)

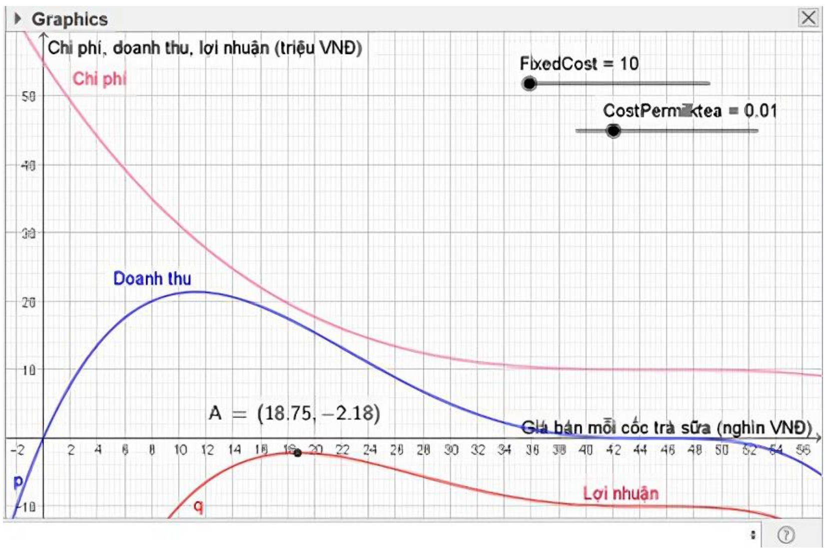

Figure 3. Maximum profit for a cubic supply function (Source: Author's own elaboration, using GeoGebra software)

The quadratic function then has the form (**Figure 4**).

Figure 5 shows that if each cup of milk tea is sold for VND 28,790, Mr. Nam earns the highest profit of VND 30.22 million/month.

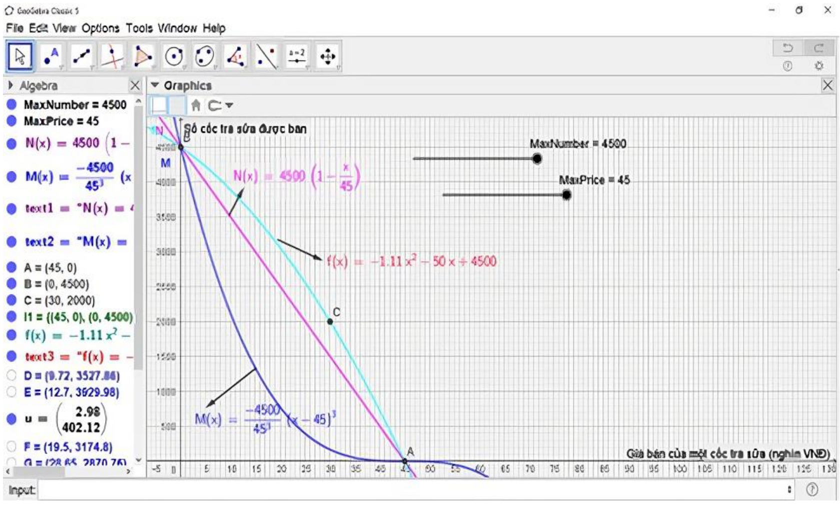

Figure 4. The quadratic function (Source: Author's own elaboration, using GeoGebra software)

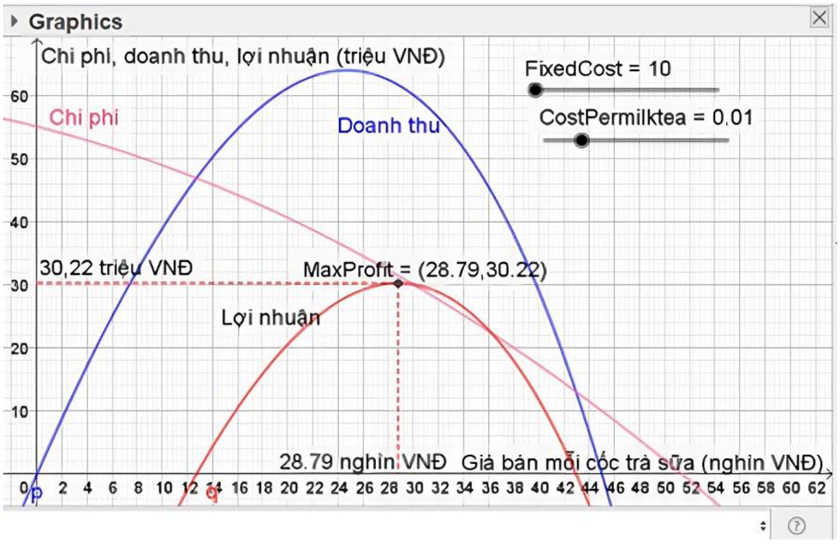

Figure 5. Profits (Source: Author's own elaboration, using GeoGebra software)

Obviously there are many flaws with this model. The supply function will fluctuate greatly over time and is also strongly influenced by competition and advertising. In fact, analytics like these aren't used much by small businesses. They are important tools of the big stores, where the economic models used are of course much more complex than this simple one. However, this simple model provides some valuable insight into the underlying economic structure and prepares students for some initial knowledge of applying derivatives to solving some optimization problems. advantage in economics, production and business.

Activity 4. (five minutes). Summarize and apply:

- Teachers re-evaluate each individual's learning process, cooperative attitude and learning spirit.
- Assess students' presentation skills and teamwork skills.
- Recalling mathematical knowledge that appears in real-life situations.
- Ask students to solve some similar situations (in homework).
- Finally, the teacher designs a number of questions to survey students' feedback on the designed situation, thereby allowing the teacher to make necessary adjustments or additions to suit the actual teaching in class.

CONCLUSIONS AND RECOMMENDATIONS

Connecting mathematical knowledge to solving some real-world problem really makes learning mathematic meaningful. In teaching mathematics, the combination between reference theory of realistic mathematics education and authentic learning methods contributes to improving and developing problem-solving capacity; mathematical modeling competence; ability to perform and communicate mathematics for students in the new program context; helps guide the analytical and experimental construction of meaningful teaching situations with students that give them intellectual meaning and contribute to the development of expected mathematical competence.

A real-world problem, if carefully designed, in line with the goals of the curriculum, provides teachers with the opportunity to learn through problem planning and implementation. It also provides students with the opportunity to connect math with their social lives. Real-world problems can be overwhelming for students when they are faced with problems they haven't heard of. Therefore, teachers need to be careful in choosing contexts and problem situations suitable for students, they can appreciate and apply problem-solving processes to connect mathematics with the world in which they are beginning to discover. More research is needed in support of such tasks, especially in differentiating tasks to meet students' different needs and problem-solving abilities.

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