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Understanding relativistic time: Exploring the concepts of spacetime and time dilation

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ABSTRACT

Relativistic time deals with concepts as spacetime and time dilation. I will try to explain this as clearly as possible. To explain the concept of spacetime is not at all impossible, although it is four-dimensional. But that means that we define it using four numbers. If we describe us through age, weight, height, and IQ, that would be a four-dimensional representation of us.

Keywords: applets, Einstein, GeoGebra, relativity, spacetime

INTRODUCTION

Important topics in relativistic time are concepts as spacetime and as time dilation. I will try to discuss and explain this as easily as possible. In physics we consider spacetime as a model combining the three dimensions of space plus one dimension of time into a single four-dimensional multiple. Spacetime diagrams can visualize relativistic effects such as wherefore various observers observe differently when and where actions appear.

Before the 20th century, physicists and others considered the three-dimensional geometry of the universe (in terms of coordinates, distances, and directions) as independent of one-dimensional time. Albert Einstein established the idea of spacetime as an important construction in his theory of relativity. Before Albert Einstein, physicists used two independent theories to explain physical events: Isaac Newton's laws of physics described the motion of massive objects, and James Clerk Maxwell's electromagnetic models described the properties of light. In 1905, Albert Einstein declared that his special relativity was built on two postulates: The laws of physics are identical in all inertial systems (i.e., non-accelerating frames of reference).

The speed of light in a vacuum is identical for all observers, despite the consequences of the motion of the light source.

The concept of spacetime is thus four-dimensional and defined by four numbers. We could describe us through age, weight, height, and place of birth, and that would result in a four-dimensional representation of us.

Now suppose we want to describe a specific event. We need to specify when it happened and where it happened. Let us consider an airplane trip to New York to meet someone dear to us in downtown New York at the position of 40.72 latitude and–74.03 longitude, and 6,370 kilometers up from the center of the Earth (the Earth radius). We meet this person at 12:30:00 GMT on November 12 in the year 2021. This event in spacetime can be described by an ordered four-tuple: (where, when) = (40.72, -74.03, 6,370, 2021.11.12.12.30.00).

There must be other events that happened at the same time in other places (many humans drank beer in London); at the same place in other times (another couple kissed at the same place in 1975); and at completely different times and places (the sea battle between Sweden and Denmark in Köge on the 4th of October at 1710). If we gather every possible location of all events ever, every combination of four numbers specifying times and places in the universe, that is spacetime. Physicists call it a manifold.

This is not so hard to understand. The mathematics for dealing with arrays of four numbers at a time is well developed. If we combine it with the concept that the speed of light is all the time constant, then we have the foundation of the theory of relativity by Albert Einstein.

Albert Einstein recognized that the velocity of light is absolute, invariable, and cannot be surpassed and he also concluded that the velocity of light was more essential than either time or space. If we travel with a velocity close to the light velocity in vacuum, then time is part of the universe and cannot exist apart from the universe. On the other hand, if the velocity of light is invariable and absolute then both space and time must be flexible and relative to hold this.

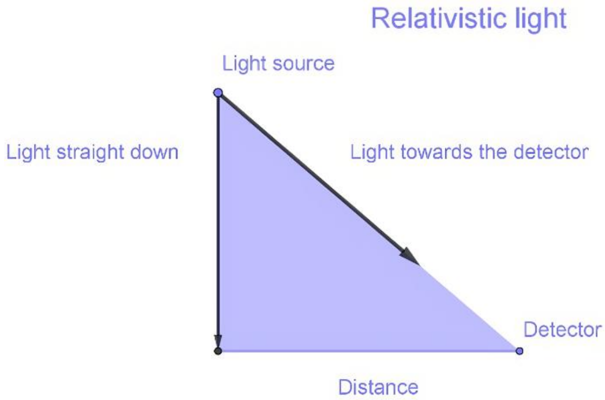

Figure 1. Relativistic time in a GeoGebra construction (which can be found at: https://www.geogebra.org/m/b6xmmhah)

Time dilation is one consequence of the theory of relativity by Albert Einstein and since rates of time actually run differently depending on relative motion an effect is known as time dilation. Two synchronized clocks will not stay synchronized if they move relative to each other. There is a similar effect in length contraction, where moving bodies are shortened in direction they travel.

Time dilation (as well as length contraction) is insignificant at everyday velocities in the world around us, although measured with very sensitive instruments. However, it becomes much more noticeable if an object's velocity approaches the velocity of light.

If a spaceship could travel at 87% of the velocity of light, an observer would see the spaceship's clock moving twice as slow as normal and the observer would also see the astronauts inside the spaceship moving in slow-motion. At 99.5% of the velocity of light, the observer would see the clock moving 10 times slower than normal. At 99.9% of the velocity of light, the factor becomes 22 times, at 99.99% 224 times, and at 99.9999% 707 times, increasing exponentially. In the largest particle accelerators currently in use, physicist can make time slow down by 100,000 times. At the velocity of light itself, if or when it is possible to achieve that, time would stop completely.

According to Albert Einstein's theory, time is relative to the observer, and especially to the motion of that observer. Time is still governed by the laws of physics and entirely predictable in its manifestations, it is not absolute and universal as Isaac Newton possibly believed.

We also have the so-called *twins paradox*, where an astronaut returns from a near-light velocity voyage in space to find his twin, who stayed at home, many years older than him, traveling in a velocity close to the velocity of light has allowed the astronaut to experience only one year of time, while ten years have elapsed on the Earth. Because of the time dilation effect, a clock in the spaceship registers a shorter duration for the journey than the clock in mission control on the Earth.

The real paradox comes from the fact that since there is no preferred frame of reference in relativity, we could consider the traveler in the spaceship as the one remaining at rest while the Earth travels close to the velocity of light. In that scenario we would expect the astronaut to age much more than the inhabitants of the Earth. But the spaceship is accelerating away at near light velocity from in the universe, whereas the Earth is not. Consequently, it is the spaceship (and its astronaut) that experiences relativistic time dilation, not the twin on Earth.

Are we on rest on the Earth? Not from a hypothetical observer out in space. The Earth is falling around the Sun through space with a velocity of 110,000 km/h. The Sun is falling in the gravitational field in the Milky Way with a velocity of 230 km per second. The Milky Way is also falling, relatively like all other galaxes, with a velocity of 400 km per second. Thus, all reference systems are relative.

This is what Albert Einstein's theory of relativity is built upon. We can summarize it as "all uniform motion is relative". A motion in which an object is travelling in a straight line with uniform velocity is labelled "uniform motion". The velocity of the object remains constant as it covers equal distances in equal intervals of time. If there is no acceleration, we can argue that we are at rest and everything else is moving. There is nothing in the laws of physics that can argue against us since it depends on what we compare us with.

Let us derive a formula for the time dilation. We start with considering a spaceship, going in a near-light velocity, and with a light source sending a beam of light towards a detector at the floor of the spaceship. An instrument is measuring the time it takes for the light beam to reach the detector and we call it t_0. During the time it took the light beam to reach the floor the detector moved forward in a near-light velocity. From the view of a hypothetical observer the light had a longer way to go and must follow the detector. By considering how a light beam is travelling in a spaceship can we derive the following relations (**Figure 1**).

We know that *distance*=*velocity*·*time* and since everything is mowing with near-light velocity we can write the distances, as in **Figure 2**.

Pythagoras's theorem: $(c \cdot t)^2=(c \cdot t_0)^2+(v \cdot t)^2$

Algebraic manipulation with $(v \cdot t)^2$: $(c \cdot t)^2-(v \cdot t)^2=(c \cdot t_0)^2$

Take out t^2 in the left part of the expression: $t^2 \cdot (c^2-v^2)=(c \cdot t_0)^2$

Divide with (c^2-v^2): $t^2=\frac{(c \cdot t_0)^2}{c^2-v^2}$.

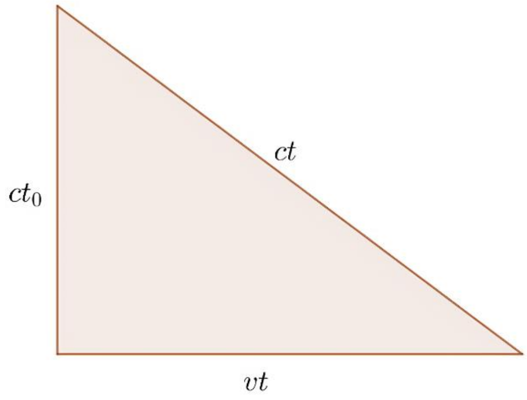

Figure 2. Pythagoras in a GeoGebra construction (which can be found at: The beam and the detector – GeoGebra)

Table 1. Velocity and time

Velocity (v)	Time on spaceship	Time on the Earth
Common velocities	1.00 seconds	1.00 seconds
50% of light velocity	1.00 seconds	1.15 seconds
80% of light velocity	1.00 seconds	1.67 seconds
87% of light velocity	1.00 seconds	2.00 seconds
99% of light velocity	1.00 seconds	7.09 seconds

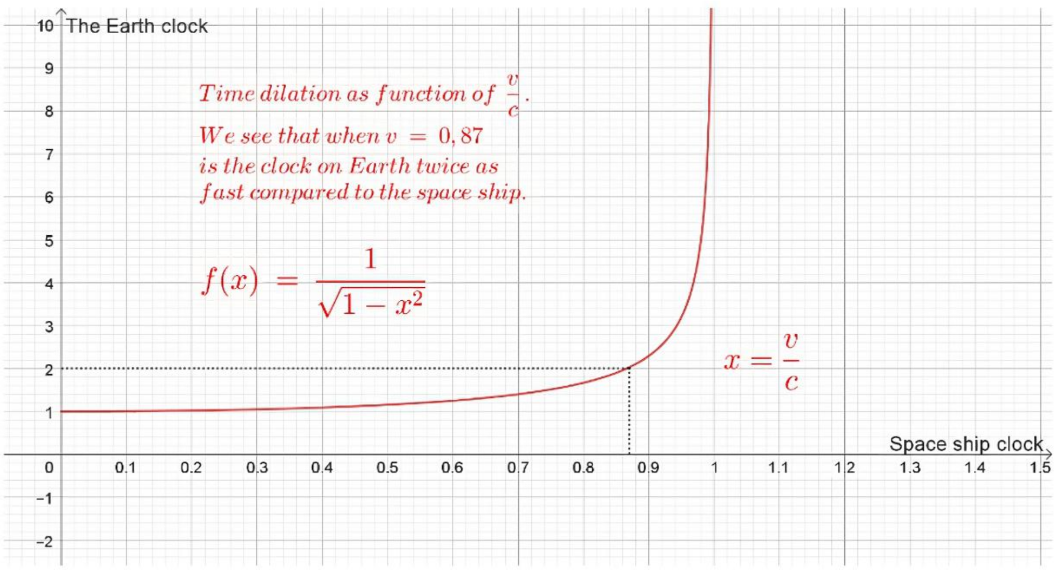

Figure 3. The graph of time dilation in GeoGebra (which can be found at: Time dilation – GeoGebra)

Divide the expression with c^2: $t^2 = \frac{{t_0}^2}{1-\frac{v^2}{c^2}}$.

Use the square root and we get: $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

This is an important relation called the time dilation. We can also use the same formula for calculation of time contraction. Let us use this formula and calculate some values for **Table 1**.

We can also graph the time dilation, as shown in **Figure 3**.

THEORETICAL FRAMEWORK

Dynamic Geometry Environment as Amplifier and Reorganizer

Pea (1985, 1987) used two metaphors for the use of technology in education, technology being an ‘amplifier’ and a ‘reorganizer’ of mental activity. A dynamic geometry environment (DGE) definitely serves as an ‘amplifier’ and a ‘reorganizer’ for enhancing students’ physical thinking. The phrase ‘amplifier’ identifies that technology performs monotonous computations fast and precisely. Therefore, students can make observations and develop insight rather than manual techniques. Here, the tool does not change students' thinking but fairly simplifies their investigations. If technology is used as a reorganizer, it assists to develop students' thinking by giving them access to deeper processes. A DGE helps students to look for patterns, identifying invariances or making and testing conjectures. In a paper-pencil environment students will spend a significant amount of time on drawing and measuring objects only. Pea's (1985, 1987) metaphors of technology as amplifier and reorganizer, will be used to analyze the teaching of relativistic time and its relation to the GeoGebra applets.

Visualization and Dynamic Geometry Environment: A Key Aspect in Developing Thinking

A necessary process in the understanding and construction of physical concepts is visualization. Since students are allowed to manipulate objects, a DGE facilitates visualization. It expands possibilities for representation and has a great impact on conceptualization of objects and internalizing their meanings (Falcade et al., 2007; Moreno-Armella et al., 2008). Aid of technology in teaching and learning is perceived as strongly linked with dynamic interactive graphical representations (Laborde et al., 2006).

A variety of representations, diagrams, drawings, and graphs are used for teaching physical concepts in a traditional classroom. Such representations facilitate and enhance student's understanding of physical concepts. DGE allows figures and shapes to be manipulated using the dragging feature, which provides a dynamic opportunity to the learning of physics. It allows students to perform investigations and affords the possibility of a dynamic visual representation of concepts in a physical sense. Such investigatory activities are hard to experience in a static environment such as paper and pencil (González & Herbst, 2009).

In a DGE can students use complex figures and easily perform in real time a wide range of transformations on those figures, so students have access to a variety of examples that can hardly be matched by non-computational or static computational environments.

Theory of Variation: Dragging as a Tool in a Dynamic Geometry Environment

Creating mental images is an important step to abstract a physical concept. For example, to help students abstract the concept of a relativistic time, the teacher may present stories of the twin paradox, hoping that students will notice some common features, among all the figures, namely, time dilation. In DGE, the student may experience how we could change the value for the detector and through this continuous process, be able to 'see' the light velocity as absolute. The dragging tool is a very powerful feature of a DGE since it allows the user to abstract an idea by observing properties of figures, which remain invariant during the process of variation. Leung (2003) aptly describes this affordance of a DGE, as follows:

> "… when engaging in learning activities or reasoning, one often tries to comprehend abstract concepts by some kind of "mental animation", i.e., mentally visualizing variations of conceptual objects in hope of "seeing" patterns of variation or invariant properties."

> "… one of DGE's power is to equip us with the ability to retain (keep fixed) a background configuration while we can selectively bring to the fore (via dragging) those parts of the whole configuration that interested us in a learning episode."

Many researchers have studied the role of dragging in DGE focusing on how it can be instrumental in helping students construct figures using their properties, explore physical problems, formulate conjectures and even proofs. Arzarello et al. (2002), identified seven dragging modalities (wandering, guided, bound, dummy locus, line, linked, and drag test) while trying to analyze conjecture-making episodes by students working on a problem.

Variation Theory

Variation is at the very heart of a DGE. Marton and Booth (1997) proposed four inter-related functions of variation, which they referred to as patterns of variation. These are as follows.

Contrast

> "… in order to experience something, a person must experience something else to compare it with."

Generalization

> "… in order to fully understand what "a graph" is, we must also experience varying appearances of "a graph", …"

Separation

> "In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant."

Fusion

> "If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously."

They advocated that variation and simultaneity play an important role in discernment of a concept. According to them, in order to discern a concept and to understand it completely, one must experience variations of it.

Three Aspects of Fidelity

If we use technology for exploring mathematical concepts and problems, we should assess its pedagogical, mathematical, and cognitive fidelity. Zbiek et al. (2007) describe mathematical fidelity as

> "faithfulness of the tool in reflecting the mathematical properties, conventions, and behaviors (as would be understood or expected by the mathematical community)" (p. 1173).

Let us have a look at the function $f(x)=(x^2-1)/(x-1)$. If a graphics calculator graphs this function, it may produce the linear equation y=x+1. This is inaccurate since the function $f(x)$ is not defined at $x=1$ and the correct graph of $f(x)$ should have a point break at $x=1$. Thus, the mathematical fidelity of the tool is compromised in relation to graphing of the function.

Zbiek et al. (2007) describe cognitive fidelity as

> "the faithfulness of the tool in reflecting the learner's thought processes or strategic choices while engaged in mathematical activity" (p. 1173).

A tool has cognitive fidelity if it produces external representations, which match the user's internal representations, enhancing their conceptual understanding. Used correctly, a DGE has good cognitive fidelity. The third kind of fidelity is that of pedagogical fidelity, which, according to Zbiek et al. (2007), is

> "the extent to which teachers (as well as students) believe that a tool allows students to act with a physics concept in ways that correspond to the nature of learning physic that underlies a teacher's practice" (p. 1187).

Pedagogical fidelity refers to the tool's ability to support students' explorations and learning. In a DGE, the dragging feature can afford this kind of fidelity. We can use a slider to vary the position of a point in the time dilation right angle triangle and observe the change in the graphical representation of the detectors place. The level and degree of the types of fidelity vary among technology tools and should be considered while selecting and evaluating appropriate tools for students' explorations. The aspects of fidelity should be kept in mind while designing exploratory tasks for students.

Theoretical frameworks may be classified into two categories. Four inter-related functions of variation focus on understanding students' thinking and reasoning in a DGE environment, while Pea's (1985, 1987) theory of amplifier and reorganizer and the three aspects of fidelity are related to design and positioning of DGE-based geometrical applets for learning physics.

CONCLUSION

I sincerely hope that you now have come to a better understanding of relativistic time and that you can use my applets and images to help someone else to understand it too.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

REFERENCES

Arzarello, F., Olivero, F., Domingo, P., & Ornella, R. (2002). A cognitive analysis of dragging practices in Cabri environments. *Zentralblatt für Mathematic* [*Central Journal for Mathematics*], *34*(3), 66-72. https://doi.org/10.1007/BF02655708

Falcade, R., Laborde, C., & Mariotti, M. (2007). Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics, 66*(3), 317-333. https://doi.org/10.1007/s10649-006-9072-y

González, G., & Herbst, P. G. (2009). Students' conceptions of congruency through the use of dynamic geometry software. *International Journal of Computers for Mathematical Learning, 14*(2), 153-182. https://doi.org/10.1007/s10758-009-9152-z

Laborde, C, Kynigos, C., Hollebrands, K., & Strässer, R. (2006). Teaching and learning geometry with technology. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 275-304). Sense Publishers. https://doi.org/10.1163/9789087901127_011

Leung, A. (2003). Dynamic geometry and the theory of variation. In N. A. Pateman, B. J. Doughherty, & J. T. Zillox (Eds.), *Proceedings of PME 27: Psychology of Mathematics Education 27th International Conference* (pp. 197-204). University of Hawaii.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Lawrence Erlbaum Associates.

Moreno-Armella, L., Hegedus, S. J., & Kaput, J. J. (2008). From static to dynamic mathematics: Historical and representational perspectives. *Educational Studies in Mathematics, 68*(2), 99-111. https://doi.org/10.1007/s10649-008-9116-6

Pea, R. D. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167-182. https://doi.org/10.1207/s15326985ep2004_2

Pea, R. D. (1987). Cognitive technologies in mathematics education. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89-122). Erlbaum.

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Information Age.