Journal of Mathematics and Science Teacher
2023, 3(2), em047
e-ISSN: 2752-6054
https://www.mathsciteacher.com/

MODESTUM

Research Article **OPEN ACCESS**

Using indigenous artefacts to support conceptual field approach of learning special trigonometric angles

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Citation: Ali, C. A. (2023). Using indigenous artefacts to support conceptual field approach of learning special trigonometric angles. *Journal of Mathematics and Science Teacher, 3*(2), em047. https://doi.org/10.29333/mathsciteacher/13698

ARTICLE INFO

Received: 31 Mar. 2023

Accepted: 11 Sep. 2023

ABSTRACT

Concerns have been expressed on the abstract nature of teaching and learning trigonometry in pre-tertiary institutions. However, studies on student-teachers mathematics learning shows that this concern could be ameliorated by using indigenous artefacts to support conceptual fields of trigonometry. With pre-/post design, the researcher selected 50 student-teachers through simple random sampling and performed experiments using indigenous artefacts in teaching and learning of 30º-60º-90º and 45º-45º-90º special trigonometry angles. This cohort has had at least two years of teaching experience in their permanent schools of work. After going through the experiments, two diagnostic tests (pre- and post-test) were administered, scored and analyzed with the SPSS software. The results of the descriptive statistics, one sample t-test, paired samples t-tests, and correlation coefficients showed that the student-teachers' performance had significantly improved. The improvements were really attributable to the deployment of the indigenous artefacts to carry out the instruction in the special trigonometric angles. We, therefore, recommended that stakeholders should adopt indigenous artefacts to support the conceptual field approach for the teaching and learning of basic trigonometry.

Keywords: conceptual field approach, indigenous artefacts, pre-/post design, special trigonometric angles, t-test statistics

INTRODUCTION

Teaching is the process of using appropriate strategies, methods, personnel and materials to reach predetermined goals, and the instruction is a planned, controlled and organized activity that occurs in classrooms (Bosch, 2022). This means the teachers' approach should be conscious, purposeful and oriented to predetermined goal in order to reach desirable behavioral change. So, teachers must focus on new but interconnected concepts to ensure complete participation, self-motivation and academic achievement. The notion of cognitive artefacts is based on the socio-historic school of Vygotsky, which drive both external process and internal process to improve logical reasoning and cognitive activities (Collins, 2023).

Kortjass (2019) argues that when teachers prepare learning activities, they introduce special tools serving as auxiliaries between the learner, the concept and the activity to construct a mental representation of the concept. These tools are called artefacts, which can be physical objects (such as tokens of varying colors, rulers, mathematical sets, and dice), or three-dimensional models or paper-and-pen drawings. In most cases, the artefacts are established objects within particular learning cultures and just act as intermediaries between the learned concepts and the internal representation of the learner (Serpe & Frassia, 2021). Trigonometry is essential in the study of calculus, geometry, vectors and mechanics. In the triangle in particular is important for everyday life processes involving sea navigation, road traffic signs, truss bridges, modern roofs and technology (Dewi et al., 2021; Jelatu et al., 2019). Even in acient times, cultural artefacts were conceived from trigonometry. In Ghana, the common artefacts enriched with trigonometry are adinkra, kentey, smock, basketry, and pottery (Ali & Davies, 2018). Modern conception and representations of trigonometry using artefacts can be a novel.

In the mathematics curriculum, student-teachers are expected to acquire knowledge of content, pedagogy and teaching techniques for trigonometric angles. Even though any three tripartite angles that sum up to 180° is justifiable, the case of 30°-60°-90° and 45°-45°-90° are exemplary (Spangenberg, 2021). Student-teachers still find it extremely difficult to handle the topic. The most viable strategy they devise is the acronym 'SOA', 'CAH', and 'TOA'. In the right angle triangle, this acronym means sine is the division of opposite by adjacent, cosine is the division of adjacent by hypotenuse and tangent is the division of opposite by adjacent. This strategy is superficial and fails to support pedagogy in mathematics. In their quest to ameliorate the canker, Dewi et al. (2021) used regenerative learning, Nabie et al. (2018) used APOS theory, and Spangenberg (2021) used mathematics knowledge for teaching. Even though they have provided qualitative solutions, they have failed to provide systematic procedures of discovering the solutions. They have not also provided an action-oriented research methodology, verifiable by heavy-laden robust hypotheses and results.

CONCEPTUAL FIELD THEORY

Conceptual knowledge is a network of knowledge in which the linking relationships are as prominent as the discrete pieces of information. This is closely linked to conceptual structures and conceptual field. Ali (2019) and Ali and Wilmot (2016) define conceptual structures as the complex networks of relationships among concepts are called the conceptual structures. Unlike conceptual structures that can be narrowed down to two, three, four, or any polygon of didactics (Ali, 2019), conceptual fields are the sets of situations, problems, relationships, contents, thoughts, and procedures that learners use to give meaning to a given topic to comprehend the real world (Helenius & Ahl, 2022).

Alfaro-Carvajal and Fonseca-Castro (2016) and Duignan (2016) offer a set of interrelated concepts to strengthen analysis and conceptual consistency between studies, while providing latitude to overlay different disciplinary perspectives. Thus, there is the existence of an established body of mathematical knowledge, comprising concepts and theorems in which concepts and relationships are inherent. The conceptual field requires the study of "bulk" concepts, which develop in relation to other concepts, through several kinds of problems and with the help of several wordings and symbolisms.

The theory of conceptual fields is a developmental theory of cognitive complexity. It is based upon the fact that student-teachers' competences and conceptions develop through experience and that there are high regularities in the difficulties students have to overcome (Vergnaud, 1992). Vergnaud (2009) proposed an improved conceptual field theory to provide coherent structures and basic principles to the study of the development of complex skills. In this study, the conceptual field theory has both a mathematical framework and a psychological perspective, notably the acquisition of concepts building on the work of Piaget, and the function of instruction building on the work of Vygotsky (Helenius & Ahl, 2022).

Helenius and Ahl (2022) allude that the theory of conceptual fields and theory of representation constitute an organized network of ideas, notions, distinctions, terms, and claims that carries the legacy of both Piaget's and Vygotsky's work. In contrast to the well-established idea of a hypothetical learning trajectory on which the students move towards new insights and higher capacity to handle mathematics, Vergnaud developed his theories from the perspective that growth in knowledge is best described as the growth of conceptual fields, which is inherently a network rather than a path (Helenius & Ahl, 2022). It can therefore be deduced that mathematics knowledge consists of conceptual and procedural knowledge. Procedural knowledge constitutes the by step-by-step procedures for solving mathematical tasks, on one hand, and knowledge of the symbolic representations used in such procedures, on the other hand. Hence, to be competent in mathematics involves not only knowledge of concepts and knowledge of procedures but also of relations (Klein, 2015).

This study interconnects the theories of Piaget cognitive levels, Vygotsky's zone of proximal distance, and Vergnaud's representation and conceptual fields. Piaget cognitive theory states that knowledge is made of up ideas of mental schemes that evolve as adaptation of the schemes, in the process of assimilation and accommodation. Vygotsky's ZPD states there is a gap between what a child can without support and what the child can learn with support from an adult. Vergnaud's conceptual fields and representation constitute an organized network of ideas, notions, distinctions, terms, and claims that carries the legacy of both Piaget's and Vygotsky's work. While knowledge in trigonometry is nonnegotiable in understanding the special angles, the application of appropriate artefacts to bridge the knowledge is equally non-sacrosanct. These interconnected theories seek to drum home the importance of artefacts in consolidating knowledge in trigonometry (Helenius & Ahl, 2022).

In this realm, the combined theory contributes to comprehend how empirical knowledge evolves into scientific one as well as how pedagogical activities support learning processes. Hence, the strength of the theory of conceptual fields in the provision of coherent approaches to research in mathematics sets, situations, invariants and representations. Among these benefits of the theories are, as follows:

1. Provision of how children acquire mathematical concepts, building on the work of both Piaget and Vygotsky.
2. Integration of epistemological elements, concept characteristics and cognitive development, into an approach to teaching.
3. Clear articulation of the role of representations required to mediate mathematical concepts
4. Assessment purposes of gauging learners in development along mathematical paths in order to design further learning experiences

SPECIAL TRIGONOMETRIC ANGLES

Trigonometry is concerned with specific functions of angles and their application to calculations (Ngu & Phan, 2020). The functions are used in obtaining unknown angles and distances from known or measured angles in geometric figures. Other measures are lengths of the sides and the areas of the triangles (Barnard & Maor, 2022). Studies (Hidayat et al., 2023; Killian et al., 2021) show that the only relationship among the lengths of sides of the triangles is the sum of any two sides always being greater than the third. In angle relationships, two peculiar relationships in the isosceles (i.e., 30^{o}-60^{o}-90^{o}) and right-angled (i.e., 45^{o}-45^{o}-90^{o}) triangles remain invincible.

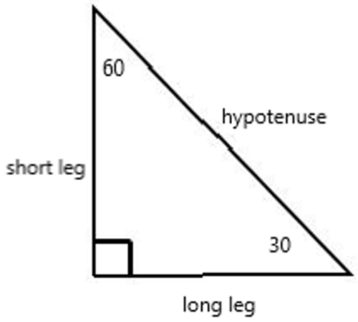


Figure 1. Parts of 30°-60°-90° triangle (extracted from Hidayat et al., 2023)

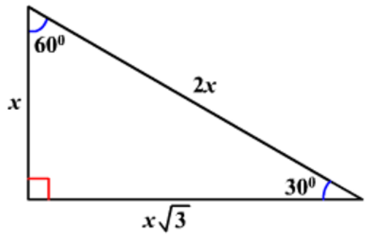


Figure 2. Relationships among 30°-60°-90° triangle (extracted from Watkins et al., 2022)

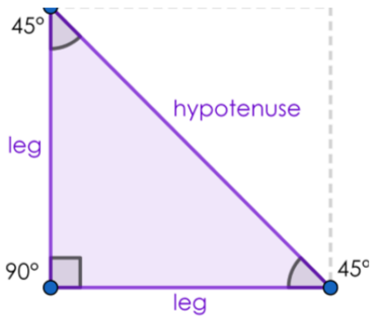


Figure 3. Parts of 45°-45°-90° triangle (extracted from Killian et al., 2021)

30°-60°-90° Triangle

Figure 1 shows the sides of a 30°-60°-90° triangle have the following parts:

1. The side opposite the 30-degree angle is called the *shorter leg* (x).
2. The side opposite the 60-degree angle is called the *longer leg (l)*.
3. The side opposite the right angle of 90° is called the *hypotenuse (h)*.

Using **Figure 1**, it is possible to find the relations among the parts (Rast & White, 2021).

Figure 2 shows 30°-60°-90° triangle, where the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. The sides of a 30°-60°-90° triangle are always in the ratio of 1:$\sqrt{3}$:2. The sides are considered to be a Pythagoras triples (Watkins et al., 2022). 30°-60°-90° triangle can be by constructed by relating one side of the equilateral triangle to obtain:

$$h = 2x \,\&\, l = x\sqrt{3} \tag{1}$$

In Eq. (1), '*h*' is the hypotenuse, '*x*' is the shorter, and '*l*' is the longer side. It can be proved that '*h*' is two times '*x*' and '*l*' is the square root of '*x*' (Bosch, 2022).

45°-45°-90° Triangle

A 45°-45°-90° triangle has two of the angles smaller than the third and is considered to be both an isosceles and right triangle (since one angle that measures exactly 90°). 45°-45°-90° triangle has the exact same shape, but they can still come in different sizes. Two 45°-45°-90° triangles put together make a perfect square, whose sides are all of equal length (Killian et al., 2021) (**Figure 3**).

45°-45°-90° triangles are also special right triangles with one 90° angle and two 45° angles (McKinsey, 2023). 45°-45°-90° triangle is a commonly encountered right triangle whose sides are in the proportion 1:1:$\sqrt{2}$. The measures of the sides are *x*, *x*, and $x\sqrt{2}$ (Watkins et al., 2022).

In **Figure 4**, 45°-45°-90° triangle has three unique properties that make it very special and unlike to all the other triangles.

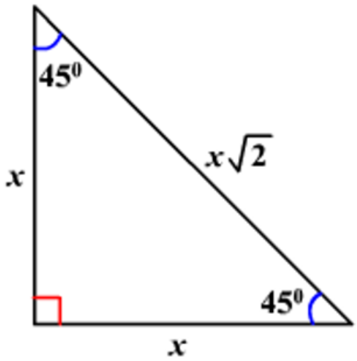


Figure 4. Relationships among 45°-45°-90° triangle (extracted from Killian et al., 2021)

1. The polygon is an isosceles right triangle.
2. The two side lengths are congruent, and their opposite angles are congruent.
3. The hypotenuse (longest side) is the length of either leg times square root of two (McKinsey, 2023).

45°-45°-90° triangle rule states that the three sides of the triangle are in the ratio 1:1:$\sqrt{2}$. So, if the measure of the two congruent sides of such a triangle is x each, then the three sides will be x, x, and $x\sqrt{2}$. On the other hand, 45°-45°-90° triangle can be constructed by relating the hypotenuse and the shorter alone, where 'h' is two times the length of either side as in Eq. (2):

$$h = x\sqrt{2} \tag{2}$$

INDIGENOUS ARTEFACTS

An artefact is an ornament, tool, or other object that is made by a human being, especially one that is historically or culturally interesting (Collins English Dictionary, 2023). Indigenous artefacts might be similar to mathematical objects and have an opportunity for sustainable education in remote areas (Schreiber & Klose, 2017; Utami et al., 2021). In this context, the indigenous artefacts are adinkra, baskets, and other symbols (Ali, 2021).

RESEARCH PROBLEM

First and foremost, several concerns have been expressed on the abstract nature of teaching and learning trigonometry in pre-tertiary schools. Nabie et al. (2018) revealed that students have difficulties in trigonometry. Some of these difficulties emanate from lack of motivation, abstractness of trigonometric concepts, lack of understanding of fundamental concepts, and students' inability to connect concepts in trigonometry. A pre-survey analysis on student-teachers' in the University of Education, Winneba clearly supported this claim and discovered that most student-teachers could not apply any alternative methods to analyze sides and angles of the right-angled triangle. The only common method they all applied was the Pythagoras theorem. And in applying the theorem, they either obtained inaccurate results or incorrect answers, especially when the lengths of sides of the triangles were not a Pythagoras triad. This was traced to student-teachers lack of relating the ideas to interrelated concepts within a particular scope of content.

Secondly, the continuous assessment scores (i.e., tagged as 'pre-intervention') in the basic trigonometry can attest to this fact. It was also observed that the students resorted to using only the relationships of the sides of the triangle, popularly referred to as "SOHCAHTOA" to solve problems related to basic trigonometry. The acronym "SOHCAHTOA" stands for 'sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent'. This produces both teachers and students who cannot relate knowledge to everyday life, and worst still, cannot solve life problems involving trigonometry. In the relations, very little conceptual fields were built in solving problems containing 30°-60°-90° or 45°-45°-90° triangles problems.

Thirdly, the revised curriculum for undergraduate post-diploma in basic education program at the University of Education, Winneba requires student-teachers to connect and relate teaching and learning to their immediate environment using their cultural artefacts and symbols. Ghana abounds in various indigenous artefacts such as adinkra, baskets, kente, smock, and traditional realia (Ali, 2021). Most of these indigenous artefacts endowed with the triangle and by large, concepts in trigonometry. These can be harnessed to handle the content in trigonometry. However, this was completely absent, and student-teachers remained disillusioned. In fact, no conceptual and procedural knowledge took cognizance of the fast growing artefacts available for teaching and learning seemingly difficult areas of trigonometry.

In order to address these concerns, the following research questions were posed:

***Research question 1*:** How would the essential elements of conceptual field framework address the teaching and learning of special angles in trigonometry?

***Research question 2*:** What artefacts do student-teachers use to support the teaching and learning of special angles in trigonometry?

***Research question 3*:** What statistically significant differences are there in the interventions with the conceptual field approach?

MATERIALS & METHODS

Research Method

The researcher used both the quasi-experimental method. In this method, an intact class of student-teachers in the university was the primary source of respondents. Then quantitative data was first generated from student-teachers knowledge in the special trigonometry angles. Qualitative data was also examined from the ways student-teachers solved the tasks with the artefacts. Themes were provided for each task. Both data were transposed and scaled on SPSS software to robust statistical analyses (Cohen et al., 2017).

Research Design

This method explores the pre-/post design, which aims to simultaneously investigate and solve the problem in the special angles (George, 2023). The resign design primarily adopted both the quantitative and the qualitative methodologies on the conceptual field's framework in identifying the classroom problem, developing and implementing a plan, collecting and analyzing data, and making instructional decisions to share the results. Kusumarasdyati (2016) opines that action research design can be either qualitative or quantitative, within the interpretative paradigm and positivism paradigm respectively. In this study, the researcher use one group rather than assigning the class into two groups and administer a pretest and a posttest to obtain information about the effectiveness (Kusumarasdyati, 2016).

In the first phase, the researcher obtained the materials from the scores of the pre-survey questions. The identification phase was the discovery of 30^o-60^o-90^o and 45^o-45^o-90^o models as innovations to improve their competencies and skills in basic trigonometry (Riel, 2020).

The development and implementing phase applied the models to 50 students, tested the students twice, scored the outcomes and analyzed the results with SPSS software. The instructional decisions and sharing the information took place in conferences, workshops, seminars, symposia and now this international reputable journal. The researcher would revisit these models periodically in order to continuously guarantee uniformity, consistency and coherence of the findings (Maphutha et al., 2022).

Sample & Sampling Procedure

The sample of 50 student-teachers consisted of the post-diploma sandwich student-teachers at the University of Education, Winneba in Ghana. The anticipated population of students was 126. 50 of them were randomly sampled and tested twice after exploring. They were selected based on their abysmal performance in trigonometry part of the mathematics test.

In collecting the data, two similar tests, designated as pre-test or control and post-test or experimental treatment, were implored on the students to assess and compare performances before and after utilizing the conceptual field models. When the first and second experiments were determined, their basic trigonometry's scores for the dependent variable were taken into consideration. If the effect of teaching with the models was investigated, the groups' equalization in gender, regional distributions, class of teaching, subject of teaching, books used for learning, second cycle attended, and tertiary school attended were crucial independent variables.

Analysis

The analysis used the SPSS software version 21 to produce both descriptive and inferential statistics. Particularly, the inferential statistics used the correlation coefficients, one sample t-tests, paired-sample t-tests and analysis of variance to determine whether there were statistically significant differences between the experimental and control groups. Cronbach's coefficient of 0.70 was used to determine the internal consistency of the instruments (Cohen et al., 2017).

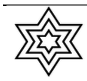
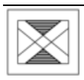
RESULTS

Research question 1: How would the essential elements of conceptual field framework address the problems in the teaching and learning of special angles in trigonometry?

This research question was analyzed by guiding student-teachers to go through step-by-step presentation and relating the concepts and the procedures. 30^o-60^o-90^o triangle rule says that *the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is* $\sqrt{3}$ *times the length of the shorter leg* (Watkins et al., 2022). Using Eq. (1) and **Figure 2**, the student-teachers derived the following relations:

$$Tan\ 30^o = \frac{Opposite}{Adjacent} = \frac{Shorter\ side}{Longer\ side} = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3} \tag{3}$$

Table 1. Adinkra symbols (extracted from Agbo, 2020)

Name of artefact	Local meaning	English meaning	Use in trigonometry
	Mmere dane	Times change	Constructing isosceles triangles
	Awurade baatanfo	Good the mother	Constructing isosceles triangle
	Mrammuo	Crossing paths	Constructing isosceles triangle
	Kyemfere	Potsherds	Constructing equilateral triangle
	Menso wo kenten	I am not carrying your basket	Constructing equilateral triangles
	Mframadan	Well-ventilated house	Constructing equilateral triangles
	Kenten ahinasa	Three stand	Constructing scalene triangle

$$Sin\ 30^o = \frac{Opposite}{Hypotenuse} = \frac{Shorter\ side}{Hypotenuse} = \frac{x}{2x} = \frac{1}{2} \tag{4}$$

$$Cos\ 30^o = \frac{Adjacent}{Hypotenuse} = \frac{Longer\ side}{Hypotenuse} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2} \tag{5}$$

Also, the student-teachers derived the following relations from the 60^o angle:

$$Tan\ 60^o = \frac{Opposite}{Adjacent} = \frac{Longer\ side}{Shorter\ side} = \frac{x\sqrt{3}}{x} = \sqrt{3} \tag{6}$$

$$Sin\ 60^o = \frac{Opposite}{Hypotenuse} = \frac{Longer\ side}{Hypotenuse} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2} \tag{7}$$

$$Cos\ 60^o = \frac{Adjacent}{Hypotenuse} = \frac{Shorter\ side}{Hypotenuse} = \frac{x}{2x} = \frac{1}{2} \tag{8}$$

45^o-45^o-90^o triangle rule states that *the three sides of the triangle are in the ratio:* $1{:}1{:}\sqrt{2}$ (Watkins et al., 2022). Using Eq. (2) and **Figure 4**, the student-teachers derived the following concepts of the 45^o angle. Since it is isosceles, the two acute angles are congruent or 45^o (in order for the angles to add to 180^0):

$$Tan\ 45^o = \frac{Opposite}{Adjacent} = \frac{Longer\ side}{Longer\ side} = \frac{x}{x} = 1 \tag{9}$$

$$Sin\ 45^o = \frac{Opposite}{Hypotenuse} = \frac{Longer\ side}{Hypotenuse} = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2} \tag{10}$$

$$Cos\ 45^o = \frac{Adjacent}{Hypotenuse} = \frac{Longer\ side}{Hypotenuse} = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2} \tag{11}$$

It follows from Eq. (3) to Eq. (11) that the values of each of the three trigonometric functions depend on the special trigonometric angles.

Question 2: What artefacts do student-teachers use and how they use the artefacts?

Table 1 displays the artefacts and their uses in trigonometry. Even though there were many over 50 different adinkra artefacts (Agbo, 2020) the major ones directly that were linked to the discussion are mmere dane, nyame baatanfo, menso wo kenten and mframadan (see their interpretations in English language in **Table 1**). While nyame baatanfo was excellent in illuminating the concept of the isosceles triangle, mframadan was just perfect in showing all the properties of an equilateral triangle. By extension, these three-dimensional figures in **Table 1** were transformed into two-dimensional figures.

Table 2. Knowledge of student-teachers in types of artefacts (Interviewed, 2022)

Artefact	Number	Percent (%)
Adinkra	32	64
Baskets	3	6
Others	15	30
Total	50	100

Table 3. One-sample test of types of artefacts & their use, 2022

Use	t	df	Sig. (2-tailed)	Mean difference	95% confidence interval of difference	
					Lower	Upper
Artefact knowledge	8.745	49	.000	2.260	1.74	2.78
Artefact general use	22.409	49	.000	1.580	1.44	1.72
Artefact conceptual use	21.000	49	.000	1.500	1.36	1.64
Artefact procedure use	17.739	49	.000	1.700	1.51	1.89

Table 4. Paired samples test of types of artefacts & their use, 2022

	Paired differences					t	df	Sig. (2-tailed)
	M	SD	SEM	95% CI of difference				
				Lower	Upper			
Artefacts type–Artefacts use	.680	1.845	.261	.156	1.204	2.605	49	.012

Note. M: Mean; SD: Standard deviation; SEM: Standard error mean; & CI: Confidence interval

Once student-teachers transformed the artefacts into two-dimensional figures, they applied the special trigonometry angles to solve tasks in trigonometry. As student-teachers discussed the rationale behind the artefacts to trigonometry, they equally discovered many other properties of trigonometry concepts enshrined in these symbols.

Table 1 details the type of artefact and the number or percentage of student-teachers who had applied it to the tasks in the special angles.

Table 2 represents the types of artefacts student-teachers were familiar with in their regions and cultures. It was found that "adinkra" (64%) dominated the respondents. Followed by "adinkra" were other artefacts not listed by the researcher and baskets came third. This goes to confirm student-teachers have a vast bulk of knowledge in the indigenous artefacts. This knowledge, if well harnessed, can be put to maximum use in solving not only the special trigonometric angles but mathematics in general (George, 2023).

Table 3 shows one sample t-test statistical test to examine whether the mean of data was statistically different from the pre-existing information. In the one-sample t-test, the variables tested were compared with the known average values based on the hypothesis to test the difference between the zero value and the average changes. The data satisfied requirements for using one sample t-test, namely interval or ratio scale data, each data in the sample is an independent value, samples were taken randomly from the population, sample variance was homogeneous and no data ass extreme (outlier).

There were three conclusions based on one sample t-test results in **Table 3**. Thus, all the t-values were greater than the t-test statistics, the p-values were smaller than the significance level (5%) and the confidence intervals contained no zero in the intervals. Based on these three indicators, the null hypotheses were successfully rejected. It means the average score of the sample was significantly different from the hypothesized zero.

The results of **Table 3** support that of **Table 2**. It was actually proved that indigenous artefacts were not the best tools for exploring the conceptual fields in the special angles, but they were excellent for solving the tasks involved.

Table 4 presents the paired samples t-test of types of artefacts and their uses. The data satisfied the assumptions of continuous variables, related groups, no significant outliers, and approximately normally distributed. The results showed participants made use of types of artefacts for conceiving the use of the artefacts (mean [M]=.680, standard deviation [SD]=1.845). The paired samples t-test found this difference to be significant, $t(50)=2.605$, $p<0.05$. This suggests that types of artefacts have contributed to the use of the artefacts for learning the special trigonometric angles, supporting the hypothesis.

In **Table 2**, we observed that the popular type of indigenous artefacts was adinkra. This results prominently espouses that adinkra should be the main artefacts for the teaching and learning of trigonometry. Even though the results did not actually delve deep into the type of adinkra, it still suffices student-teachers to derive maximum conceptual knowledge from adinkra.

Question 3: What statistically significant differences are there in the interventions with conceptual fields?

Table 5 shows the results obtained after the student-teachers used the artefacts to transition into the conceptual field approach in the two special angles. This question was answered by paired t-test statistics.

Table 5 presents the paired samples t-test of types of artefacts and their uses and satisfied the same assumptions in **Table 4**. The paired t-test was run to determine whether there was a statistically significant mean difference between the conceptual knowledge of student-teachers compared to procedural knowledge in special trigonometric angles. It was revealed that student-teachers steadily progressed in their procedural knowledge after learning the conceptual knowledge (-0.300 ± 0.789 marks); a statistically significant increase (95% confidence Interval, -0.524 to -0.076), $t(50)=-2.689$, $p<.05$, showed participants made use of the conceptual fields.

Table 5. Paired samples tests of conceptual field progression, 2022

		Paired differences					t	df	Sig. (2-tailed)
		M	SD	SEM	95% CI of difference				
					Lower	Upper			
Pair 1	Artefact–Conceptual	.080	.274	.039	.002	.158	2.064	49	.044
Pair 2	Conceptual–Procedure	-.300	.789	.112	-.524	-.076	-2.689	49	.010

Note. M: Mean; SD: Standard deviation; SEM: Standard error mean; & CI: Confidence interval

Table 6. Paired samples tests of 30º-60º-45º angles , 2022

		Paired differences					t	df	Sig. (2-tailed)
		M	SD	SEM	95% CI of difference				
					Lower	Upper			
Pair 1	30º-60º-45º	.180	.388	.055	.070	.290	3.280	49	.002

Note. M: Mean; SD: Standard deviation; SEM: Standard error mean; & CI: Confidence interval

Table 7. Pearson correlations of pre- & post-test scores, 2022

Test		Pre-test	Post test
Pre-test	Pearson correlation	1	0.898
	Sig. (2-tailed)		0.000
	Number (n)	50	50
Post-test	Pearson correlation	0.898	1
	Sig. (2-tailed)	0.000	
	Number (n)	50	50

Table 8. Paired samples t-test of pre- & post-test scores, 2022

		Paired differences					t	df	Sig. (2-tailed)
		M	SD	SEM	95% CI of difference				
					Lower	Upper			
Pair 1	Pre-/post-test	-9.020	6.944	.982	-10.994	-7.046	-9.185	49	.000

Note. M: Mean; SD: Standard deviation; SEM: Standard error mean; & CI: Confidence interval

Table 6 shows results from the paired samples t-test of the two groups of special angles as the assumptions were satisfied. The mean was 0.180, with a standard deviation of 0.388. The 95% confidence interval excluded zero. The t-statistic was 3.280 on 49 degrees of freedom. The p-value of 0.002 was less than 0.05. The null hypothesis was therefore rejected and that the two means were not equal in the population. Therefore, using indigenous artefacts really supported conceptual field approach to learning special trigonometric angles.

Table 7 shows the tests of the Pearson's correlations among the scores of pre- and post-test scores of the student-teachers in basic trigonometry. We discovered a high coefficient of about 0.90 between the two tests we administered. This means there was a high relationship between the two test scores to justify we used similar procedures and instruments in administering, scoring and analyzing the tests. Furthermore, with correlations tested at 0.05 significance level (2-tailed), we confirmed that the tests did relate with each other generally. There is therefore the need to use the 30º-60º-90º and 45º-45º-90º models to explore student-teachers' competencies and skills in angles in basic trigonometry, in order to consolidate their knowledge and understanding to promote effective teaching and learning of trigonometry.

Table 8 shows paired-samples tests of significance of the pre- and post-test scores in basic trigonometry. As seen in **Table 7**, the paired samples t-tests indicated that participants differ in their scores in the special trigonometric angles t(50)=-9.185, p=0.000. Given these results, it was really necessary to control for knowledge of student-teachers prior to the main analysis. The manipulation indicated that participants reported higher levels of conceptual understanding (M=-9.020, standard error [SE]=0.982) relative to the in 30º-60º-90º and 45º-45º-90º special angles.

DISCUSSION

On research question one, the Eq. (3) to Eq. (8) show that the 30º-60º-90º triangle rule could be derived from *the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is* $\sqrt{3}$ *times the length of the shorter leg*. The Eq. (9) and Eq.(11) also show that the 45º-45º-90º triangle rule could be derived *the three sides of the triangle are in the ratio:* 1:1:$\sqrt{2}$ (Watkins et al., 2022). These qualitative derivations amply demonstrated enough evidence to conclude that the two rules could be harnessed to explain the special trigonometric ratios (Maphutha et al., 2022). It therefore follows from Eq. (3) to Eq. (11) that the values of each of the three trigonometric functions depend on the special trigonometric angles. The genesis of the success could be attributed to the deployment of indigenous artefacts. The findings of Ali (2021) opine that Ghanaian indigenous artefacts have the potential of catalyzing the teaching and learning of mathematics. Bergsten et al. (2017) recontextualized and revealed that conceptual approach to mathematics is more essential than a procedural approach. It was therefore not surprising that the outcome of this question chalked a huge success.

On research question two, **Table 1**-**Table 4** show the types, uses and knowledge of the artefacts in teaching and learning trigonometry. Serpe and Frassia (2021) proposed artefact signs, mathematical signs and pivot signs. In this study, I proposed artefacts such as adinkra, baskets, kente, and many more cultural signs of the indigenous people. Even though majority of the student-teachers used only adinkra symbols, the results conformed to George (2023) that student-teachers have a vast bulk of knowledge in the indigenous artefacts. This knowledge can be put to maximum use in solving not only the special trigonometric angles but mathematics in general. In particular, Maphutha et al. (2022) equally performed a two-dimensional trigonometry test on pre- and post-test to measure learners' performance and found significant differences between the two tests (t[100]=.3. 95; p=.05) with Cohen d=.79. It was evidently clear that the outcome was attributed to the conceptual activities.

The results in **Table 3** were emphatically clear that all the t-values were greater than the t-test statistics, the p-values were smaller than the significance level (5%) and the confidence intervals contained no zero in the intervals. The results of **Table 2** proved that indigenous artefacts were not only the best tools for exploring the conceptual fields in the special angles but for solving the tasks involved. Dewi et al. (2021) discovered students' constraints in performing algebraic operations on trigonometry are normally due to a lack of understanding of the concepts in algebraic operation materials. Therefore, the smooth sail of the tasks was the confidence the tasks endowed the student-teachers with. This explains why the results of the paired samples t-test in **Table 2** clearly supported the fact that every task in the special angles was solved with the aid of the artefacts in **Table 1**.

On research question three, the results in **Table 6**-**Table 8** significantly improved with the use of the conceptual field approach as a whole. The paired t-test statistically significant increased to showed participants made use of the conceptual fields (Maphutha et al., 2022). The results also showed that using indigenous artefacts really supported conceptual field approach to learning special trigonometric angles in **Table 8**. Again, the results in **Table 8** confirmed that the tests on the use of 30^o-60^o-90^o and 45^o-45^o-90^o related to each other (Kusumarasdyati, 2016). Drawing on the research findings of Ngu and Phan (2020), mental representations of three variant theories and mapping of two twin concepts in trigonometry is the best milieu for classroom discourse. The sequencing of the trigonometric equations, culminating into the dialogue with the artefacts became the best didactical contract for teachers and student-teachers when handling the special angles in trigonometry. In fact, this actually consolidated their knowledge and understanding of the conceptual field approach to trigonometry.

CONCLUSIONS

On addressing essential elements of conceptual field framework in the teaching and learning of special angles in trigonometry, it was concluded that sequential and interconnected theories are the best bet. Even though Piaget and Vygotsky's theories were called into action, it was realized that the conceptual field approach was the central enabler in equipping student-teachers to transition from Eq. (1) to Eq. (11) with ease. As a result of, it was found that 30^o-60^o-90^o and 45^o-45^o-90^o models effect not only on the student-students' achievements but also on the competencies and pedagogical skills in solving tasks the basic trigonometry angles with indigenous artefacts. On artefacts student-teachers did use to support the teaching and learning of special angles in trigonometry, it was revealed that adinkra, baskets and other (kentey, smock, and other relia) are essential. In fact, the dominance of adinkra was really powerful in consolidating both conceptual and perceptual knowledge of student-teachers in the concepts of triangle and subsequent special trigonometry angles. On the apparent statistically significant differences in the interventions with the conceptual field approach, the findings showed diverse significant differences. These significant differences were contained in **Table 5**-**Table 8**. Even though few minor deviations were found, the large proportion of the student-teachers really agreed that they have gained new knowledge to equip them to handle special trigonometry angles at any level of education. In accordance with these results, the following implications could be advanced:

1. Student-teachers should be provided the opportunities to use diverse artefacts to support conceptual field approach to learning special trigonometric angles. In realizing this goal, "adinkra" artefacts should be redesigned and promoted at the national and regional levels of education to improve upon their design and quality, fit for the purpose of solving of trigonometric tasks.
2. Student-teachers should be taken through the conceptual field approach to learning 30^o-60^o-90^o and 45^o-45^o-90^o trigonometric angles. The focus should be general knowledge of artefacts, general uses of the artefacts, the conceptual knowledge of the artefacts and then the procedural knowledge of the artefacts. This step-by-step processes may help student-teachers not only to learn the special angles but also to propose on the knowledge to their school learners.
3. Student-Students should be encouraged to explore and apply 30^o-60^o-90^o and 45^o-45^o-90^o angles to other problems of basic trigonometry. This would give them the opportunity to enhance and consolidate their knowledge, competencies and skills in trigonometry in general.
4. Student-teachers should learn framework and given opportunities to prepare lesson plans and notes. These affirmative actions would provide opportunities for pre-service and serving teachers to learn to use and apply them in real life.

Funding: No funding source is reported for this study.

Ethical statement: The author obtained an informed consent from the university's authorities through the head of department to grant the ethical approval before collecting this data for the study. Permission to conduct the research in the selected group was also obtained from each and every student in the group, with assurances of confidentiality, anonymity and trustworthiness of their responses and diagnosis scores. Therefore, issues of internal threats to validity, reliability and validity were discussed and certified.

Declaration of interest: No conflict of interest is declared by the author.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the author.

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