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The concept of computational thinking in mathematics education

Eloisa Rosotti Navarro [1]*, Maria do Carmo de Sousa [2]

[1] Federal University of Paraná (UFPR), Curitiba, BRAZIL
[2] Federal University of São Carlos (UFSCar), São Carlos, BRAZIL
***Corresponding Author:** eloisa-rn@hotmail.com

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ABSTRACT

This article is part of an investigation that aimed to develop, from conceptual nexuses and assumptions of cultural-historical theory, the theoretical concept of computational thinking for its development in mathematics education. Therefore, a study of the logical-historical movement of the term "computational thinking" in mathematics education was carried out, seeking to explain how and under what conditions the term was used in academic and documental productions in Brazil. In summary, the results indicate that studies corresponding to computational thinking in mathematics education are still incipient in Brazil and, until then, there was no concept for this term in the area. Thus, this concept was developed in order to establish three conceptual nexuses: problem-solving, algebraic thinking, and algorithmic thinking, which are in constant motion as they are dialectical, historical, logical, and cultural.

Keywords: mathematics education, cultural-historical theory, logical-historical movement, conceptual nexus, computational thinking

INTRODUCTION

A careful look at academic productions in the field of mathematics education reveals that the use of the term "computational thinking" is present in studies, in most cases, with an emphasis on programming and computer language, using digital information and communications technologies (ICT) as a means for this kind of thinking to be developed. But what is computational thinking? How to develop this kind of thinking in the classroom? Is the use of ICT necessary for this end? Such questions led us to the following research problem: what are the possible conceptual nexuses that contribute to the development of computational thinking in the context of mathematics education?

In order to answer this guiding question, we carried out a study of the logical-historical movement of the term "computational thinking" in mathematics education, which initially took us to the mid-1960s, when the term was first mentioned by Papert (1996), one of the creators of the "logo" language, who, when discussing his approach to geometric thinking, stated that "the goal is to use computational thinking to forge ideas that are […] more accessible and more powerful."

However, a few years later, the "logo" language and, together, the term "computational thinking", were relegated to the background, due to studies and research that aimed to investigate and understand how ICT could be implemented in mathematics education, as a resource for solving conceptual and theoretical problems.

As of 2006, one of the first definitions of the term "computational thinking" for education was architected by Wing (2006). Through the precepts of this author, there was an academic movement related to researchers in mathematics education who, for their part, are interested in employing computational thinking in school teaching practices.

Although this term is still not well theoretically delineated and, equally, what would be its specificity regarding the type of activity it encompasses and/or the methodological procedure in which it could be inserted, computational thinking was instilled in the curriculum of some countries, including in Brazil, through base nacional comum curricular (BNCC).

In this context, we focus our analysis on BNCC, dissertations, theses, and articles, through bibliographic state-of-the-art research. In order to do so, we used the coordenação de aperfeiçoamento de pessoal de nível superior (Capes) platform searching for articles, dissertations, and thesis and the scientific electronic library online (Scielo) platform searching for articles. The period established was from 2009 to 2019, with the objective of analyzing how the term has been used in studies that deal with mathematics education. The results indicate that there are studies that use the term and are opposed in relation to the definition of computational thinking and its characteristics.

In addition to the study of the papers, the analysis of the aforementioned documents allowed, under the aegis of the cultural-historical theory (CHT), to determine three conceptual nexuses of computational thinking, which may be configured in the context of mathematics education and, consequently, in school mathematics, which are: problem-solving; algebraic thinking, and algorithmic thinking. These conceptual nexuses are, essentially, in constant movement, as they are dialectical, historical, logical, and cultural. Based on them, we developed the concept of computational thinking in mathematics education, which considers the essentiality of developing computational thinking as a potential means of expanding the abilities of problem-solving, interpreting reality. and expanding the forms of action of the students in their sociocultural context, either in a plugged (ICT) or unplugged way. In this sense, the definition of the concept aims not only to think about "what it is" (external nexuses), but "how we can use it in everyday school life" and "how it can be developed to interpret and solve problems in reality" (internal nexuses).

LOGICAL-HISTORIC MOVEMENT OF THE TERM "COMPUTATIONAL THINKING"

Investigating the logical-historical movement of the term "computational thinking" means to defend that this type of thinking is in constant movement and social transformation, becoming, therefore, a historical category. The logical-historical unit corresponds to the systematization produced by Kopnin (1978), who, in turn, defended the inherence between the historical and the logical, since a logic devoid of the objective/subjective doing of the human being would be impracticable. Studying this dialectical unit makes us understand the conceptual nexus of the concept; in this case, the term "computational thinking" acquires another status.

The starting point of this logical-historical movement in this research was related to the experience and the need to understand the term "computational thinking" and, consequently, the execution of a state-of-the-art investigation, which culminated in the analysis of theses, dissertations, articles, and BNCC. This kind of research is used as a form of inventory, systematization, and evaluation of existing scientific production, "seeking to identify trends and describe the state of knowledge of an area or topic of study[1]", as pointed out in the studies by Fiorentini and Lorenzato (2012, p. 103).

State-of-the-art studies, as understood by Romanowski and Ens (2006), address investigations in a single sector of publications on a given topic, as in our case in which articles, theses, and dissertations were covered with the aim of mapping academic production regarding the use of the term "computational thinking" in mathematics education.

The term "computational thinking" was used for the first time, in the context of mathematics education, in 1967, with a view to the "logo" programming language, built by Seymour Papert, Cynthia Solomon, and Wally Feurzeig. In this period, the purpose of programming was to provide an understanding of how computational language works, with the initial concern of developing children's thinking regarding "how" and "why" to program.

Papert and his collaborators elaborated experiments, with the aim of analyzing the ways in which children controlled the movement of a robot, through instructions described by the "logo" language. In summary, the robot had a pen attached to its body that, when activated, highlighted its path on a paper, drawing geometric shapes, for example.

With the realization of these experiments, Papert and his team were able to build a theory of learning, naming it Constructionism, which is a branch of Piaget's constructivism. In general terms, this theoretical approach considers that the students are able to build their knowledge based on "doing", in the creation of concrete and shareable objects with the guidance of a teacher (Papert, 1985).

In 1980, experiments with the "logo" language were carried out in Brazil, under the guidance of researchers such as José Armando Valente and Léa Fagundes, in partnership with the public and private education systems. In contrast, constructionism and the programming language (logo) that supported this theory did not gain notoriety in school practice. It happened due, among other factors, to the availability of laboratories in schools and the model for using computers (Valente, 1996).

During the 1990s, the greatest concern of scholars was focused on the following question: what would ICT be used for? In other words, the focus in this period was not the programming itself but applying tools that were already ready. Thus, the term "computational thinking" and the language that involved it lost strength in the field of mathematics education, being rejected in later years and replaced by the internet and various applications. However, computer scientists turned their attention to this theme, understanding it as an area of study.

diSessa (2000) argues that computers may be the technical basis of a new type of literacy, thus coining the term "computational literacy". For him, unlike "computer literacy", "computational literacy" refers to the natural use of the computer, in accordance with new personal and professional needs that emerge on a daily basis.

Given this process, the computer has become an indispensable tool in people's daily lives and, at the same time, useful for the development of other languages and concepts. In this perspective, Blikstein (2008) proposes that scientific and mathematical concepts, for instance, may be simplified through symbolic representation and computational language.

As a consequence, ICTs had new functions and perspectives in the context of mathematics education. Researchers began with analyses about the importance of reflecting at school on "how" and "why" we use certain ICTs in the development of our daily goals and actions. This argument gains even more notoriety with an article published by Jeanette Wing, in 2006 in which the author synthesizes the essentiality of thinking with technology and not simply using it to achieve something (daily necessity).

Wing (2006) infers, through computational thinking, the possibility of "solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science" (p. 33). In another article, published in 2011, the author defended the importance of accessing this thinking, as they are "[…] thought processes involved in

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[1] Our translation from Portuguese. Original: "buscando identificar tendências e descrever o estado do conhecimento de uma área ou de um tema de estudo".

formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent” (Wing, 2011).

Along these lines, some studies were carried out in addition to Wing (2006, 2011), in an attempt to define the term “computational thinking”. By way of example, in 2012, Royal Society (2012, p. 29) summarized computational thinking as a “process of recognising aspects of computation in the world that surrounds us and applying tools and techniques from computer science to understand and reason about both natural and artificial systems and processes”.

These definitions are operational but demonstrate that the understanding of computational thinking is closely linked to “[…] developing an approach to computational thinking that is suitable for K-12 students”, according to the *K-12 computer science teachers association* (CSTA) report (Seehorn, 2011, p. 10).

In some countries, such as the United States, England, and Italy, although there is no specific concept related to computational thinking, the relevance of the development of this thinking in education is already being researched. There are, for example, several publications that recognize the benefits and scope in education that this type of thinking may offer (Barr & Stephenson, 2011; Denning, 2009; Hu, 2011; Wing, 2006 , 2008, 2011).

In the bibliographical research, when seeking to define an approach to the term “computational thinking” in the studies, we noticed that, although there are already some studies that elucidate the importance of developing this type of thinking in Education, the term continues without a definition and/or a common approach in all works.

In some investigations, the term “computational thinking” is totally linked to computer science; however, in other cases, such as the studies proposed by Wing (2006, 2011), there is an attempt to move away from computational thinking as a property of computer science alone. Through a differentiated focus, these studies aim to provide the insertion of the development of this way of thinking in education. In this way, programming, which was a fundamental part of computational thinking, is no longer the most significant feature, as it was in the “logo” language.

Faced with this difficulty in defining and building a common concept of computational thinking, the CSTA and the *International Society for Technology in Education* (ISTE) proposed nine essential concepts linked to computational thinking, which are: data collection, data analysis, data representation and analysis, problem decomposition, abstraction, algorithms and procedures, automation, parallelization, and simulation (CSTA, 2011).

According to the above-mentioned authors, the term “computational thinking” is associated with the use of computers and thinking with technologies. Although these concepts may be useful for the development of activities that encompass computational thinking, there are still no studies that indicate, which and/or how many of these concepts need to be present, in the context of school teaching practice. In other words, giving the teacher resources to provide students with opportunities for the development of computational thinking.

In summary, even with so many attempts to define the term, there is no conceptual consensus regarding what computational thinking really would be and, therefore, how it would be developed in education. Although this term is present in the curriculum of some countries, including BNCC in Brazil, all references to the term are located in mathematics.

Hence, we carried out a search for articles, theses, and dissertations on Capes and Scielo platforms using “computational thinking” as a descriptor, in the period from 2009 to 2019. 15 theses, 68 dissertations, and 42 articles were found, totaling 125 works. We read all the works in full, aiming to select the studies concerning the area of mathematics education. Thus, we found two theses, 12 dissertations, and two articles, totaling 16 works.

Established the research *corpus*, we started a summarizing process to screen the data. First, we focused on a critical reading of the work in its entirety. Then, in the studies, we aimed at how computational thinking was approached, in terms of discourses and practices. That said, we built analysis categories, with the scope of guiding the treatment of data with regard to categorization and interpretation of results.

According to Gibbs (2009), in state-of-the-art research, it is substantial to trace categories aiming at “funneling” the data set, selecting, which ones will be analyzed, and which ones will be rejected. In this context, categorization is a data reduction technique, that is, it is a means of “passing” data through a sieve, selecting those that will contribute to the problem and preconceived objectives for the research.

After a whole process of summarizing and re-reading the embodied research material and referring to the guiding principles suggested by Gibbs (2009), we chose to define three categories of analysis: computational thinking and programming language (algorithmization and robotics); computational thinking and problem-solving; and computational thinking in BNCC.

Finally, we do not intend, in this article, to break down the research production in its entirety. More than that, we want to contribute towards making more accessible information related to the production of theses, dissertations, and articles related to computational thinking in mathematics education.

SYNTHESIS REGARDING THE STUDIES AND DATA ANALYSIS

The studies carried out reveal that research concerning computational thinking in mathematics education, in our understanding, is still incipient, given the scarcity of dissertations, theses, and articles dealing specifically with this topic in this respective area of knowledge. In **Table 1**, it is possible to verify the 16 aforementioned studies, according to the year of publication.

Analyzing the aforementioned studies and BNCC, we noticed that there is a gap about the concept of computational thinking, since the definitions presented are different and multifaceted, and, equally, the way in which this thinking would develop in the

Table 1. Works about computational thinking in mathematics education

	Author	Year	Title	Type
1	Nunes, C. B.	2013	Introdução à computação: Uma proposta para o ensino básico [Introduction to computing: A proposal for basic education]	Dissertation
2	Barcelos, T. S.	2014	Relações entre o pensamento computacional e a matemática em atividades didáticas de construção de jogos digitais [Relations between computational thinking and mathematics in didactic activities for building digital games]	Thesis
3	dos Santos Lummertz, R.	2016	As potencialidades do uso do software *Scratch* para a construção da literacia digital [The potential of using *Scratch* software to build digital literacy]	Dissertation
4	de Morais, A. D.	2016	O desenvolvimento do raciocínio condicional a partir do uso de teste no *Squeak Etoys* [The development of conditional reasoning from the use of testing in *Squeak Etoys*]	Thesis
5	Costa, E. J. F.	2017	Pensamento computacional na educação básica: Uma abordagem para estimular a capacidade de resolução de problemas na matemática [Computational thinking in basic education: An approach to stimulate problem solving ability in mathematics]	Dissertation
6	Mestre, P. A. A.	2017	O uso do pensamento computacional como estratégia para resolução de problemas matemáticos [The use of computational thinking as a strategy for solving mathematical problems]	Dissertation
7	de Morais, A. D., de Azevedo Basso, M. V., & da Cruz Fagundes, L.	2017	Educação matemática & ciência da computação na escola: Aprender a programar fomenta a aprendizagem de matemática? [Mathematics education and computer science at school: Does learning to program foster math learning?]	Article (Scielo)
8	de Carvalho, F. J. R.	2018	Introdução à programação de computadores por meio de uma tarefa de modelagem matemática na educação matemática [Introduction to computer programming through a mathematical modeling task in mathematics education]	Dissertation
9	Egido, S. V.	2018	Educação matemática e desenvolvimento do pensamento computacional no 3º ano do ensino fundamental: Crianças programando jogos com Scratch [Mathematics education and development of computational thinking in the 3rd year of elementary school: Children programming games with Scratch]	Dissertation
10	Silva Jr, A. M.	2018	Microgênese do desenvolvimento sociocultural do raciocínio [Microgenesis of the sociocultural development of reasoning]	Dissertation
11	Barbosa, L. M.	2019	Aspectos do pensamento computacional na construção de fractais com o software GeoGebra [Aspects of computational thinking in the construction of fractals with GeoGebra software]	Dissertation
12	Evaristo, I. S.	2019	O pensamento computacional no processo de aprendizagem da matemática nos anos finais do ensino fundamental [Computational thinking in the process of learning mathematics in the final years of elementary school]	Dissertation
13	Guarda, G. F., Goulart, I. F., dos Santos Gonçalves, C., & Cunha, L. R. R.	2019	O circuito quatro desafios–atividade lúdica apoiada pelo pensamento computacional [The circuit four challenges–playful activity supported by computational thinking]	Article (Capes)
14	Nascimento, R. M.	2019	A matemática e o visualg: Lógica de programação no ensino médio [Mathematics and the visual: Programming logic in high school]	Dissertation
15	de Lima Pereira, J. P.	2019	Programação e pensamento computacional no 8º e 9º ano do ensino fundamental: Um estudo de caso [Programming and computational thinking in the 8th and 9th grade of elementary school: A case study]	Dissertation
16	Ribaldo, S. M. O.	2019	A linguagem de programação Scratch e o ensino de funções: Uma possibilidade [The Scratch programming language and the teaching of functions: A possibility]	Dissertation

Note. Source: Author's research (2020)

classroom was not outlined, specifically in school mathematics. In the sense presented, in all these attempts at definitions, the term "computational thinking" is totally concatenated with the use of computers and thinking with ICT; furthermore, most of the cited authors are from the computer science field.

Regarding the analysis categories, in the category "computational thinking and programming language (algorithmization and robotics)", we analyzed a total of 12 works. These works indicate that computational thinking, whether as a plugged or unplugged activity, is in accordance with the theoretical precepts by CSTA (2011) and Wing (2006, 2014). However, it is worth pointing out that these works, so far, have not developed or built any kind of concept/definition of computational thinking. In summary, such studies proposed some characteristics and abilities of computational thinking, tending to a possible interconnection between problem-solving, algorithm, and programming language.

We understand that these studies were mainly based on Wing (2006, 2014) as a theoretical reference, demarcating computational thinking as a thinking process, which leads to the problem formulation and solving, using the computer for this action. For this reason, such works are in line with the idea that computational thinking is analogous to computer programming language.

In order to justify this assertion, we argue that most of these studies we analyzed used Scratch programming language as a way of working and prescribing computational thinking as a programming language in the context of the field of school mathematics. Based on constructionism, it is possible to notice in these studies that the basic theoretical support was Piaget's constructivism and Papert's (1985) constructionism. Added to this, there are the studies by Blikstein (2008), Brackmann (2017), and Wing (2006, 2014), and, which deal with computational thinking in the development of algorithms with the use of computers and/or other ICTs. Furthermore, the studies analyzed suggest that computational thinking is a creative, critical, and strategic capacity (or ability) to apply the fundamentals of computation in mathematics.

In this bias, when analyzing the 12 studies in this category of analysis, we found that they are based on the precepts of computer science in order to seek a possible definition of computational thinking. Thus, we found that computing, more specifically programming language, algorithmization, and robotics, was present in these studies and computational thinking, as a result, was shortened to basic actions of computing and programming. Therefore, we infer that in this category of analysis, research deals with computational thinking from the perspective of programming language.

In the second category, "computational thinking and problem-solving", we analyzed a total of four studies. Although the works focus on problem-solving, we consider that they are still based on a relationship between mathematics and computer science. In summary, the authors of these studies analyzed which competences and skills of mathematics and computational thinking may be mobilized and developed with the use of the computer as a primary resource of mediation.

In addition, we found that such studies structured a discourse on computational thinking with problem-solving, through work with logical reasoning, modeling and simulation, problem decomposition, robotics, creation of digital games, and the use of programming software.

Basically, in this category of analysis, we understand that there were attempts to justify and develop computational thinking in mathematics education through the aspect of problem-solving designed by Polya (1978). It should be said that this aspect of problem-solving was linked to programming, making use of algorithms, algebra, arithmetic, and logical reasoning. Thus, problem-solving served as a motto to defend a work with computational thinking in school mathematics. However, what we found was a practice of exploring the potential arising from ICT through the use of problem-solving skills. It means that programming, abstraction, unplugged computing, and algorithmic thinking must turn to problem-solving, in the context of school mathematics; however, without abandoning the theoretical precepts of computer science that subsidize computational thinking.

While emphasizing that computational thinking is aligned with problem-solving, the studies still underlie programming language. That is, the discourse on computational thinking in the context of problem-solving is linked to the use of computers and thinking with ICT, thus linking authors who deal with problem-solving in school mathematics with authors in the field of computer science.

In general terms, we may state that all these works analyzed in both categories of analysis provide some explanations regarding the term "computational thinking". However, these are explanations that link the development of this thinking with computer science, not as an area that will be included in the curriculum, but as another area introduced in mathematics.

In addition, we note that there is a theoretical gap regarding the concept of computational thinking, given that the definitions raised are disparate and multifaceted. Likewise, the researchers were not concerned with building concepts or assumptions that show how this thought would develop in the classroom, namely, in school mathematics.

Regarding our category "computational thinking in BNCC", we found that there seems to be an attempt at the correlation between algebra and computational thinking; however, at no point in the document is this relationship exemplified or pedagogically structured. We understand that this way of thinking makes it impossible for the teacher, for instance, to create or specify how and when computational thinking is present in the organization of mathematics teaching. Furthermore, BNCC's theoretical framework does not include at its core studies or research that systematize computational thinking as a curricular theme or even a theoretical reference that explains at least one definition of the term.

BNCC seems to implicitly agree that computational thinking must be tied to the algorithm, as well as some kind of standardization and problem situations. In short, the way the term appears distributed in the text of BNCC takes place out of context and hardly contributes to its development in the classroom, due to this decontextualization.

It is worth noting that the application of the precepts of computational thinking along these lines, conveyed by the studies and by BNCC, at the heart of mathematics teaching, may be occurring superficially. In other words, if we continue to develop computational thinking in a shallow way, without a clear understanding of what such development means for students, we tend to develop in students only the external conceptual nexuses (visible and practical) and this will only provide partial results to the student's learning process. Now, these damages may be confirmed not only in the student's lack of subjectivity, as well as in the formation of theoretical thought, understanding it as a means of generalizing the concept (de Sousa, 2014).

It is in this perspective that we defend that school mathematics is not synthesized in the acquisition of a set of information. More than that, it is one of the sources of development and world reading. Therefore, when reflecting on computational thinking, we defend that its objective is to promote students' development, providing them with instruments, intellectual-affective operations, and the ability to use knowledge in different sociocultural contexts.

That said, we conceived that computational thinking, in the context of school mathematics, has the function of helping students to produce mathematical knowledge (algebraic and algorithmic thinking), develop research and problem-solving skills, such as world reading, etc. In this way, it is necessary to break with the empirical-discursive, technicist and mechanistic paradigm, which preaches the training of techniques, the memorization of formulas, and the reproduction of proofs and axioms, through, for instance, a work based on programming language as a product.

It is necessary to ask: What is "thinking"? What are conceptual nexuses?

In summary, the analysis of the research and the BNCC indicated that there are conceptual divergences among the authors themselves, although most of them are based on the studies by Wing (2006). Given this, we need to answer the questions asked above so that we may develop the concept of computational thinking in the context of mathematics education, considering CHT.

WHAT IS "THINKING"?

When dealing with the concept of "thinking" from the perspective of CHT, it is essential to also consider the role of language, since they are constituted as a dialectical unit. According to Vygotsky (2000), language (word, speech, gestures, images, etc.) is a phenomenon of thinking, insofar as thought itself is related to language and is materialized in it. In this understanding, we may state that the author assumes that it is essential to analyze thinking in an inter-functional way with language (set of signs), since it is from the signification (appropriation and use of signs) that thinking, and language (especially the speech) unite. Therefore, thinking and language form an inseparable unity.

In this way, thinking is not only manifested by words, images, actions (conjunction or not of gestures with words, sounds, images), but it is language itself, that is, thinking is constituted by the interrelationship among these semiotic systems. In summary, thinking is like a unit of communication and generalization (language functions).

Analyzing from a CHT perspective, thinking may be considered as awareness, the conscious act of the subjects to gather information, images, knowledge, and concepts from their experiences, turning to themselves (reflect) or trying to act in their medium. Thinking turns to concrete action, based on higher psychic functions, such as logical and associative memory, voluntary attention, skills to interpret and synthesize information, behavior control, imagination, and perception.

Thus, when we think, we set in motion higher psychic functions, such as images and sensations that come to us from perception, imagination, logical memory, etc. We also put into action the signs, the language. When we think, we understand the meaning of words, we concatenate meanings, some coming from our sensitive experience, others coming from our reasoning. As a result, our thinking is permeated by images, words, memories and previous ideas, abstractions, problems. Thus, thinking gives us the opportunity to solve problems, compare, analyze, separate, evaluate, dialogue, interpret, decipher, reflect, interrogate, that is, thinking is our conscious act in face of reality.

From the above, in order to understand how the consolidation of these intellectual processes contributes to the development of the concept of computational thinking in the context of mathematics education, it is necessary to first understand how the formation of concepts takes place in the cultural-historical perspective.

WHAT AND WHICH ARE THE CONCEPTUAL NEXUS OF COMPUTATIONAL THINKING?

According to Vygotsky (2000), the concept is, in psychological terms, a means of generalization. That is, it denotes awareness or formation of a higher type of concept. This author infers that awareness happens through a system of concepts.

For this reason, the formation of concepts conceives the role of language not only as a communication tool, but as an essential way to formulate concepts. The word, for example, is a way of representing, classifying, abstracting, and generalizing reality through complex mental activities (Vygotsky, 2000, 2014).

> A concept is much more than the sum of certain associative connections formed by memory, it is more than a simple mental habit; it is a real and complex act of thinking that cannot be learned through simple memorization, only being performed when the child's own mental development has already reached its highest level. The investigation teaches us that, at any level of its development, the concept is, in psychological terms, an act of generalization. The most important result of all investigations in this field is the firmly established thesis that psychologically conceived concepts evolve as word meanings[2] (Vygotsky, 2000, p. 246).

Kopnin (1978) analyzes that the concept is formed by a network of meanings composed of the internal sense to which it refers, together with causal nexuses and/or the indispensable relationships among its elements, principles, consequences, causes, and effects of what it refers to. Thus, the concept provides us with the essential meaning of something, its cause, its consequences, or even its effects, its ways of being and/or acting.

According to Kopnin (1978), for individuals to build a concept (element as a whole), their thinking needs to recognize different relationships of microelements, called conceptual nexuses, representing "[...] links that help us to build concepts, continuously" and "[...] are made up of sociocultural, historical, and philosophical aspects" (de Jesus & de Sousa, 2011, p. 115).

In this way, the conceptual nexuses make up the logical-historical aspect of the concept, encompassing a movement of ascension from the abstract to the concrete. Furthermore, according to Davydov (1982) and Kopnin (1978), the conceptual nexus of an element (concept) is divided into internal and external nexuses.

The internal nexuses are different from the external ones, as the latter focus on the perceptible elements of the concept. The internal nexuses, on the other hand, constitute the logical-historical aspect of the concept. In other words, external nexuses are limited to language (verbal designations) because they are formal and compose an abstract unit, delimited by a set of observable characteristics of the concept.

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[2] Our translation from Portuguese. Original: "Um conceito é muito mais que a soma de certos vínculos associativos formados pela memória, é mais do que um simples hábito mental; é um ato real e complexo do pensamento que não pode ser aprendido por meio de simples memorização, só podendo ser realizado quando o próprio desenvolvimento mental da criança já houver atingido seu nível mais elevado. A investigação nos ensina que, em qualquer nível de seu desenvolvimento, o conceito é, em termos psicológicos, um ato de generalização. O resultado mais importante de todas as investigações nesse campo é a tese solidamente estabelecida segundo a qual os conceitos psicologicamente concebidos evoluem como significados das palavras".

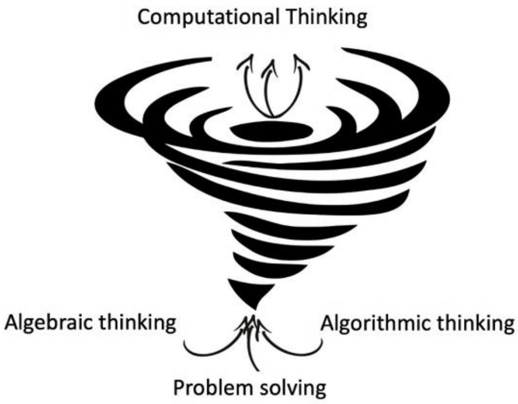

Figure 1. Conceptual nexuses of computational thinking (elaborated by the author, 2021)

As an example, when analyzing computational thinking in mathematics education, the external nexus would refer to the formal characteristics of the object (computational thinking), that is, its particularities, its structure, its formality; thus, focusing on the process of description and classification of knowable objects. In summary, the external nexuses of computational thinking are a kind of communication language of the concept.

The internal nexuses that base the concepts have the logical, historical elements and the formalizations of human thinking in view of knowledge. Therefore, the internal nexus is the relationship between the ways of thinking the concept and the formality of the concept (external nexuses–verbal designations/characteristics) (Davydov, 1982; Kopnin, 1978).

The internal nexus of computational thinking, in this perspective, is about generalizations of the subject using the concept. In other words, the internal nexus of computational thinking is a kind of operationalization of the external nexus. This denotes the use of the concept in everyday life to solve problems, understand situations, interpret reality, elaborate synthesis processes, research, etc.

Thus, we infer that, at the heart of the conceptual nexuses of computational thinking, the internal nexus starts with the external nexus (formalized concept), from the moment a subject uses it, aiming to understand a given situation and its application in different contexts and objects of reality, thus enhancing the subject's awareness in the context of action. We then defend that the internal nexus of computational thinking, although it has a formalized conceptual dynamic (external nexus), is based on the concrete action of the subject in the objective world. Appropriating the concept of computational thinking subsidizes not only socializing senses and meanings about it, but allows the act of signification, which corroborates for the subject to use computational thinking in particular situations, which demand its use, systematizing their forms of action.

Based on these assumptions, in order to build the concept of computational thinking, we considered: the concept of thinking in the cultural-historical perspective, as this theory is directly linked to the logical-historical perspective; the logical-historical movement of the term computational thinking; and, finally, the logical-historical movement of computational thinking in the context of mathematics education.

This aforementioned triad and its dialectical relationships helped us to define the historically organized conceptual nexuses for computational thinking. Furthermore, it subsidized our understanding of the scientific knowledge of the concept of computational thinking, serving as a foundation for understanding this thinking in the theoretical ambit.

It should be emphasized that the development of a concept of computational thinking, within the scope of mathematics education, needs to give rise to its development in the classroom. In this sense, it is intended to transpose the teaching of mathematics in a technicist and fossilized way (Ernest, 1991), it is essential to organize teaching under the aegis of the logical-historical movement of concept formation. In this way, this perspective may help to overcome a purely formal and empirical teaching (focused only on external nexuses).

We emphasize that all this survey and theoretical studies, systematic and careful reading and re-reading, appreciation, and criticism of the data, considering the problem and objective of this research as the guiding axis, led us to list three conceptual nexuses (external and internal nexuses), which present themselves in the concept of computational thinking:

1. Problem-solving in CHT,
2. Algebraic thinking, and
3. Algorithmic thinking.

Such conceptual nexuses, essentially, are in constant movement, are dialectical and, therefore, are historical, logical, and cultural.

In **Figure 1**, we illustrate the fact that for the development of computational thinking in basic education to be possible, it is necessary to develop the three conceptual nexuses, which are simultaneously and dialectically interconnected. For they are not tight, linear, or ready, but are in constant movement, culturally and socioculturally, as well as computational thinking. In other words, computational thinking is constituted by the three conceptual nexuses, with interdependence between them.

Therefore, in order to develop computational thinking, the student needs, from a given problem situation, to study four movements for its understanding:

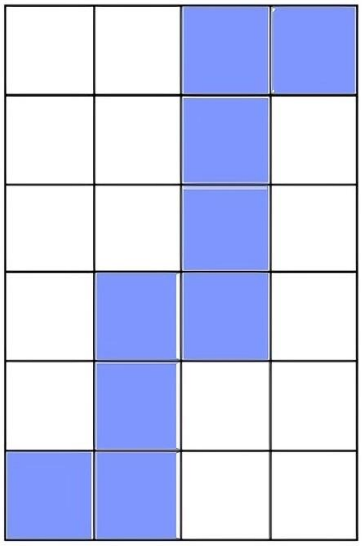

Figure 2. Path of the problem situation "what is the path?" (elaborated by the author, 2021)

1. To interpret data, classifying and ordering them for further analysis and synthesis,
2. To raise and systematize hypotheses, using different languages and building models,
3. To search for regularities, and
4. To appropriate abstractions that present themselves in some types of generalizations, according to the assumptions by Davydov (1982).

By way of example, we will illustrate these movements with a problem situation titled "what is the path?" (**Figure 2**). First, it is interesting that the teacher presents students with a path taken on squared paper (4×6), like the one in the image below, and invites students to describe the path taken on this grid.

Ideally, the teacher would listen to as many students as possible, so that each one may interpret their data and the data of the others. Thereby, it is possible to carry out the first movement, which is to interpret data, classifying and ordering them for further analysis and synthesis.

Next, that the teacher would ask students to draw a grid (4×6), a path different from the one previously shown, and, on a sheet, separate from the drawing, tell them to describe this path. It is important, at this point, to make it clear to the students that the objective is to describe a path so that another student, through just reading the description of that path, can draw the same drawing. At this time, the exchange of ideas between students is not allowed, so that the path drawing and/or possible tips do not influence the description.

After drawing a path on one sheet and the description of that path on another sheet, students must save the drawing, identify themselves (write their names) on the path description sheet, and exchange it with another student. And, only based on the description (without hints, or orality), the students will try to draw their colleague's path. With this, it is possible to implement movement 2: to raise and systematize hypotheses, using different languages and building models.

In order to continue the problem situation, students must return the drawings they made so that they may be checked against each student's original path.

It is important that the teacher listens to the students and reflects with them if they all managed to solve the problem situation, if it was easy, if they all used the same language, if they all indicated, where to start, if they considered the square (step), where they were in the count, and, finally, if someone used the Cartesian plan (ordered pairs). In this way, it is possible to implement movement 3: to search for regularities.

It is noteworthy that this problem situation may facilitate the approach of the movement from "rhetorical" and "syncopated algebra" to "symbolic algebra" (de Moura & de Sousa, 2005, p. 11), arguing about the standardization of the writing of ordered pairs (x, y) and the importance of recognizing regular movements that lead to pattern definition, so that we may use the same language. Here, there is the possible implementation of movement 4: to appropriate abstractions that present themselves in some types of generalizations, according to the assumptions by Davydov (1982).

Thus, problem-solving, algebraic thinking, and algorithmic thinking, when associated with the teaching process, in the search for the student to understand the four movements for comprehension, listed above, may concatenate in the development of learning how to use computational thinking, with the development of concepts and knowledge in these areas to achieve a pre-established objective.

The subsequent understanding of learning such concepts (reflection, abstraction, generalization) will consequently lead to the development of computational thinking, establishing a set of mental operations that facilitates the understanding of reality.

FINAL CONSIDERATIONS: AFTER ALL, WHAT IS THE CONCEPT OF COMPUTATIONAL THINKING IN THE CONTEXT OF MATHEMATICS EDUCATION?

In this article, we show our concern regarding the concept of computational thinking, as this term is inserted in BNCC in Brazil and in the curricula of other countries; however, without a pedagogical indication of how, when, where, and what resources, especially didactic ones, basic education teachers may develop it in the classroom.

In this sense, we defend the essentiality of developing a concept of computational thinking for mathematics education, thus recognizing the possibility of organizing a pedagogical practice, overcoming the mere transmission of knowledge, technological technicality, and skills training, as we see in the theoretical bases of computational thinking under the aegis of computer science. Our intention, then, was to contribute to the constitution of (internal and external) conceptual nexuses for computational thinking, in order to be aligned with the mathematical contents.

Therefore, we found that computational thinking, in the context of mathematics education, was thought of in a fragmented way, not contemplating in a dialectical and combined way the nine (9) characteristics, which were suggested and traced by several authors (Barr & Stephenson, 2011; CSTA, 2011; Wing, 2006, 2014). On the contrary, these characteristics were elaborated as a segmented structure, but without building articulations or conceptual nexuses between them, as exposed in the studies we analyzed.

It is not possible to consider computational thinking as isolated, fragmented, and mechanized knowledge, through certain procedures and stagnant rules, tending solely to the training of skills or the development of a programming language. We should not preconceive computational thinking, as shown by most of the authors we investigated, as thinking based on the reproduction of rules, axioms, models, and schemes, which contribute little to students' ability to solve problems significantly. It is necessary to consider that mathematical knowledge is a way of apprehending and organizing reality, explaining phenomena, systematizing data, instigating the formation of critical and creative thinking. In summary, mathematics is a way of world reading.

In these terms, computational thinking, under the above terms, is shown to be disconnected from the students' reality and from the development of psychic functions linked to the problem-solving process. Thus, important elements such as creativity, decision-making, intellectual autonomy, imagination, and logical-strategic reasoning are devoid of value.

On this basis, in contrast to these studies, we defend a dialectical perspective of computational thinking. Now, we assume the dialectics under the aegis of the Logical-Historical (Kopnin, 1978; Kosik, 2002) and CHT (Vygotsky, 1996, 2000, 2012, 2014), as a way of analyzing the continuous contradictions in what corresponds to reality and the ways of understanding it as something contradictory and in constant transformation. Dialectics, according to Kopnin (1978), is a movement that intends to penetrate the process of acquiring knowledge itself, that is, into the movement of thought itself and the way in which objective reality is reflected.

From this perspective, we summarize that computational thinking is a type of thinking composed of three conceptual nexuses (problem-solving, algebraic thinking, and algorithmic thinking). Therefore, it demands articulations between some psychic functions responsible for the organization, discovery, and decoding of relationships and dynamic-causal nexuses between our actions in the face of concrete reality. Thinking, in the sense of computational thinking, is establishing links between ideas and hypotheses, which are abstract representations of concrete reality, but which materialize in the language (in signs).

Thus, we understand that computational thinking, in the context of school mathematics, plays the role of generalizing thinking. This means that its (external and internal) conceptual nexuses act in the conceptual ordering and categorization of reality into sets of objects, data, information, hypotheses, situations, given phenomena, etc., which are fundamental to the interpretation and resolution of problem situations.

In this sense, computational thinking is a process of solving plugged or unplugged problem situations, which include the interpretation and organization of data, analysis and synthesis, generalization, abstraction, and implementation (produced from mathematical knowledge). Thus, we consider computational thinking as a type of thinking that expresses a sociocultural practice in which students have theoretical and practical domains (explanatory systems) to understand and act in their concrete reality.

In other words, we conceive that computational thinking, in the context of mathematics education, is formed by the interdependence between the conceptual nexuses of problem-solving and algebraic and algorithmic thinking, aiming at the production of mathematical knowledge in the classroom context. Thus, in its conceptual configuration, computational thinking is a dialectical movement of thinking, which aims to guide students in the actions of interpreting, analyzing, questioning, exploring, investigating, decomposing, reflecting, observing regularities, and producing syntheses, tending to the creation of resolutions and/or strategies using algebraic and/or algorithmic language.

From this point of view, we consider that computational thinking is conceptually consolidated through inter-functional relationships between thinking and language, which represent a set of sequenced phases to solve problems, perform tasks, or organize data. In other words, it may occur via different forms of languages (oral, written, gestural, etc.), symbolized by signs, operations, and mathematical rules. These computational thinking languages are used aiming at generalization, mainly, in the movement of analysis and interpretation of information, decomposition and synthesis, in studies on structures, in problem-solving, in the solution and execution of actions, etc.

We argue that the use of computational thinking is related to the search for a certain result, that is, a generalizable pattern, a regularity, a logical sequence, a feasible resolution, a disposition in steps, etc. In summary, it is a type of ordered mathematical reasoning, based on individual (mathematical thinking) and external situations (materialization of strategies through mathematical language).

For this reason, we support the need to understand computational thinking not in an instrumental, utilitarian way, only worked with the use of ICT (plugged). More than that, we defend computational thinking as a potential means of expanding problem-solving capabilities, interpreting reality, and expanding the students' forms of action in their sociocultural context, either in a plugged (ICT) or unplugged way.

That said, we believe that computational thinking, in the context of school mathematics, has the function of helping students to produce mathematical knowledge (algebraic and algorithmic thinking) and to develop research and problem-solving skills, thus favoring the development of students, as well as expanding their world reading through the act of thinking dialectically, that is, of understanding reality in its entirety.

Finally, we conceive that computational thinking in mathematics education is constituted by the inseparable link between the (external and internal) conceptual nexuses of problem-solving and algebraic and algorithmic thinking, with a view to the production of mathematical knowledge. Therefore, computational thinking is a dialectical movement of thinking, which aims to guide students in the actions of interpreting, analyzing, questioning, exploring, investigating, decomposing, reflecting, observing regularities, and producing syntheses, tending to the construction of systematizations, resolutions and /or strategies, using mathematical language.

We hope that these studies and, consequently, this article will significantly contribute to future research and teaching practice, given that the developed concept of computational thinking in the context of school mathematics admits, *a priori*, the school as a critical, dialectical, complex, and dynamic socio-cultural space, composed of cultural-historical subjects.

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