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Research Article

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Strengthening pre-service mathematics teachers' knowledge of students' thought processes, students' misconceptions and text analysis skill

Wasiu Ismaila Otun [1]*

[1] Department of Science & Technology Education, Faculty of Education, Lagos State University, Ojo, NIGERIA
***Corresponding Author:** otun_w@yahoo.com

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ABSTRACT

Mathematics teachers may not be able to fix students' errors and misconceptions unless they are trained on how to identify, recognize, respond and effectively remediate students' thought processes and students' misconceptions. This study determines the effects of solve-reflect-pose strategy (SRPS) on prospective mathematics teachers, students thought processes ability, students' conceptions and misconceptions ability and their text analysis skill level (TASL). The pre- and post-test research model was used. There were two groups, which were named experimental group (consisted of 92 participants taught with the SRPS) and control group (consisted of 90 participants taught using the modified conventional method (MCM). The quantitative data was collected through an instrument subdivided into three, namely: knowledge of students' thought processes test, knowledge of students' conceptions and misconceptions test, and TASL test; and interview protocol was used to collect qualitative data. The formulated research questions were analyzed using descriptive statistics while independent sample t-test was used to analyze the hypotheses. Results showed that the effects of SRPS instruction were statistically significantly different in the mean post-test achievement scores on knowledge of students' thought process test, students' misconceptions test and analysis skill level test.

Keywords: solve-reflect-pose strategy, students' thinking processes, students' conceptions and misconceptions, text analysis skills

INTRODUCTION

In mathematics classrooms, many teachers are facing the challenges of what could possibly be the factors responsible for their students' failure despite all their efforts toward improving their students' achievement in mathematics. Successful mathematics teachers apply multiple approaches or methods, which enhance meaningful understanding of the subject and promote students' thinking process skills and eliminate misconceptions (Ake et al., 2013; Pournara et al., 2016; Zuya, 2014). Successful application of multiple approaches could directly impact on students' creativity, enhances students' access to important mathematics concepts, and gradually uplifts students mind from basic challenges and difficulties to higher order thinking skills. The teaching of mathematics is generally believed to be knowledge-intensive. During teaching, if a teacher could imagine what it takes to struggle in order to perform perceived difficult mathematics tasks, such a teacher could be said to be a successful teacher.

It is a common knowledge that when a student is perceived to be a weak learner in algebra, such a student may continue to struggle or experience difficulties in learning other mathematical concepts (Makonye & Stepwell, 2016). Studies have reported that majority of secondary school students experienced algebraic conceptual difficulties, weak algebraic thinking processes and they are overwhelmed by algebraic errors and misconceptions (Booth et al., 2014; Chow, 2011; Ling et al., 2016; Mulungye et al., 2016; Pournara et al., 2016; Zuya, 2014). Many students have quit trying because they perceived that mathematics tasks especially algebra are simply too complex and difficult. The joy of every teacher is to help their students' experience deep understanding of perceived difficult mathematics tasks. A successful teacher will put himself in the place of a student in order to understand what the student learns and the way he/she understands it. Such a successful teacher would rely on various domains of knowledge, in which, knowledge of students is pertinent.

Knowledge of students could positively impact mathematics teacher's "knowledge of students' thought processes (KSTP)", "knowledge of students' conceptions and misconceptions (KSCM)", and the "text analysis skill level (TASL)" (Gunawardena, 2011, Shulman, 1987). Mathematics teachers that are well equipped in the knowledge of students and content, discover mathematically promising students, students who are mathematically creative and with higher order thinking skills (Gunawardena, 2011; Tanisli & Kose, 2013). KSTP, KSCM, and TASL encompass a range of forms including the ability of mathematics teachers to carefully scrutinize the written product of students to understand the logic behind the thinking that led to an error, be able to predict students' misconceptions and proffer remediation. Faulty KSTP, KSCM, and TASL are detected by analyzing students' written work. Many scholars placed knowledge of students at the center of pedagogical content knowledge, and it is viewed as one of the key components of pedagogical content knowledge (Park & Oliver, 2007; Shulman, 1986).

Previous studies on teachers' knowledge of students and contents found out that mathematics teachers and pre-service mathematics teachers have incomplete and inadequate knowledge of students in general (An et al., 2004; Chick & Baker, 2005; Chick et al., 2006; Zuya, 2014). Experience has shown that students' difficulties and lack of understanding of algebra cannot be attributed to students own limitations. Teachers' knowledge of students and contents could also have influences on students thinking processes (Cengiz et al., 2011). It is only few teachers that are aware that students sometimes over-generalize what they learn about algebraic concepts and this could cause them to make errors in the learning of successive algebraic concepts. Everybody thinks but the quality of our thinking varies. It is possible for students to experience faulty thinking processes. There is need to guard students' thinking process and ensure their thinking is operating at its optimum for better achievement.

Teacher's KSTP and KSCM are issues that have attracted attention to many researchers in the field of mathematics education, because they are very paramount to promote such thinking and remediate misconceptions at different levels of education (Ake et al., 2013; Carraher & Schliemann, 2007; Filloy et al., 2008; Kieran, 2007; Zuya, 2014). Teacher's teaching method or approach may aid or hinder the development of algebraic thinking processes and misconceptions of students in the classrooms (Ake et al., 2013; Zuya, 2014). Reports from literature show that students' algebraic thinking processes, conceptions and misconceptions have implications for teacher training in primary, secondary and tertiary education (Ake et al., 2013; Cai & Knuth, 2011; Carraher & Schliemann, 2007; Filloy et al., 2008; Godino et al., 2014; Kieran, 2007; Zuya, 2014). Teachers' TASL is the ability of teachers to identify, analyze and respond appropriately to students' thinking processes, conceptions and misconceptions about some algebraic concepts could help identify characteristics of mathematical practices in which teachers could intervene to gradually increase the thinking processes of students and eliminate misconceptions in algebraic activity.

The KSTP, KSCM, and TASL contribute to the development of mathematics teachers' content knowledge and pedagogical content knowledge (Batanero & Diaz, 2011; Beswick, 2012; Depaepe et al., 2013; Grossman, 1990; Tanisli & Kose, 2013; Tichá & Hošpesová, 2009; Toluk-Ucar, 2009). Teachers could acquire this knowledge from teaching and other experiences outside the classroom. The KSTP, KSCM, and TASL influence how teachers act with students in the classroom as they engage students in studying algebra. Mathematics teachers who could convert some classroom incidents into learning opportunities could be said to have possessed the right sorts of knowledge, since their interventions in the classroom would lead to greater students' achievement and when teachers do not possess this sort of knowledge, their students' achievement suffer (Akinsola, 2013; Ball et al., 2005; Batanero & Diaz, 2011; Beswick, 2012; Tanisli & Kose, 2013).

One way to facilitate teachers' KSTP, KSCM, and TASL development is by deepening their conceptual, procedural and flexible procedural understanding and be familiar with students' thinking processes, conceptions, and misconceptions through professional development experiences or effective teaching methods (Kajander et al., 2006; Otun & Olaoye, 2019; Zerpa & Kajander, 2008). There is a growing recognition that, more researches are necessary to explore teachers' KSTP, KSCM, and TASL in the field of mathematics education, because it is very paramount to promote such knowledge at different levels of education (Ake et al., 2013; Carraher & Schliemann, 2007; Filloy et al., 2008; Kieran, 2007; Zuya, 2014). Teacher's method of teaching may aid or hinder the development of algebraic knowledge of students' and content (Ake et al., 2013; Zuya, 2014).

Reports from literature show that, teachers' KSTP, KSCM, and TASL have implications for teacher training, both in primary and secondary education (Ake et al., 2013; Carraher & Schliemann, 2007; Kieran, 2007; Cai & Knuth, 2011; Godino et al., 2014; Filloy et al., 2008; Zuya, 2014). Students' incorrect thinking processes, misconceptions and its errors in algebra among junior and senior secondary school students have been documented (Egodawatte, 2011; Makonye & Stepwell, 2016; Seng, 2010). The ability of teachers to identify, analyze and respond appropriately to students thinking processes about some algebraic concepts could in-turn help identify characteristics of mathematical practices in which teachers could intervene to gradually increase students' algebraic activity.

Pre-service mathematics teachers in the Nigerian colleges of education are exposed to the teaching and learning of universal basic education mathematics curriculum through the junior secondary school (JSS) mathematics content courses (NCCE, 2009). These JSS mathematics method courses are meant to develop the pre-service teachers' subject-matter knowledge and pedagogical content knowledge. These courses are meant to equip pre-service teachers with different approaches or methods of teaching some difficult junior secondary school mathematics concepts. Despite the fact that pre-service teachers are exposed to these method courses, the findings from previous studies show that pre-service teachers are weak in knowledge of students and content (Ashikhia, 2010; Bessong et al., 2013; NCCE, 2009; Salman et al., 2012).

Teachers may have insufficient knowledge on how to handle students' errors and misconceptions unless they are trained on the techniques of identifying, recognizing, responding and effectively remediating students thought processes and students' conceptions and misconceptions. There are few mathematics teaching methods that are very effective in exposing mathematics teachers to strategies that could make them identify, analyze, and remediate students' thought processes, conceptions, and misconceptions in mathematical concepts. The objective of effective teaching strategies is to improve students' achievement and active interaction in mathematics classrooms, therefore, there is need for pre-service mathematics teachers to be equipped with teaching strategies that will allow them discharge their knowledge. Solve-reflect-pose strategy (SRPS) is an instructional strategy that impacts positively on pre-service mathematics teachers' conceptual knowledge; procedural knowledge and flexible procedural knowledge (Otun & Olaoye, 2019).

SRPS is informed based on the researchers' knowledge of cognitive psychology and constructivist theories. Problem solving, problem posing and reflective thinking are embedded within the cognitive psychology and constructivist theories. Cognitive and constructivist learning theories researches provide direction about teaching in general and some guidance about how to increase students' knowledge and teach them to be critical and effective thinkers. The first strategy is that, the pre-service teachers engage in problem solving activities with the help of their teacher educator. They are guided into Polya's problem solving steps. The second strategy is reflective thinking. The teacher educator guides the pre-service mathematics teachers into different methods of algebraic solutions, various students' algebraic misconceptions and errors. This would allow the pre-service mathematics teachers gain in-depth knowledge of the principle or processes underlying the given algebraic tasks.

The third strategy is activities on regeneration, reformulation of similar problem. SRPS is an active learning and teaching strategy which enables the student to become aware of, determine his/her problem solving and posing abilities and critical thinking skills, in order to build necessary domains of knowledge, develop critical thinking skills and to perform group works. SRPS is the combination of range of pedagogical approaches that are learners-centered. It is an active learning and teaching strategy which enables the student to determine his/her problem solving, reflective thinking competence and problem posing abilities, in order to build necessary domains of knowledge. It could also be used to develop mathematics teachers' ability to identify, analyze, interpret students thinking processes, to solve and pose problems (Otun & Olaoye, 2019). SRPS is not simply as a way of providing more teaching strategy, but more importantly, to allow teachers understand and reconstruct what they know and can teach with more depth and meaning. The main aim is to enable mathematics teachers (male and female) to acquire a deeper and also a more connected understanding of mathematical concepts and the learners.

There are many studies on pre-service teachers' knowledge of students (An et al., 2004; Baker & Chick, 2006; Chick et al., 2006). A consensus has not been reached on the most effective method that could impact positively on pre-service mathematics teachers' KSTP, KSCM, and their TASL. And as there is dearth of literature on pre-service teachers' KSTP, KSCM, and TASL, the researchers decided to embark on this area of research. Hence, this study aims at unpacking pre-service teachers' KSTP, KSCM, and TASL involved during a similitude classroom setting that allows pre-service teachers convert some classroom incidents into learning opportunities. Thus, it becomes pertinent to look for interventions that could be manipulated in order to find their effects on pre-service teachers' knowledge of students' thinking processes, knowledge of students' conception and misconceptions and text analysis skills. Based on this, the researchers used SRPS as an instruction in teaching pre-service mathematics teachers' KSTP, KSCM, and their TASL and compared its effects with modified conventional method (MCM). In mathematics education community, gender is still an issue and recent studies on gender posited that the attention on gender has declined (Grevholm, 2011; Lubienski & Garley, 2017). One of our aims is to ascertain if gender differences appear among the male and female pre-service mathematics teachers after been to exposed to SRPS within the same classroom settings.

Research Questions

Major research questions that guided the study are, as follows:

1. What is the difference in the KSTP, KSCM, and TASL of pre-service teachers in algebra between SRPS and MCM?
2. What is the difference in the KSTP, KSCM, and TASL of male and female pre-service teachers in algebra between SRPS and MCM?

Research Hypotheses

1. There are no significant differences in the KSTP, KSCM, and TASL of pre-service teachers in algebra between SRPS and MCM.
2. There is no significant difference in the KSTP, KSCM, and TASL of male and female pre-service teachers' in algebra between SRPS and MCM.
3. There is no correlation among KSTP, KSCM, and TASL of pre-service mathematics teachers taught using SRPS and conventional method in algebra.

RESEARCH METHODS

Research Design

This study used a mixed-method approach of quantitative and qualitative methods. Quantitative and qualitative data were collected during the study and information gathered was integrated into the interpretation of the overall results. A pre- and post-test control group (2×3×2) factorial design was employed. In this design, two teaching strategies (SRPS and MCM) were crossed with pre-service mathematics teachers' TASLs (high, average, and low in algebraic word problems) and pre-service mathematics teachers' gender (male and female). In using this design, one experimental (treatment) and one control group were used. Intact classes from two colleges of education were assigned to the experimental group and control group based on their pre-test results. Quasi experimental design was considered most appropriate in this study since intact classes were used and no randomization was done in the selection of subjects. Pre- and post-test of pre-service teachers' knowledge of students algebraic thinking processes in algebra scores were obtained before and at the end of treatment, which lasted for 10 weeks.

Treatment Procedure

The two groups used in this study consisted of participants in the experimental group (E) who were exposed to SRPS intervention while the participants in the control group (C) were treated with the lecture teaching method.

Solve-Reflect-Pose Strategy

Subjects in experimental group (E) were exposed to algebraic concepts through SRPS. The facilitator (mathematics education lecturer) engaged the pre-service teachers in algebraic tasks resulting from solve-reflect-pose group activities. In order to achieve the research objectives, the facilitator allowed the pre-service teachers to enjoy some freedom of action. The pre-service teachers chose the number of working sessions, the junior secondary school mathematics textbooks, some algebraic concepts, and perceived difficult tasks

Table 1. Breakdown of sample according to gender, teaching strategy, and text analysis skill level

School	Teaching strategy	Gender	High	Average	Low	Total
School 1	SRPS	Male	1	18	18	37
		Female	0	28	27	55
School 2	MCM	Male	11	33	2	46
		Female	8	33	3	44
Total			20	112	50	182

were discussed during the working sessions. During the working sessions, the facilitator guided the groups toward the complementation of conditions on the design of the SRPS flow chart. Additionally, they discussed on the solution of the tasks, students thinking and reasoning processes, possible students' misconceptions, analyzed possible underlining causes of students errors and misconceptions, reflected on algebraic nature of them, reflected on alternative methods of solutions and posed questions on the difficulties that students could face when solving the tasks. The following activities also took place: discussing and summarizing how the participants would teach algebraic concepts (most especially algebraic word problems) through SRPS; discussing and summarizing how they would identify, analyze and response to students thinking processes, misconceptions and errors through SRPS; sharing and comparing their individual reflections on alternative methods of solutions and their findings from the algebraic concepts tasks, preparing summaries of key words of the group reasoning to correspond with the questions under individual reflection and developing a flowchart of the SRPS; generating a new problem or reformulating similar problems; whole-class sharing of small-groups' findings. The facilitator listens to the group summaries which consisted of their collective algebraic reasoning resulting from the solve-reflect-pose group activities. The summaries reflected more depth in their understanding of algebraic concepts and the SRPS. For example, one group's description of a 'good' problem included: "different methods and techniques, focus on SRPS - not tedious calculation, and students ability to relate to the problem." Another group's: "Should make students thinking process, be challenging." Their description of the SRPS was also enhanced, particularly in terms of the flowcharts, which showed the need to move back and forth as opposed to taking a linear path to a solution. The flowchart is simplified to fit the available space; it contains appropriate boxes and arrows (**Appendix A**).

Control Group

The pre-service mathematics teachers in the control group (C) were taught algebraic concepts with the conventional method. The facilitator (mathematics education lecturer) in this group used conventional method to introduce the pre-service teachers to algebraic concepts, problem solving and posing strategies. The pre-service teachers listened; avoided interaction with other subjects but sought assistance from the facilitator only in the learning process.

Sample and Sampling Technique

The study involved one hundred and eighty-two year two pre-service mathematics teachers from two colleges of education in Lagos State. These two colleges of education were purposively sampled from Lagos state of Nigeria. The colleges of education selected involved a federal and a state college of education. The sample size was not too large because these participants were observed during their classes. The researcher tried to ensure that the subjects were representative of the pre-service teachers who majored in mathematics and who have undergone the initial levels of pedagogical content knowledge. The ability level of the participants refers to the entry knowledge of the pre-service teacher before the treatment. It was determined by pre-service teacher's score obtained in the pre-test score of the achievement test on algebraic knowledge of student and content achievement test (AKSCAT). There were three ability levels in this study.

1. Average ability level: Pre-service teachers with scores ranging from 14-19 in AKSCAT pre-test score.
2. High ability level: Pre-service teachers with scores of 20 and above in AKSCAT pre-test score.
3. Low ability level: Pre-service teachers with scores below 14 in AKSCAT pre-test score.

The breakdown of the sample is given in **Table 1**.

Research Instruments

One research instrument AKSCAT with three levels of subdivision, namely: KSTP, KSCM, and TASL tests were used for the quantitative data and interview protocol for qualitative data.

Algebraic Knowledge of Student and Content Achievement Test

This instrument was designed to assess the impact of SRPS on pre-service mathematics teachers' KSTP, KSCM, and TASL. The AKSCAT is an hour test. It was developed by the researchers to cover all algebraic word problem expressions and equations leading to algebraic equations and simultaneous equations. The items consisted of sub-categories, which are solving for the unknown, explaining, or demonstrating incorrect answers that may be given to some questions by students and prediction of possible students' errors and misconceptions. These items were intended to measure pre-service teachers' knowledge of students' algebraic thinking processes.

These equations were written in English language and they were presented in different word problem structures. Four of such questions were in form of problem structures such as

> "Ade, Faith, and Juliet were all working to solve a question, which states that the difference between square of 9 and its square root, and the result divided by two. What is the number?
>
> Ade's solution: 9^2-(√9)÷2=81-3/2=189/2=94½.

Faith's solution: $(9^2-\sqrt{9})\div 2=(81-3)\div 2=78/2=39$.

Juliet's solution: $(\sqrt{9^2})\div 2=9/2=4½$.

For the student(s) who were incorrect, identify their mistakes.

For the student(s) who were incorrect, try predict reasons for their mistakes.

For the student(s) who were incorrect, explain how you will correct their mistakes."

Another three questions took problem structures such as

"I am thinking of two numbers. The larger of the two numbers is less than twice the smaller number by 3. The sum of the two numbers is eighteen; 'What kind of incorrect answers may be given to the questions above by your students? Try and predict."

Another three items were presented as algebraic equations, with each instructed the pre-service teachers on formation of word problems on the given algebraic equation. These questions take problem structure such as

"Given the equation, $\frac{x}{3}+\frac{4x}{5}=8$, what error(s) may student(s) exhibit as they answer this question."

Another three of these questions took problem structure such as

"Kemi solved a word problem which states that I am thinking of two numbers. The larger of the two numbers is less than twice the small number by 3. The sum of the two numbers is eighteen. What are the numbers? Kemi got the first and second unknown numbers to be 17 and 11, respectively. She thought that she was right, but when she checked her answer, it did not work and she did not know why."

With different sub-categories such as

"Determine the correct values of the unknowns; identify Kemi's possible mistake(s), give reasons for her mistakes, and explain how you will correct her mistakes."

Three other questions took problem structure such as

"Ngozi, Kunle, and Jane argue on the correct answer to a question, which states that 8 is added to a certain number and then doubled the result and the result divided by four. The answer is 27, find the number."

If they all used letter *x* to represent the unknown, A worked example of the students' solutions were given, such questions have sub-categories such as

"For the student(s) who were incorrect, identify their mistakes'; 'try predict reasons for their mistakes'; 'explain how you will correct their mistakes."

These items were intended to measure the knowledge of students' algebraic thinking processes, KSCM, and TASL of pre-service mathematics teachers', relating to algebraic concepts. The reliability coefficient of the AKSCAT was determined using a test-retest. It was found to be 0.73.

Interviews Schedule

The interview protocol was designed by the researcher to collect qualitative data through in-depth interviews with the pre-service mathematics teachers. The interview schedule was made up of questions which were posed to pre-service mathematics teachers in the experiment group. The questions sought for the perceptions of pre-service mathematics teachers on SRPS used for instruction, the ease of the use of the SRPS in teaching and learning of algebra and potency of SRPS on the pre-service mathematics teachers' knowledge of students' thinking processes, knowledge of students' conception and misconceptions and text analysis skills. The researcher was directly involved in the interview process to ensure consistency and uniformity of purpose.

In order to determine ways to identify pre-service mathematics teachers for interviews and to get an adequate and a manageable number of pre-service mathematics teachers, the researchers randomly selected the 15th test and we also followed the theoretical sampling strategy while analyzing pre-service mathematics teachers' answers in the test. Therefore, the researchers selected interview participants by thoroughly examining their answers to the test. In all, six pre-service mathematics teachers (3 male and 3 female) were selected for the interview. Data was collected using video recorder. This was done to prevent loss of information provided by the pre-service mathematics teachers.

Training of Facilitators

The method course lecturers in the selected colleges of education were the facilitators for the study. These facilitators were trained on the use of the instructional strategies used in the study. The meeting for the training sessions lasted for about three weeks due to the busy schedules of these lecturers. The first week was introduction to the training session and the second week was used for in-depth study of the SRPS and the conventional method.

Table 2. Mean and standard deviation of pre-service teachers' scores in KSTP, KSCM, and TASL

Dependent variables		Experimental group		Control group	
		n=92		n=90	
		Pre-test	Post-test	Pre-test	Post-test
Knowledge students thinking processes	Mean	26.0870	81.0326	28.5222	61.2778
	SD	6.52746	9.38019	6.79595	13.86373
knowledge of students conceptions & misconceptions	Mean	29.2391	79.7609	27.2667	63.3444
	SD	5.22330	12.33718	5.52787	13.43161
Text analysis skill level	Mean	1.5109	2.6304	2.1778	2.2111
	SD	0.52403	0.48533	0.55238	0.52988

The researchers with the help of the facilitators administered the test to the pre-service mathematics teachers at the beginning and collected the test scores same day. Thereafter, the facilitators taught the pre-service teachers in the respective colleges of education using the instructional approaches assigned to each of them. This was done using lecturer guide jointly prepared by the facilitators and the researchers. The other materials used were the students' mathematics textbooks and past external examination questions for junior secondary school. The teaching lasted for about six weeks. The researchers also with the help of the facilitators, administered the test to the pre-service teachers at the end of the lesson session and collected the scores test same day.

Data Collection and Analysis

During treatment session, all the participants received instruction on algebraic word problems; knowledge of students' algebraic thinking processes; they were exposed to how to identify, analyze, and respond to students' thought processes; they were exposed to various students' difficulties in solving algebraic word problems; various students' errors and misconceptions. The researchers began by giving the pre-test to both the treatment and control groups. The pre-test scores of the pre-service teachers in AKSCAT were used to categorize the pre-service teachers in each group according to their TASL and also to assess the strengths and weaknesses of pre-service teachers in algebra.

Descriptive statistics (mean and standard deviations) and inferential tests (independent-sample t-test) were used to analyze the data after the data collected satisfied the assumptions of independent samples t-test. The differences between the experimental group and the control group for the pre- and post-tests were analyzed using a t-test at 0.05 level of significant. The independent samples t-test was considered more suitable for testing the hypothesis. In addition, responses from semi- structured interviews from the groups (experimental group and the control group) were analyzed thematically to understand the perceptions of pre-service mathematics teachers on SRPS used for instruction, the ease of the use of the SRPS in teaching and learning of algebra and potency of SRPS on the pre-service mathematics teachers' algebraic conceptual knowledge, procedural knowledge and flexible procedural knowledge. Audio recorded data were transcribed verbatim and transcripts were analyzed using open, axial and selective coding (De Vos et al., 2011).

RESULTS

The pre-test result revealed that the average score of the control group was higher than that of the experimental group and the significant values for the experimental group, control group and the F-test for both, are greater than 0.05. Hence, the use of independent samples t-test was appropriate.

Findings Relating to KSTP, KSCM, and TASL

Research question 1: What is the difference in the KSTP, KSCM, and TASL pre-service teachers in algebra between SRPS approach and MCM?

Table 2 shows that the pre-service teachers in the experiment group performed better than the control group in their knowledge of students' thinking processes, KSCM, and TASL. Specifically, the results of the pre-test of the pre-service teachers knowledge of students' thinking processes for the experimental group performance (mean=26.09, SD=6.53) and the results for the control group (mean=28.52, SD=6.80). After teaching for eight weeks, the experimental group performance was higher (mean=81.03, SD=9.38) compared to that of the control group (mean=61.28, SD=13.86). Moreover, in **Table 2**, the results of the pre-test of the pre-service teachers KSCM for the experimental group performance (mean=29.24, SD=5.22) and the results for the control group (mean=27.27, SD=63.34),the post-test result after treatment showed that the experimental group performance was higher (mean=79.76, SD=12.34) compared to that of the control group (mean=70.21, SD=16.52).

Furthermore, the results of the pre-test of the pre-service teachers, TASL for the experimental group performance (mean=1.51, SD=0.52) and the results for the control group (mean=2.18, SD=0.55),the post-test result after treatment showed that the experimental group performance was higher (mean=2.63, SD=0.49) compared to that of the control group (mean=2.21, SD=0.53).

H_{01}: There are no significant differences in the KSTP, KSCM, and TASL of pre-service teachers' in algebra between SRPS and MCM

In **Table 3**, it shows that there was a significant difference in the KSTP scores test of pre-service mathematics teachers' taught algebra using SRPS and those taught using MCM $t(180)=11.28$, $p<.001$. There was a significant difference in the KSCM of pre-service mathematics teachers' taught algebra using SRPS and those taught using MCM $t(180)=8.59$, $p<.001$. Furthermore, there was a significant difference in the TASL scores of pre-service mathematics teachers' taught algebra using SRPS and those taught using MCM $t(180)=5.57$, $p<.001$.

Table 3. Independent samples tests of effects of KSTP, KSCM, and TASL achievement tests

Algebraic knowledge of students & content	Levene's test for equality of variances	t-test for equality of means							
		F	Sig.	T	df	Sig. (2-tailed)	Mean diff	Std. error diff	H_0
Post-test of KSTP	Experimental & control groups	10.954	.001	11.281	180	.000	19.75483	1.75119	Rejected
Post-test of KSCM		.084	.772	8.590	180	.000	16.41643	1.91105	Rejected
Post-test of TASL		1.593	.208	5.569	180	.000	.41932	.07529	Rejected

Table 4. Mean and standard deviation of gender on KSTP, KSCM, and TASL test

	Gender	N	Mean	SD
post-test of KSTP	Male	37	79.3784	7.84344
	Female	55	82.1455	10.20424
post-test of KSCM	Male	37	75.9189	11.47310
	Female	55	82.3455	12.32164
post-test of TASL	Male	37	2.5946	0.49774
	Female	59	2.6545	0.4799

Table 5. Independent samples tests of effects of KSTP, KSCM, and TASL of male and female pre-service achievement tests

		F	Sig.	t	df	Sig. (2-tailed)	Mean diff
post-test of KSTP	Equal variances assumed	3.750	.056	--1.395	90	.157	-2.7671
	Equal variances not assumed			-1.457	88.319	.146	-2.7671
post-test of KSCM	Equal variances assumed	.054	.818	-2.521	90	.013	-6.4265
	Equal variances not assumed			-2.557	81.021	.012	-6.4265
post-test of TASL	Equal variances assumed	1.179	.281	-.579	90	.564	-.0560
	Equal variances not assumed			-.575	75.437	.567	-.0560

Research question 2: What is the difference in the KSTP, KSCM, and TASL of male and female pre-service teachers' in algebra between SRPS and MCM?

From **Table 4**, it could be seen that there is a significant difference in the post test scores of knowledge of students' thinking processes scores test of male (mean=79.38, SD=7.84) and female (mean=82.15, SD=10.20) indicating that the mean score of the knowledge of students' thinking processes of the female pre-service mathematics teachers is greater than the mean score of the male students. There is also a significant difference in the post test knowledge of students' conception and misconception of male pre-service teachers (mean=75.92, SD=11.47) and female (mean=82.35, SD=12.32) indicating that the mean of the female pre-service mathematics teachers KSCM of algebra is greater than the mean of the male pre-service mathematics teachers. Likewise, there is a significant difference in the post test scores of TASL scores of male (mean=2.59, SD=0.50) and female (mean=2.65, SD=0.50) indicating that the mean of the female is higher than the mean of the male pre-service mathematics teachers as presented in **Table 4**.

H_{02}: There is no significant difference in the KSTP, KSCM, and TASL of male and female pre-service teachers' in algebra between SRPS and MCM

Table 5 reveals that (t(90)=-1.46, p=0.146) p-value is greater than 0.05level of significant. The table also shows that (t(90)=-2.56, p=0.012) *p* value is less than 0.05level of significance. Furthermore, **Table 5** reveals that (t(90)=-0.58,p=0.567) p-value is greater than 0.05 level of significance. The implication of this is that, there is no statistically significant difference in the KSTP knowledge of students' thinking processes and TASL of male and female pre-service mathematics teachers' in algebra in the experimental group... Thus, the hypothesis of no significant difference is not rejected. But there is a statistically significant difference in the KSCM level of male and female pre-service mathematics teachers' in algebra in the experimental group.

H_{03}: There is no correlation among KSTP, KSCM, and TASL of pre-service mathematics teachers' taught using SRPS and conventional method in algebra

The correlation in **Table 6**, shows positive relationship r(180)=0.805, p<.001 between pre-service teachers' knowledge of students' thought process and KSCM. The correlation value shows significance at p<.001. Implying that, there is correlation between pre-service teachers' KSTP and KSCM. Moreover, table 6 shows positive relationship r(180)=0.411, p<.001 between pre-service teachers' KSTP and TASL. The correlation value shows significance at p<.001. Implying that, there is correlation between pre-service teachers' KSTP and TASL

Furthermore, **Table 6** shows positive relationship r(180)=0.346, p<.001 between pre-service teachers' KSCM and TASL. The correlation value shows significance at p<.001. Implying that, there is correlation between pre-service teachers' KSCM and TASL. Therefore, the null hypothesis which states that there is no significant correlation among KSTP, KSCM, and TASL of pre-service mathematics teachers' taught using SRPS and conventional method in algebra is rejected.

Qualitative Analysis

The qualitative portion of the study provided an in-depth explanation on the impact of SRPS on the KSTP, KSCM, and TASL of pre-service teachers in algebraic word problems. The results were used as lens to infer and support findings from the AKSCAT data. Based on information from the quantitative analysis, participants performed better in the post test on items measuring the effect of solve-reflect-post strategy on pre-service teachers' among KSTP, KSCM, and TASL. Consequently, analyzing the level of their post-test AKSCAT scores was deemed important.

Table 6. Pearson correlation coefficient of pre-service mathematics teachers' KSTP, KSCM, and TASL

		Correlations			
		Post KSTP scores	**Post KSCM scores**	**Post TASL score**	**Strategy**
Post KSTP	Pearson correlation	1	.805**	.411**	-.644**
	Sig. (2-tailed)		.000	.000	.000
	n	182	182	182	182
Post KSCM	Pearson correlation	.805**	1	.346**	-.539**
	Sig. (2-tailed)	.000		.000	.000
	n	182	182	182	182
Post TASL	Pearson correlation	.411**	.346**	1	-.383**
	Sig. (2-tailed)	.000	.000		.000
	n	182	182	182	182
Strategy	Pearson correlation	-.644**	-.539**	-.383**	1
	Sig. (2-tailed)	.000	.000	.000	
	n	182	182	182	182

In addition, all the six mathematics pre-service teachers interviewed successfully explained the process of how they applied the SRPS to skilfully analyzed students thinking process, errors and misconceptions. The insight gained qualitatively, helped explained the pre-service teachers' TASL. Female pre-service teachers' was coded as F and male pre-service teachers' was coded as M in the conversations from the interviews.

The findings from the tasks showed that pre-service teachers in the SRPS demonstrated greater KSTP, KSCM, and TASL in algebra. All the interviewees were able to apply the SRPS flow chart correctly. Obviously, when the thinking process is deficient, this could lead to misconceptions and misconceptions that will lead to systemic errors.

Explaining that students' possible misconceptions and come up with past experiences of different misinterpretations due to misunderstanding of some key words, and that some students may experience some peculiar difficulties in mathematical representation of algebraic word problems, majority of these pre-service mathematics teachers displayed ability to predict errors that students may exhibit as they interpret algebraic word problems. When asked to explain the misconceptions that may lead to the errors mentioned, almost all the pre-service teachers pointed out that some mathematics teachers are the major cause. By explaining that, teachers also contribute to errors and misconceptions, the pre-service mathematics teachers proposed that some misconceptions originate from experiences in school, students interaction with teachers being one of the experiences. For example:

Task: *Suppose you ask your students to solve this algebraic question: I am thinking of two numbers. The larger of the two numbers is less than twice the first number. The sum of the two numbers is eighteen what are the numbers? What errors and misconceptions do you think students may exhibit as they answer this question?*

The findings reveal that in the pre-test scores of the pre-service teachers, they could not attempt the above posed question, but after the exposure to SRPS they found it easier to suggest possible reasons that lie behind students' thought process, errors and misconceptions. For example, see what they all stated:

> F1: I think many students will encounter difficulties comprehending words such as 'less than', 'twice', 'the result is', and 'certain two numbers.'
>
> F2: Many students will not understand that they need to let the unknown quantities be x and y or any other two letters.
>
> F3: Solving simultaneous linear equations is certainly difficult for me and other students; therefore, majority of the students may experience text comprehension difficulties.
>
> M4: Making sense of phrases like twice, product, increased, consecutive, two certain numbers will confuse many students.
>
> M5: Many students will not remember that when a number is less than twice the first number means let say y=2x. Some words are so confusing in word problems.
>
> M6: I guess the students will not get 'the larger of the two numbers is less than twice the first number' right. They will be faced with text comprehension difficulties.

The participants' responses demonstrated the level of their KSTP. This could be noticed in the manners in which they were able to suggest possible reasons that lie behind students' thought process, errors and misconceptions. The participants demonstrated improvement in their ability to identify, analyze, and interpret students thinking processes.

SRPS has enabled pre-service mathematics teachers (male and female) to acquire a deeper and also a more connected understanding of mathematical concepts and the learners, this could be observed in their responses in other tasks given to them, For example:

Task: 'Kemi solved a word problem which states that "I am thinking of two numbers. The larger of the two numbers is less than twice the same number by 3. The sum of the two numbers is eighteen. What are the numbers?" Kemi got the first and second unknown numbers to be 17 and 11, respectively. She thought that she was right, but when she checked her answer it did not work. *What underlying mathematical misconception(s) or misunderstanding(s) might lead the student to the error you envisaged in this item and how might the student have developed the misconception(s)?*

See what they all stated:

F1: I think the misconceptions might be because many students experienced difficulties in comprehending variables in algebra. These misconceptions or misunderstandings might be because many teachers used letters to represent different meanings in different situations to their students.

F2: The misconceptions or misunderstandings might be because many students experience difficulties in dealing with algebraic expression, especially when it involves two unknowns. It's foundational; I mean it depends on how the students were introduced to algebra.

F3: Many students cannot solve simple linear equations talk less of simultaneous equations. To solve equations correctly, one must know the applications of rules of simplifying algebraic expressions. Many students are not aware that equal sign is used to express the equivalent between two sides of the equation.

M4: Many students experience difficulties in word problems because they cannot express the word problems into appropriate mathematical statement or expression. Many teachers do not introduce algebra to students through proper representation of word problems.

M5: The errors might occur due to difficulties in representing and translating key words in word problems. Many students are not familiar with these key words in problem statements.

M6: The errors might be due to misinterpretation of key words in word problems. It is due to misconceptions in simplifying algebraic terms.

The participants' responses displayed the level of their KSTP, KSCM, and TASL. Other tasks presented to the participants further corroborated the above the impact of the treatment on the participants' level of KSPT, KSCM, and TASL. For example:

Task: *From the second task, what further question(s) can help to understand the student's misconception(s) and what would you do next?*

See what they all stated:

F1: I may ask the students to tell me what they can say about x if 2x+4=10?

F2: I will ask the students to simplify (x+y)/(x-y)=3/5.

F3: I will ask the students to solve these equations: a=3b+2; a=4b – 5.

M4: I will ask them to tell me what they understand by these words: sum, difference, product, quotient, less than, more than, increase by, decrease by, and so on.

M5: I will ask them to give me their interpretation of key words such as: double, twice, triple, and so on.

M6: I will just ask them to interpret this statement "when 2 is added to twice a certain number and the result is double the number."

The participants' responses have further shown their deep understanding of algebraic knowledge for teaching. This fact could also be observed in the next task. For example:

Task: *Ngozi, Kunle and Jane argue on the correct answer to a question which states that: "8 is added to a certain number and then doubled the result and the result divided by four. The answer is 27, find the number?" If they all used letter x to represent the unknown.*

Ngozi's solution: 2(x+8)÷4=27

(x+8)÷2=27

x+8=54

x=54-8=46.

Therefore, the unknown number is 46.

Kunle's solution: (x+8)/4+(x+8)/4=27

x+2+x+2=27

2x+4=27

x=23/2=11½

Therefore, the unknown number is nine and a half.

Jane's solution: (x+8)÷4=2(27)

(x+8)÷4=54

x+8=4(54)=216

x=216-8=208.

Therefore, the unknown number is 208.

For the student(s) who were incorrect, identify their mistakes.

For the student(s) who were incorrect, what underlying mathematical misconception(s) might lead the student(s) to the error presented in their solutions?

What further question(s) might you ask to understand the student's misconception(s)?

What steps would you use during the mathematics lesson that will help eliminate the identified students' errors and misconceptions?

M6: Kunle's solution contains error of dividing 8 by 4 in the algebraic fraction while Jane's solution contains error of misinterpreting 'double the result.'

F1: The error and misconception in Kunle's solution could be interpreted as common arithmetic error, which is, dividing 8 by 4. Jane's solution is an error and misconception of misunderstand and misinterpreting the given task.

M5: In order to further understand the students' misconceptions, I would further test Kunle's algebraic conceptual knowledge on algebraic fractions. I would investigate further; Jane's previously learned arithmetical procedural knowledge. These might be the hindrance to the development of algebraic concept knowledge.

F2: Obviously, many algebraic problems are difficult for students because solving algebraic word problems require conceptual and procedural knowledge of fractions. Therefore, understanding students' previous knowledge would allow the teacher understand the reasons behind students' errors and misconceptions and their thinking process.

The participants' responses are clear demonstration of the depth of their KSTP, KSCM, and TASL, the participants were of the opinion that the quality of students thinking process varied from one student to another. The participants suggested some errors and misconceptions such as "error and misconception of basic algebraic facts", 'error and misconception of interpretation", and "errors and misconceptions of misunderstanding the task" errors might occur due to difficulties in representing and translating key words in word problems. The participants further said, many students are not familiar with the key words in problem statements. According to the participants, majority of the students often encounter difficulties when applying simple algebraic rules. They suggested that they would ask questions that would elicit the students' previous knowledge and that this would allow them understand the reasons behind the misapplication and inadequate procedure that led to these students' faulty thought processes. The pre-service teachers explained the various questions they would ask the students about their errors and misinterpretations, and from their responses the pre-service teachers expected to identify their students' thinking processes and misconceptions. They also explained that they would ask the students to tell them their understanding of certain algebraic key words; some said that they would ask the students to represent some word problems in algebraic form. They believe some students do not understand some algebraic key words.

By explaining that students do confuse and have translation problems, these pre-service mathematics teachers have displayed understanding of students' thought processes and misconceptions. It is obvious that these pre-service mathematics teachers have comprehensive KSTP, KSCM, and TASL in relation to algebra.

DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

This study explored the effects of SRPS intervention on pre-service teachers' KSTP, KSCM, and TASL in algebra. The KSTP, KSCM, and TASL demand that a teacher responds appropriately to students' during teaching and learning of mathematics concepts. The aim of this study is to equip pre-service mathematics teachers with the knowledge needed in the classroom setting that will allow mathematics teachers to convert some classroom incidents into learning opportunities. A change in the ways teachers listen to their students and manage the specific mathematics incidents in a classroom goes a long way in proving how teachers unpacked the knowledge of content and students.

The knowledge of student is a domain of mathematics knowledge for teaching. Students' thinking process plays a vital role in how students accommodate and assimilate mathematics information. Instructions are said to begin when the teacher learns from the student. A teacher who successfully puts himself in the place of a student may understand what the students learn and the way they understand it. What students learn is always less than what we teach. How much they learn is determined by the match between students thinking process, conceptions, misconceptions and teachers' instructional strategy. Teachers might not be able to do much about other factors militating against learning but teachers can maximize student's learning through the selection of appropriate instructional methods in order to remediate and eliminate faulty students thinking processes, errors and misconceptions.

This study revealed that there is a differential effect of SRPS and MCM on KSTP, KSCM, and TASL scores of pre-service mathematics teachers in algebra. The results show that those pre-service teachers exposed to SRPS had the higher mean score than their counterparts in the MCM group in the post-test KSTP, KSCM, and TASL scores test in algebra. The evidence represented by pre-test points to weak KSTP, KSCM, and TASL of pre-service teachers as contributing factors to their weak in algebraic knowledge of students and content.

The findings for KSTP of the pre-service teachers are consistent with the previous findings of differential effects of student-centered teaching strategies on mathematical knowledge for teaching of pre-service mathematics teachers (Latterell, 2008; Lim & Guerra, 2013; Moyer & Milewicz, 2002; Schoenfeld, 2007; Tanisli & Kose, 2013; Zuya, 2014). According to Tanisli and Kose (2013) and Zuya (2014), who suggested that teachers' teaching methods of mathematics need to be refocused because teachers must be exposed to strategies that will equip them on how to skilfully analyze the depth of their students' thought better. The findings from this study, therefore imply that teachers with deficiencies in KSTP, KSCM, and TASL, would not be able to teach mathematics effectively, as they are not able to evaluate the thinking process of their students in depth.

The result shows that the pre-service mathematics teachers in the SRPS group demonstrated greater KSCM than their counterparts in the MCM group. In a similar vein, in the studies done by (Moyer & Milewicz, 2002; Schoenfeld, 2007; Tanisli & Kose, 2013; Zuya, 2014) found that there were recent teaching strategies that focus on pre-service teachers' understanding of students' thinking processes which provide evidence of growth in the emergence of pre-service teachers' mathematical knowledge.

The results of this study showed a greater improvement in the pre-service mathematics teachers' TASLs and this proved that pre-service teachers benefitted from the use of SRPS. It suggested that teacher educators should endeavor to apply student-centered instructions on pre-service teachers. They should be taught with and exposed to student-centered instructions as this will enhance their KSTP, KSCM, and also improve their TASL in algebra. It must be stressed here that, few studies have really investigated the use of solve-reflect-pose instruction as a teaching strategy and the effects of SRPS on pre-service teachers' KSTP, KSCM, and TASL scores in algebra. Therefore, this calls for further investigation.

It can be seen from the findings that there is a significant difference in the post test KSTP, KSCM, and TASL scores of male and female indicating that the mean of the female pre-service mathematics teachers' algebraic knowledge for teaching scores is greater than the mean of the male pre-service mathematics teachers. This study found a significant gender difference in the KSTP, KSCM, and TASL of pre-service teachers in the experimental post-test. In other words, this strategy did differentiate between genders when it comes to pre-service teachers' KSTP, KSCM, and TASL level in algebra. The fact that female pre-service teachers outperformed males contradicts earlier studies that found that male students outperformed female students at the college levels (Bassey et al., 2009; Isiksal, 2016). Ma (2004) found that these gender gaps in mathematics performance could be characterized as being universally small. In the present study, KSTP, KSCM, and TASL scores of female pre-service mathematics teachers may be attributed to the strong correlation among the three constructs (Isiksal, 2016).

The findings on correlation among KSTP, KSCM, and TASL of pre-service mathematics teachers taught using SRPS and conventional method in algebra of pre-service teachers in algebra are in corroboration with Badru (2016) and Ishola (2002) whose findings on treatments have no significant influence on the student scores. The findings on effect of treatment and gender on knowledge of students' algebraic thinking processes and algebraic problem posing skills of pre-service teachers in algebra finding are not in agreement with the study of Tremblay et al. (2000). This could be as a result of differences in concepts investigated and the different environment and classroom settings. It may also be that the sample size, the sampling techniques and statistical tool used are equally different.

Final Remarks

This study demonstrates that students' thinking process and misconceptions could be uncovered in students' written solutions. The findings from this study revealed that pre-service mathematics teachers could acquire a deeper and a more connected understanding of mathematical concepts and the learners, if they are exposed to effective instructional approaches. From the findings of the study, SRPS impacted positively on KSTP, KSCM, and TASL.

Although students' thinking processes and remediating students' misconceptions are challenging tasks, the findings from this study revealed that pre-service teachers in the experimental group increased their KSTP, KSCM, and TASL better than their counterparts in the control group. The findings also show that SRPS could be used effectively in a mixed gender class to teach mathematical concepts. In respect of the findings in this study, SRPS is proved to be an effective pedagogy and its philosophy should be integrated into the pre-service teachers' curriculum at the teacher-preparation institutions.

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APPENDIX A

Flowcharts of solve-reflect-pose process: